COMMERCIAL ECONOMY IN STEAM AND OTHER THERMAL POWER-PLANTS

Commercial Economy

IN

Steam and other Thermal Power-Plants

AS DEPENDENT UPON

Physical Efficiency, Capital Charges and Working Costs

BY

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With Numerous Diagrams by

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PREFACE

This book would be ineffective without the Diagrams and Tables which illustrate it. The Author wishes therefore to gratefully acknowledge his great indebtedness to Mr. H. M. Hodson and to the Publishers for a finer collection of diagrams than has ever before been brought together to elucidate the practical science of Heat Power-Production. Nearly all of these drawings have been made by Mr. Hodson, who has also made a large proportion of the numerical calculations involved in the Tables. All the numerous charts of Costs in Chapter III represent actual ascertained recent costs in Britain. The physical diagrams of Chapter II are mostly reproductions of large-scale drawings used for many years by the author's students and found of great practical utility.

These Diagrams and Tables are not mere examples of the laws explained in the text: in most cases they are essentially the full graphical and numerical exhibition of all the practical results of these laws. The Author has long been convinced that engineering science ought not, in the face of difficulties due to the inherent and unavoidable complexity of practical business problems, to take refuge in erroneous approximations whose one only merit is simplicity; but ought, on the contrary, to eliminate difficulty by plotting on accurate squared paper the results of all complex calculations. Those who wish to calculate results to one-tenth of one per cent. minuteness will not be satisfied with diagram readings. But those who know that physical and commercial data are liable to variation within a range from ½ to 5, and sometimes even 10, per cent., will

be well content to see on a scaled diagram not only the whole meaning, but also all the useful practical results, of a complex law of the truth of which he is reasonably and intelligently confident, but the mathematical form of which is too difficult for him to utilise in the midst of the press of technical and commercial work.

The main endeavour of this work is to persuade the mechanical engineer to advance from the primitive view that engineering science can guide him only in the physical construction and the physical dynamics of his machinery to the much more complete idea that scientific method must also be applied to his reckonings of cost and of value produced. The ultimate triumph of practical science must be evidenced in its demonstration of the means to attain Maximum Economy. Physical Efficiency is one factor in Economy. But the only true economy is an all-round economy, and mechanical pride in the difficult attainment of purely physical excellence must leave unshaken our readiness to sacrifice mechanical perfection to low cost in such degree as yields the best economic ratio of resulting value to working expenditure.

To adapt scientific method to practical uses the first need is the means of measurement; and an exact Measure of Economy is the first essential in any section of technico-commercial science. The Economy-Coefficient investigated in Chapter I is, so far as the author knows, here recommended for the first time. It is applicable to all kinds of productive industry, inclusive, probably, of the industry of distribution and exchange. By a simple combination of the three factors Cost, Value and Speed of production, it aims at giving due value to all the essential elements of Commercial Economy.

In practical Industry and Commerce there is continuously at work the effort to balance the advantages of excellence of finished product and low cost of production against the disadvantages of high capital outlay. In electrical engineering alone has this effort, so far, been guided to any visible extent by scientific principle and method. It is fortunately inevitable that the same procedure shall force itself into the practice of every other department of engineering. The generation of Power by means of Heat is of all of them that

in which these principles have by far the greatest, and indeed overwhelming, importance.

The most elementary serious consideration of economic problems leads the practical engineer to recognize the ubiquitous influence of the *speed-element*. In the hands of the mathematicians thermodynamics has been developed mainly as a static science and mainly as the science of temperature. This treatise insists strongly that the Mechanical Resilience produced by heat-action is alone of any ultimate value to the power-engineer, and condemns utterly the academic fetish of Reversibility as an ideal unworthy to be striven after and approximated to. It demonstrates that all Thermal action is Irreversible: that only the dynamic element in complex thermodynamic action yields its due proportion of reversibility. The all-important Speed-element in heat-power engineering acts wholly in the direction of Irreversibility.

The analysis of Chapter VI of thermodynamic action into its purely mechanic and purely thermal elements leads to the use of the Dynothermic Coefficient, which is now published in book form for the first time, although the Author has himself used it for many years past. This coefficient gives the ratio of the new mechanical resilience to the heat-flux which generates it.

The closer examination of the influence of the time-factor has led to two results of very great physical interest. The first (see Chapter V) is the entire dependence of the modulus of Elasticity or Resilience upon the *ratio of two time-rates*, namely the *speed of expansion* or other unstraining and the *speed of the heat-flux* or internal generation or destruction of heat. The second result, which is in reality only a practical translation of the first into terms of boiler duty, is the interdependence of Furnace Temperature and Working Speed. This is dealt with in Chapter XI. The attempt made here to deduce most economic limits of furnace temperature and of working speed is one of great difficulty; and the Author does not profess that the chapter gives more than a theoretic outline of the solution of this problem. Nevertheless, the reader who follows the argument will find the Tables and Diagrams of this chapter both interesting and instructive in the study of an intensely practical

question. In work, such as the Generation of Power, in which there are so many influential elements of design which may be independently specified by the designer, the problem of finding those combinations yielding the greatest possible commercial economy is essentially and unavoidably complex. The graphic methods given in Chapters VIII and IX are strongly recommended as the best for attacking special cases of this problem. Attention is drawn to Fig. 64 in Chapter VIII as a really simple and rapid diagram useful both for estimating purposes and also in the search for maximum-economy combinations. The Author will feel grateful for any communications of well-authenticated values of the pressure-constants of this diagram.

R. H. S.

March, 1905.

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Chapter XI

INTRODUCTORY

Chapter I

INTRODUCTORY

Product, Profit, and Economy. Commercial Economy Co-efficient.

Commercial and Industrial Economy is dependent on the value of the product as compared with its cost. The products concerned are in general those of industrial and commercial activities, and those with which this book deals are the results of work done in engineering manufactures. In particular it deals with the machinery whereby mechanical power is generated by the utilization of heat.

Products and their costs are conveniently evaluated in money, but not necessarily so. They may be evaluated in terms of any other product that is useful and that is capable of being measured quantitatively. The total cost of any product is, however, the sum of a great number of items of different kinds—labour, material, rent, management, interest on capital outlay, depreciation of the plant employed—and in order to sum these up they must, perforce, be first reduced to some common measure. By far the most convenient common measure is money. Still it is well never to lose sight of the fact that the money,by which all the items are measured, is nothing more than this convenient, almost conventional, scale applicable to the measurement of all sorts of values. The money is itself no more the real value than the mercury of a thermometer is the heat whose temperature it measures ; no more than the dial-scale of a watt-meter is the electrical energy whose time-rate it indicates. Of course, money has equivalent real intrinsic value, but this intrinsic value is of practically no consequence so far as concerns the purpose of this treatise.

All industrial and commercial work is undertaken for the sake of profit. Fortunately the particular scope of this book leaves the meaning of "profit" wholly unambiguous : it does not require us to face the difficult question as to whether the real total profit of any industrial activity includes or excludes the wages of labour

and remuneration of other personal services involved. In our particular problem, namely the generation of mechanical power, wages and personal services are parts of the cost and are to be deducted from the value of the product in order to arrive at the profit. This agrees with the technical definition of profit almost universally applied to all kinds of production.

But it may be well, in passing, to suggest that this orthodox idea of profit does not in the least correspond with the primitive and natural notion implied in the word, namely the advantage or benefit yielded by production to those who take part in producing it. This ordinary technical business meaning has been given to the word " profit " in consequence of the whole profit being divided up into sections, the manufacturer's profit, the wholesale trade profit, the retail profit, and so on. The manufacturer's profit is, of course, that shown by the balancing of his accounts ; and, as a matter of course, the wages he pays is no part of his profit : it is as much a deduction from it as is the money he pays for material. But the manufacturer's profit is quite a different thing from the profit of the manufacture. The income derived by the owner of a factory, if he be also its manager, may be regarded as the sum of two parts, namely interest on his invested capital and remuneration for his services as manager. After paying interest on his capital, etc., etc., he may for years derive from it a personal income which, reckoned on a reasonable scale, is an insufficient remuneration for his personal exertions ; but, if his whole income from it be sufficient to enable him and his family to live comfortably and happily, he will think it advantageous to continue to run the factory. If profit were thought of as advantage, or profitableness, then the whole of his personal income from his factory without the deduction of any interest on his own capital or of management salary, would be his real profit. The adoption of this definition of profit would, of course, by irresistible logic lead to the inclusion of the whole wages and salaries paid to workmen, clerks, managers, etc., etc. in the sum total of the profit gained from the manufacture (not by the manufacturer or owner). It would also follow unquestionably that the interest earned on capital would be included in this total profit, because capital value is the value of accumulated work; but *not* the depreciation on capital outlay or sinking fund contribution, because the lost capital value must be reproduced before any profit or benefit appears.

If this generalized benefit be called " total gain " to distinguish it from technical " profit," and none of the total gain be included

in costs, then the costs are restricted to the materials used and the depreciation on capital outlay.

The distinction between the narrow technical meaning of the word profit and this enlarged idea in the sense of "derived or resultant material benefit" or total gain is one which it is highly important to bear in mind in considering industrial questions; and there can be no intelligent comprehension of the mainsprings of industrial and commercial activity, or of the actual forces which guide and direct it, without a full recognition of gain in the larger sense. For instance, if the real only object of carrying on a given industry were to win profit in the technical meaning of the term, that industry would certainly die out of existence in a few months. What keeps it alive is the stimulus of the immensely greater bulk of profit in the larger sense of total gain, which includes all the wages and the personal incomes of all those engaged in the various parts and various stages of the industry, as well as the interest on borrowed money and dividends on shares.

Another proof of the importance of an intelligent perception of this broader profit to all engaged in the production, is derived from a rational consideration of what has recently come to be called "dumping." If the productiveness of certain plant has become greater than enough to supply the demand at ordinary standard prices, either by increase of the productiveness of the plant or by decrease of the market demand, and if this excess is deemed likely to be permanent and not of such temporary character as can be met by increasing stocks; then the question unavoidably arises whether it is more advantageous to throw out of operation a certain portion of the plant or to increase the demand either by a general lowering of prices or by a spread of the market into new hitherto untouched regions by sales in these new regions at specially low prices. If the former expedient were adopted, as it often actually is, then, whether it be done nominally in the accounts or not, there must be—or to the certain knowledge of the owners there ought to be—written off depreciation of the plant thrown out of work. It should be written down to the price at which it could be sold, which price is naturally low in the circumstances assumed because the market is already overstocked with that kind of plant. If it be not actually so written down in the accounts, it loses real value none the less to the full amount to which it ought to be written down, and intelligent owners do not deceive themselves on this point even though they may not put down the loss in black and white in their capital accounts. The loss due to throwing part of

a plant out of work is greater than the simple depreciation. With no reduction of price obtained per unit of produce, the *gross revenue* is diminished in proportion to the plant thrown out, but the expenditure does not decrease in nearly the same ratio because the establishment charges are lessened little or not at all.

Now if this loss by stoppage of plant thrown out of work be ascertained to be greater than that incurred by selling at lower than ordinary prices in extended markets, then, as a matter of course, the lesser loss will be accepted as the best solution. These lower prices do not, even in extreme cases, mean selling at below real cost; they mean selling at a profit upon the depreciated value of the plant. If this were not so, then the other alternative, namely that of throwing parts of the plant out of operation, would be adopted. The choice between the alternatives is, perhaps generally, decided in accordance with the exclusive interests of the shareholders, that is, without reference to those of the debenture holders or of the employés. In other words it is decided in favour of profit in the narrowest technical sense of the term. But even so, the throwing out of work of a staff of specially trained employés who cannot afterwards be replaced except at inordinate expense, is a material consideration exercising influence upon the decision; an influence precisely similar to that of the stoppage of the plant. If now the choice of alternatives were decided in the interest of profit in the fullest and largest sense of the term, it is clear that there would be much less plant shut down and much more "dumping" than there actually is. This is what would happen if the shareholders and the employés were identical persons; if the choice were decided in the interests of all those who are actually and humanly interested. It is because the power to decide such questions necessarily rests mainly in the hands of the capitalist owners, that the decision is usually made for the best advantage of technical profits and not for that of profit in the broad sense of total gain or benefit.

As regards the rapid reduction of net income with diminution of output, fairly full statements of its causes and laws may be found in the present writer's papers upon "The Mathematical Laws of Industrial Profits" in *Feilden's Magazine* of February, 1902; and in *The Engineer* of September 8 and 15, 1899.

But, as has been said above, the particular subject of this book does not require us to consider profit in this larger sense. We have to deal with a wholly material and strictly isolated product, and have to consider the profit derivable from it and the economy of its production only in the narrowest technical sense of these terms.

All the wages of labour involved in the generation of mechanical power, and all the interest on capital outlay on the plant, are strictly *costs*; and the only profit is the surplus of value in the power generated over and above these and other costs.

The economical and profitable character of any production may be judged and measured in more ways than one. One measure is the ratio of annual profit to the capital value employed, commonly stated as a percentage. This most usual form of stating the result is obviously that adapted to let the shareholder, who supplies the capital value, understand in what degree the work is advantageous to him. It has the great advantage of being a *time-rate* measurement. It is an error to consider this measure as a pure number, a simple ratio between two quantities of the same kind, that is, a simple amount of money per £100. It is in reality, of course, a pure number divided by a time. It is—

$$\frac{\text{Profit per Year}}{\text{Capital}}$$

the year being unit of time. The profit and the capital are of the same dimensions, both money values, but the numerator of this ratio is divided by time while the denominator is not so divided. If the employed capital be turned over thrice per year, and there be a 4 per cent. profit in each turn-over, the annual profit is 12 per cent. The measure is not a ratio of profit to capital: it is the time-rate of profit-making in ratio to capital. The 4 per cent. per 4 months is the same time-rate as the 12 per cent. per year. This measure is excellently adapted to its special purpose, but it is too little recognized that its purpose is special, and by no means covers the whole question of economy. The capital costs are only one part of the whole expenditure incurred in attaining the result, and the full consideration of economy must bring into account all the costs.

A second measure is—

$$\frac{\text{Profit}}{\text{Cost}} = \frac{\text{Value of Product} - \text{Cost}}{\text{Cost}}$$
$$= \frac{\text{Product}}{\text{Cost}} - 1$$

Here, in order to take the difference forming the numerator, it is essential to evaluate the product and the cost in the same common measure, say, both in money. Therefore, unavoidably, both numerator and divisor are in the same common measure, and the fraction is a pure number. It has no reference to time, and remains

unaffected directly whether the work be done slowly or quickly. Whether the cost be incurred, and the profit gained, in a week or a month or a year, this measure of the advantage resulting from the work remains the same; it indicates in no way the rapidity with which the advantage has been gained. This is a great defect in this mode of measurement because the rapidity of gain is of all things the most important factor in its influence upon human well-being.

Indirectly this measure, as applied to profit in the narrow technical sense, is influenced by time, because part of the cost is the interest on capital employed, and this interest is proportional to the time during which the capital is devoted to the work; interest being charged, at say, 6 per cent. per year, not at any fixed percentage for the time occupied in the turn-over. If the whole cost be the sum of the Working Expenditure F, and Capital Charges, Kit, where K is capital, i rate of interest per unit of time, and t time spent in realizing the value produced; then—

$$\frac{\text{Profit}}{\text{Cost}} = \frac{\text{Product}}{F + Kit} - 1.$$

But Kit is of the same dimensions as K: it is simply a sum of money. Usually a large part of F also increases steadily with lapse of time. Still the fraction remains a pure number; not a time-rate in any sense. And it is only because certain parts of the cost have by custom secured the position of being chargeable in proportion to time elapsed, whether the time have been utilized vigorously or the reverse, that its numerical value is at all influenced by time. If the whole of the costs were impartially secured in the same position, it would be seen at once how bad a form of measurement this is of the advantage derived from any sort of industrial activity. Yet some engineers are inclined to be well content if they make a large profit, say, 20 per cent., on an operation which they have allowed to drag on so that perhaps six or seven years have elapsed before its completion. In order to secure a larger non-time-rate profit, operations are often purposely delayed. The non-time-rate profit can, for example, be increased by pushing the work actively only when the market prices of labour and of materials are low and stopping it, or slowing it down, when these are high. Most materials also can be bought at specially low rates if the delivery is, by contract, spread uniformly over a prolonged period. The very different results of this policy upon the different classes of persons engaged upon the productive work are illustrated very effectively in the example set forth in the annexed Table I, and the corre-

TABLE I

Capital Outlay on Plant, Land, Buildings, £20,000. Interest 5%. Depreciation 2% for whole work done, plus 2% per year spent on it.

Years Spent on Work.	Product or Value Realized.	Costs. Interest.	Costs. Depreciation.	Costs. Materials, etc.	Costs. Wages and all Personal Services.	Costs. Total Cost.	Costs. Total Cost per Annum.	Fixed Capital Benefit. Profit.	Fixed Capital Benefit. Profit per Annum.	Fixed Capital Benefit. Profit / Capital.	Fixed Capital Benefit. Profit per Annum / Capital.
1	50,000	1,000	800	20,000	25,000	46,800	46,800	3,200	3,200	·16	·16
2	48,000	2,000	1,200	17,000	21,000	41,200	20,600	6,800	3,400	·34	·17
3	46,000	3,000	1,600	15,000	18,000	37,600	12,533	8,400	2,800	·42	·14
4	44,000	4,000	2,000	14,000	16,000	36,000	9,000	8,000	2,000	·40	·10
1	2	3	4	5	6	7	8	9	10	11	12

	Personal Benefits. Wages and Services per Annum.	Personal Benefits. Total Gain.	Personal Benefits. Total Gain per Annum.	Economy. Profit / Cost.	Economy. Profit per Annum / Cost.	Economy. Cost / Product	Economy. Product / Cost.	Product per Unit of Cost / Time per Turn-over. *or* Product per Annum / Cost per Turn-over.	Years × Cost / Product.	Product per Annum / Fixed Capital.
1	25,000	29,200	29,200	·0684	·0684	·936	1·068	1·068	·936	2·5
2	10,500	29,800	14,900	·165	·0825	·858	1·165	·582	1·716	1·2
3	6,000	29,400	9,800	·223	·0743	·817	1·224	·408	2·451	·77
4	4,000	28,000	7,000	·222	·0555	·818	1·223	·306	3·272	·55
	13	14	15	16	17	18	19	20	21	22

sponding diagram Fig. 1, which clearly exhibits the influence of the time-element upon the problem of profit-making. A capital of £20,000 laid out in land, buildings, and plant, is taken as employed in the production of contract work for which the full price, £50,000, is paid if the work be completed in one year. For each one year of delay beyond one year, £2,000 is deducted from the price paid. A penalty of this kind is more usually arranged per month or per day of delay in delivery; but the principle is precisely the same, and at this stage we can more simply deal with the year as unit of time. The £2,000 per year is at the rate of $\frac{1}{13}$th of 1 per cent. of £50,000 per week. To get the work finished in one year incurs considerable extra expense in all ways except in depreciation and interest upon capital outlay. Both materials and wages, etc., are reduced by extending the time over two years; the materials from £20,000 to £17,000, and the wages, salaries, commissions, and other personal services from £25,000 to £21,000. The interest is taken at 5 per cent. per annum, and the depreciation is here reckoned at 2 per cent. per annum, plus an extra 2 per cent. for the amount of work done whether it be done slowly or quickly. In extending the work of completion over three and four years further saving in materials and wages is effected, in diminishing ratio as shown in the 5th and 6th columns of the table. The total cost thus goes down by successive steps of £5,600, £3,600, and £1,600, in spite of the increase of interest and depreciation included in this total. As the price obtained is reduced by £2,000 each year, the last saving in costs of £1,600 is less than enough to balance the loss in price. The total profit found in column 9 thus *increases* at first rapidly, being more than doubled by spreading over two instead of one year, and then more slowly, while the extension from three to four years *diminishes* it from £8,400 to £8,000. The profit *per year* at first *increases* slightly, but, as column 10 shows, *decreases* by £6,000 and £8,000 in the two subsequent year-delays. As percentages on fixed capital outlay, these profits follow the same variations, as seen in the next two columns. 16, 17, and 14 per cent. per annum are the profits on fixed capital from the work spread over 1, 2 and 3 years respectively. Of course, the 34 per cent. in two years is to be preferred to 16 per cent. in one year, and there are few who would not prefer 42 per cent. in three years, or 14 per cent. per year, secured for three years, to 16 per cent. for one year. Many might prefer 42 per cent. in three years to 34 per cent. in two years, because it secures 8 per cent. for the third year. The extension from three to four years is seen to be pure loss from every point of view.

If the penalty for delay be taken as $\frac{£2,000}{12}$ per month, and the other quantities be supposed to vary gradually per month, these results may be studied very simply by help of a diagram such as

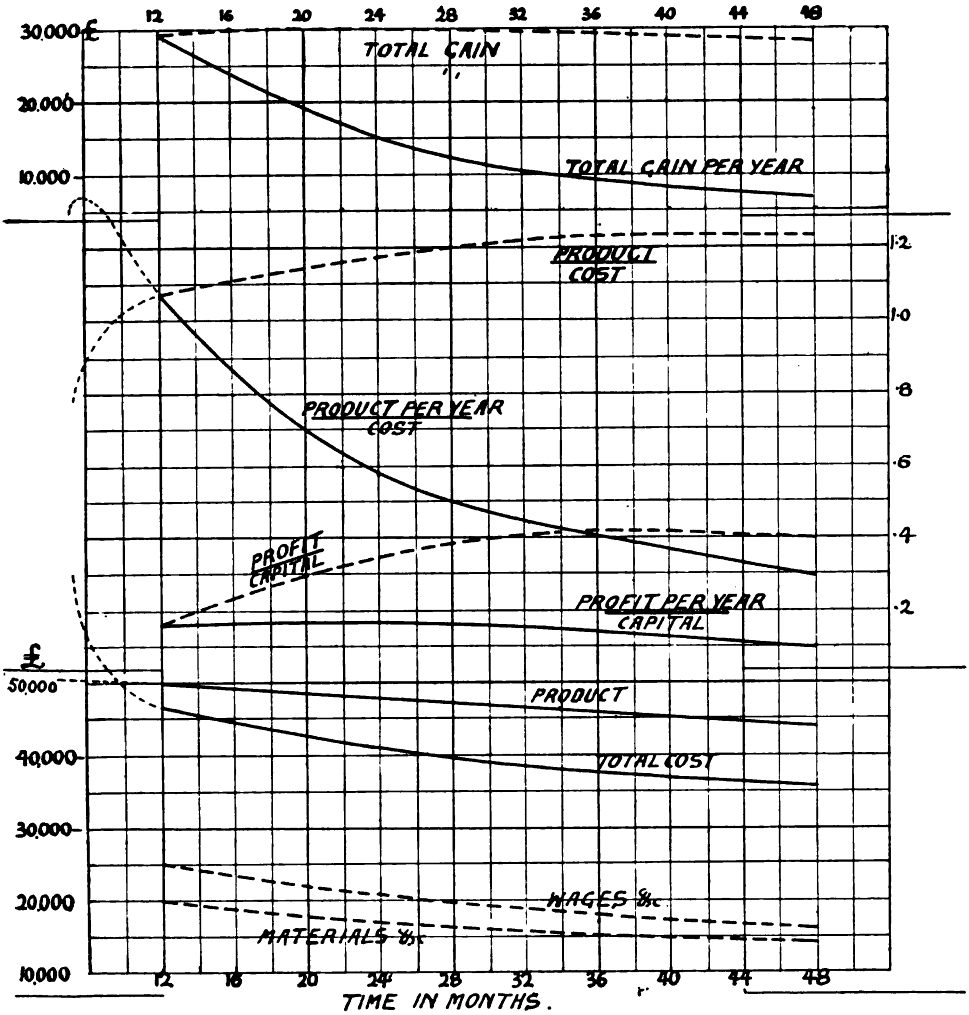


FIG. 1.—Costs. Benefits. Economy.

Fig. 1, where they are set out as curves showing the monthly variations.

Perhaps most managers of capital would recognize clearly enough the disadvantage of slowing down the completion of the work from

two to three years in order to gain 42 per cent. instead of 34 upon fixed capital. But when the question is between different lengths of time spent under a year, for instance, as between three or four or five months, it is very frequently the case that the advantages of quick work remain wholly unrecognized and neglected.

Passing now from capital benefits to a consideration of personal benefits, the 13th column of the table shows how rapidly the payments of wages, salaries, etc., per annum fall. These do not constitute the only personal gains because the interest paid upon the capital, whether this be borrowed or not, is, of course, income to the capitalist, and the profits cleared are also income to the shareholders or owners. What is here called total gain and printed in the 14th column is the sum of the Wages, etc., Interest and Profit. This total gain is in fact the value of the product realized lessened by depreciation and cost of material. This total personal gain is seen to be slightly greater for completion in two years than for completion in one; but further delay beyond two years effects a diminution of it. The decrease is not large except in the fourth year. But as this total gain is the means of subsistence of individuals from year to year it cannot be rationally considered except as reduced to per annum. This reduction is shown in the 15th column, and it is seen how rapid and serious is the fall resulting from slow work.

The last section of the table gives ratios and coefficients which enable one to see the various results viewed in the light of pure material economy irrespective of personal interests. The first column gives the ratio of total profit to total cost. This increases at first rapidly and then more slowly until it remains stationary from three to four years, being $22\frac{3}{10}$ per cent. at the end of the three years, and $22\frac{2}{10}$ at the end of four years. This measure of economy $\frac{\text{Profit}}{\text{Cost}}$ is an eminently useful one for many purposes; but it must not be forgotten that it contains no time-element in its composition. If the profit be taken per year or per month or per week it would seem natural to take the costs for the same period of time. If this be done the time-factor cancels out from the ratio. But the time-rate of profit making is of the greatest importance in industry, because it is this that maintains from day to day the people engaged in the enterprise in healthy and comfortable life.

If this ratio be modified as in column 17 by reducing the profit to per unit of time without similarly reducing the cost, it will be seen that the resulting coefficient follows a somewhat similar variation

to that of the profit per annum as a percentage of fixed capital, although the variation up and down is more rapid.

The next column gives the ratio of cost to product, the cost being the total inclusive of interest, etc., and the product taken at its money value with the time-penalty deducted. This is an ordinarily used coefficient of economy; but it suffers the disadvantage of going down in numerical value as the economy becomes greater. It will be noted that delay in completion up to three years decreases this coefficient, while the addition of a fourth year slightly raises it.

The reciprocal of this ratio is put in the next column. As a measure of economy it is preferable in as much as its numerical value increases as the economy becomes greater. The information it conveys is, of course, precisely the same as that in the preceding column. Its value goes up to the third year, and drops very slightly by the addition of the fourth year.

Neither of these measures contains any time-factor, and they do not indicate in any way speed of production. A productive activity of £1,000 per three months at a cost of £700 is the same time-rate activity as £4,000 per year at £2,800 cost, and in either case the ratio is a pure number, $\frac{10}{7}$. But the same $\frac{10}{7}$ is the ratio given by £1,000 produced per year at £700 cost, although the time-rate of this activity is only one quarter as much.

In column 20 the product is reduced to per year, and this is divided by the whole cost without reference to the length of time over which the expenditure is spread. The precise meaning of this coefficient may, perhaps, be more clearly realized by considering its reciprocal, which is put down in the following column. This reciprocal is the number of years spent in completing the work multiplied by the whole costs and divided by the value of the product.

Here the years spent indicate the undesirable slowness of the work done, while the ratio cost to product indicates the undesirable kind of relation between these quantities. The reciprocal, "Product per unit of cost divided by time taken to execute work," indicates, therefore, what is desirable in the character of the work both as regards speed and otherwise.

The coefficient of column 20 has this further advantage that the product and the cost are not necessarily measured in the same terms. The cost may be stated in money and the product valued otherwise than in money.

The form of expression $\frac{\text{Product per Year}}{\text{Cost}}$ would be, without explanation, deceptive. It does not mean the product of one year's

work divided by the cost of that work. Its exact meaning will be more clearly understood if it be written—

$$\frac{\text{Product}}{\text{Cost} \times \text{Time spent in executing completed work.}}$$

Suppose the product be one engine, and the time spent from first to last be four months $= \frac{1}{3}$ year, and suppose the total cost to be £600. The fraction becomes $\frac{1 \text{ Engine,}}{200}$ using £1 and one year as units in the divisor. Now if in another shop, or at another time, seven similar engines (each of same value) be built in the same time, four months, at precisely seven times the cost, i.e. $7 \times 600 = £4{,}200$, then the fraction remains the same as before, or

$$\frac{7 \text{ Engines}}{£4{,}200 \times \frac{1}{3}} = \frac{7 \text{ Engines}}{1{,}400} = \frac{1 \text{ Engine}}{200}$$

If, in the first case, this operation were repeated each four months, then in one year there would be finished three engines, and the cost of these would be $3 \times £600$; so that the ratio of all the engines built per year to their cost is the same as that of one engine to its cost. But in framing the co-efficient, its value must *not* be taken as $\frac{3 \text{ Engines}}{£1{,}800 \times 1 \text{ year}} = \frac{1 \text{ Engine}}{600}$; but as $\frac{3 \text{ Engines}}{£1{,}800 \times \frac{1}{3} \text{ year}} = \frac{1 \text{ Engine}}{200}$ as before. The time-divisor is the time spent on any piece of work before its finished value is realized; in other words, the time of "turn-over" of capital and labour combined. In the larger shop where seven engines per four months are finished simultaneously (not one after the other), the yearly account is—

$$\frac{3 \times 7 \text{ Engines}}{3 \times £4{,}200 \times \frac{1}{3} \text{ year}} = \frac{1 \text{ Engine}}{200}$$

the same fraction as before. But, if, in any case, one or any number of engines of the same finished value be built at the same cost in three instead of four months, then for these engines the coefficient becomes $\frac{1 \text{ Engine}}{£600 \times \frac{1}{4} \text{ year}} = \frac{1 \text{ Engine}}{150}$, or $33\frac{1}{3}$ per cent. greater than when the divisor is 200.

A larger engine of the same kind and same quality of workmanship is a product of greater value, it costs more, and it takes (under like conditions) a longer time to build. To take first the simplest case, suppose that the finished value, the cost, and the time are each doubled. Then this coefficient, $\frac{\text{Product}}{\text{Cost} \times \text{Time}}$, is halved. The reason

is that the time-rate of turnover is halved, although the product per unit of time is unchanged, and the time-rate of expenditure on costs remains also unchanged.

If by doubling the time-rate of expenditure on total costs—for instance, by doubling the number of employés and doubling the fixed capital—this larger engine can be built in the same time as the smaller one, the proportion of cost to produced value being the same, the coefficient under discussion will be the same for the large as for the small engine; and this is the result, although the production per unit of time has been doubled.

Thus this coefficient is not affected by simply enlarging the works and doing more work per day. It also remains unaffected whether it be calculated with the year, or month, or day, taken as unit of time. It depends upon, and is the product of, two factors. The first is ratio of product to cost. The second is time-rate of turn-over of working capital employed in paying the working expenditure, in this expenditure being included interest and depreciation of fixed capital. This time-rate of turn-over is the reciprocal of the time spent from the date of starting work to the date at which the value of the work is realized.

It should now be clear that "working costs" or "running expenses," are not a time-rate of expenditure, not an expenditure per day or per week; but are of the same nature as capital. For the work set out in Table I with the intention of completing it in two years, it is certainly not necessary to start with a working capital of £41,200 in addition to the fixed capital of £20,000. Nevertheless, £41,200 has to be provided somehow before the value of the product is realized. It is, indeed, not essential for the manufacturer to provide himself with this capital before undertaking the work for two reasons. Firstly, the expenditure is incurred gradually. More material will need to be bought during the first three months than in any subsequent equal period of time, but on the other hand it is not generally paid for in full until some time after delivery. The expenditure on wages, salaries, interest, depreciation, is incurred at a more or less uniform rate over the whole period: its time-rate will usually be less towards the beginning than in the middle and after the middle of the work; probably lessening again towards the end. On the whole, the total costs may be taken roughly as spent at a uniform rate throughout the whole period of working. Secondly, as the work progresses, money for the payment of wages, etc., etc., may be borrowed on the security of the value of the portions of the work finished, or half

or quarter finished, although that value has not yet been realized. But the borrowing of money by the manufacturer is simply shifting the burden of the supply of capital from his own to other shoulders. The capital has none the less to be supplied and spent—supplied by outside capitalists and spent by the manufacturer himself. The same holds if such borrowing be avoided by arranging in the contract for the work for part payments by the purchaser at intervals in proportion to the progress made towards completion. These various arrangements are mere personal bargains in respect of the supply of the necessary capital. They lay a greater or less share of the duty of furnishing it on this or the other party interested in the enterprise. They do not alter the amount of capital needed. All the costs, by whomsoever defrayed, form a capital fund advanced on the security of the as yet unrealized value of the final product. The manufacturer takes a larger percentage profit on the share of capital he himself supplies because he advances it on small, or zero security. If he has borrowed for working expenditure, the security required for the loan being, say, double the cash amount of the loan, his borrowing powers are exhausted, so far as the security created by his contract goes, long before the necessary expenditure is completed.

It is to be noted that we do not here count partly finished work as realized value, even in the case of part payments being made by the purchaser on instalments of completely or partially finished portions of the whole work. They are not actually realized value until they come into use, and this is so whether they become the property of the purchaser or remain the property of the maker. It must be remembered that we are dealing with the question of economy of production apart from personal interests : quite independently of the sectional share of the resulting profits going to one or other party concerned. These partly finished portions of the work would have some selling value if the work were stopped uncompleted ; but this is irrelevant to the problem in hand, which has no reference to such conditions, but has sole reference to the conditions under which the work goes on to completion and is not sold or otherwise utilized until it is complete.

It may be here also noted that if the product has to be kept in stock after being finished for any period, short or long, before its value is realized by being sold and put in use, then this period ought to be added to the time which is the divisor in the economy-coefficient being now discussed. This period during which the finished product has to be kept in stock is always necessarily in

greater or less degree a matter of speculation beforehand. If the product be stocked by middlemen, then this prolongation of the time-divisor of this coefficient may be wholly disregarded by the manufacturer, on whom none of its expense falls.

An important point to observe is that in the Total Cost, which is the Working Capital that must be supplied, is included the interest and depreciation on the fixed capital. Since these fixed capital charges are added to the other parts of the costs, this indicates that wages, expenditure on materials, etc., etc., are of the same nature as interest on fixed capital. These costs are capital, namely working capital, but of a different order to the fixed capital. The essential difference is that the capital fixed as plant, buildings, etc., is *lent* only for the work and returned on its completion. The working capital is *spent* upon it; it is sunk in the creation of the new value in the product; it is not returned; but in its place arises another thing of greater capital value. The fixed capital is not wholly returned; the part of it represented by the depreciation is absolutely *spent* in the same way as the rest of the costs. Similarly the interest on fixed capital represents the *spent* value of the *use* of the plant, etc. *All* the items of the working capital are *spent* values, and it is this characteristic that distinguishes working from fixed capital, the latter not being spent.

Little consideration is required to perceive that interest on working capital falls logically in the category of costs, that is, of working capital itself. This would be more easily recognized if financial custom allowed us to think of the whole operation, involving the supply of both fixed and working capital, as being carried out with borrowed money. This involves charging interest upon the interest of fixed capital. Such interest would, of course, be charged if the interest on fixed capital were paid with borrowed money. The working capital employed, however, only accumulates gradually through the period of time spent on the work; so that interest is not paid for the full final amount of cost expenditure on the whole time. If the expenditure be taken as uniform throughout the time, the time-average of capital employed is half its final or full amount, and this interest must therefore be charged either for half time or on half the full amount. This is provided for by multiplying the whole costs, reckoned without interest upon them, by the factor $\left(1+\frac{it}{2}\right)$ where t is the whole time spent in finishing the work, and i is the rate of interest per unit of time.

We will now set forth in Table II and the diagram Fig. 2, another

TABLE II

Time Spent on Work. — Weeks.	Price Paid. — £	Cost. Interest and Depreciation on Fixed Capital. — £	Cost. Materials, Labour, etc. — £	Cost. Interest on Working Capital. — £	s.	d.	Cost. Total Cost. — £	Profit. Nett Profit. — £	Profit. Profit per Year. — £	Profit. Profit per Year. / Working Capital.	Economy. Product. / Cost.	Economy. Product per Year. / Working Capital.
10	1,200	100	900	5	0	0	1,005	195	975	·970	1·194	5·97
11	1,197	110	880	5	8	11	995·4	201·6	916·4	·920	1·203	5·46
12	1,194	120	860	5	17	7	985·8	208·2	867·5	·880	1·211	5·05
13	1,191	130	840	6	6	1	976·2	214·8	826·1	·848	1·220	4·69
14	1,188	140	820	6	14	5	966·6	221·4	790·7	·818	1·229	4·39
15	1,185	150	800	7	2	6	957·0	228	760	·794	1·238	4·13
16	1,182	160	780	7	10	5	947·4	234·6	733·3	·774	1·248	3·90
17	1,179	170	760	7	18	1	937·8	241·2	709·4	·756	1·257	3·70
18	1,176	180	740	8	5	7	928·2	247·8	688·3	·741	1·267	3·52
19	1,173	190	·20	8	12	11	918·6	254·4	669·5	·729	1·277	3·36
20	1,170	200	700	9	0	0	909	261	652·5	·717	1·287	3·22
1	2	3	4	5			6	7	8	9	10	11

numerical example of different character to the last. Here it is assumed that the choice of length of time spent upon execution of the work ranges between ten and twenty weeks. If completed in ten

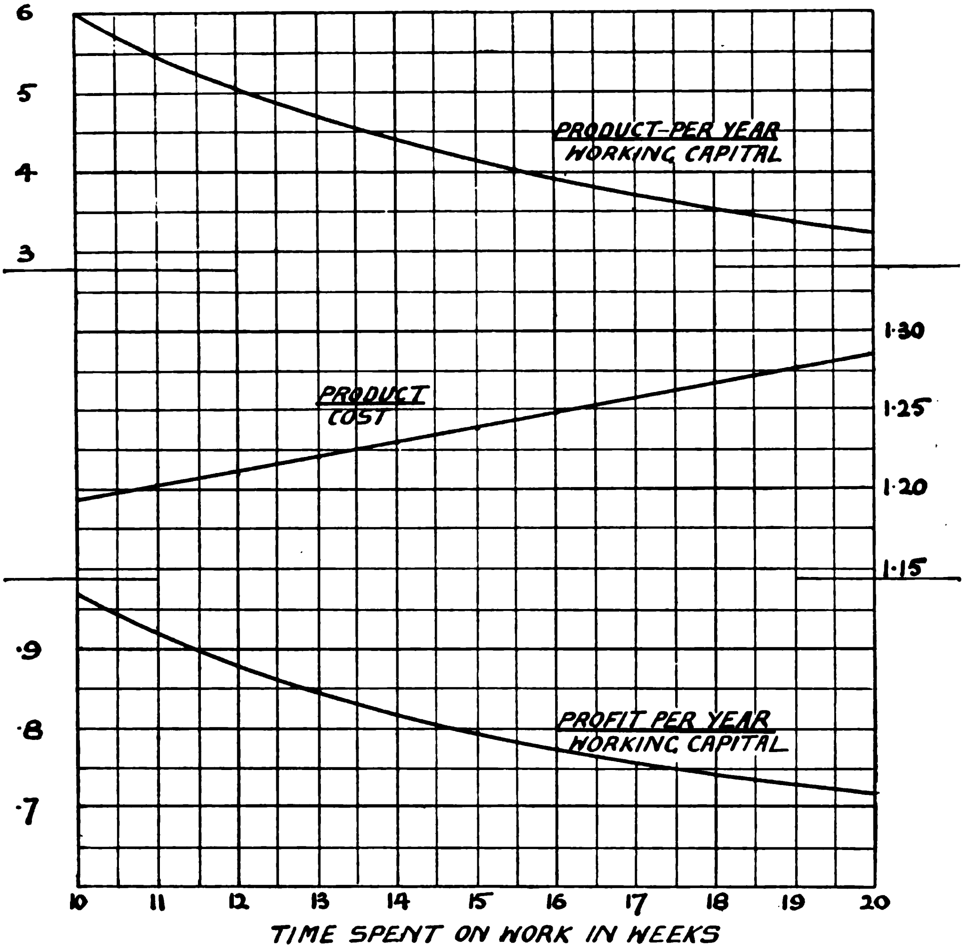


FIG. 2.—Cost. Profit. Economy.

weeks the total costs or working capital is £1,000, and the realized value is £1,200. For each week of delay in delivery beyond ten weeks a deduction of £3 or $\frac{1}{4}$ of 1 per cent. is made from the price paid, this being at the rate of $\frac{52}{4} = 13$ per cent. per annum. A fixed capital of between £5,000 and £6,000 is employed, for the interest

and depreciation of which £520, or nearly 10 per cent. per annum, is charged, this amounting to £10 per week. In ten weeks this means £100 or 10 per cent. of the total cost, and each week of further delay gives 1 per cent. increase of cost spent in interest and depreciation on plant, etc. On the other hand delay within the limits ten to twenty weeks permits economies in expenditure on materials, labour and otherwise at the rate of £20 = 2 per cent. of £1,000 per week of delay. The resulting clear profit, irrespective of interest on working capital, thus increases by £20 − £13 = £7 per week of extension of time. All these rates of variation are here taken as uniform, not in decreasing ratio as in Table I. The rates of variation are more rapid in this example than in Table I. The numerical results from the Table II are set out in diagram form in Fig. 2. In column 5 is put down the exact interest for half the time spent on the work at 5·2 per cent. per annum on the total cost or working capital. This varies not quite uniformly, the increase for the first extra week being 8*s.* 11*d.*, while that for the last of the ten extra weeks is 7*s.* 1*d.* The mean is 8*s.*, and as the difference is trifling, it is taken as uniform and equal to this average in the subsequent calculations.

Each extra week of prolongation raises the clear profit uniformly by £6 12*s.*; but this increasing profit reduced to per year falls from £975 to £652, that is, in the ratio 3 to 2, the fall being at first 3½ times more rapid than at the end of the scale. The profit per year in proportion to the total expenditure in finishing the one turn-over, goes down similarly, but not in so large ratio. As already explained, this total cost is the working capital employed and the profit per year is the single profit multiplied by the number of times this working capital is utilized in one year.

If the fixed capital be taken at £5,000, the ratio of annual profit to this sinks from 19½ per cent. for execution and delivery in ten weeks, to 13 per cent. for the same in double the time. It may be noted that these percentages differ in no very great degree from the average in Table I, *see* column 12; but the law of variation is here quite different. The percentage drops steadily all through the range of time-extension. The law of variation in Table II is similar to that from two to four years in Table I, although the curves in the two diagrams have opposite curvature.

The economy-ratio, product divided by cost, varies much in the same way as in Table I, but lies at a higher level throughout, ranging from 1·19 for ten weeks to 1·29 for twenty weeks, and increasing steadily by about 1 per cent. for each week of extension.

This, however, divided by the time spent in execution (measured as a fraction of a year) falls from about 6 to 3¼, the fall being nearly four times as rapid at the beginning as at the end of the range. In columns 8, 9 and 11, the reduction has been made to a year of 50, not 52, working weeks.

The lines on Fig. 2 show the character of these variations, the straight line showing the uniform rate and the curves those in which the rate changes.

It is not difficult to frame a fairly simple algebraic formula showing how the economy-coefficient given in column 11 of Table II falls off with extension of time. Neglecting the very small deviation from uniformity of decrease in column 5, so that the Total Cost is taken as decreasing uniformly, this formula is—

$$\text{Time Economy-Coefficient} = \frac{\mathrm{P}}{\mathrm{C\,T}} \cdot \frac{1-p\,t}{1-w\,t}$$

where P = value realized in minimum time of execution;
C = Total Working Cost, or Working Capital employed, for minimum time of execution;
T = actual time spent on work;
t = extension of time beyond the minimum;
p = the ratio to P of the reduction of realized value per unit extension of time;
w = the ratio to C of the reduction of working capital needed per unit extension of time.

If p and w were both zero, or equal, then the coefficient would vary in exact inverse ratio to T.

If w be not considerably greater than p, then there cannot be even the appearance of temptation to extend the time.

For small extensions of time t beyond the minimum, a close approximation to the coefficient is—

$$\frac{\mathrm{P}}{\mathrm{C\,T}} \left\{1 + (w-p)\,t\right\}$$

The rate at which this *decreases* with t may be found, by an elementary application of the differential calculus, to be proportional to $\left\{1 - (w-p)\,\mathrm{T}_{\mathrm{min}}\right\}$, and this is a positive decrease, unless $\mathrm{T}_{\mathrm{min.}}$ be greater than $\dfrac{1}{w-p}$.

In Table II, $w = {\cdot}0096$ per week and $p = {\cdot}0025$ per week, so that $\dfrac{1}{w-p} = 141$ weeks. With these rates of variation, therefore, any

extension of time must decrease this economy-coefficient unless the minimum time allowed is greater than 141 weeks = say 2·8 years.

Both the coefficient in column 9 in terms of Profit and that in column 11 in terms of Product are useful. The former measures solely the benefit to those who supply capital. The latter is more general and comprehensive. It measures the benefit resulting to all concerned, including both the capitalists (those who supply fixed capital and those who supply working capital), the managers and all who get salaries or other income from the work, and the labourers and artizans. It measures the whole benefit irrespective of how it is distributed between the different classes engaged in the production.

Similar coefficients in terms of the fixed capital have also their special use; but they cannot be called economy-coefficients, because they have reference only to one portion of the capital engaged.

The coefficients in column 20 of Table I and column 11 of Table II have identical meanings, and are calculated in identically the same manner; but they are given, in the two tables, different titles. These different titles describe the coefficient in different aspects, and they are not *obviously* identical in meaning. These different aspects are useful, because they arise naturally, under different industrial conditions. It is of great importance to recognize very clearly that they are merely varied aspects of identically the same thing. This, perhaps, may be most easily and thoroughly done by writing the following identical equations—

$$(1) \quad \text{Economy-Coefficient} = \frac{\text{Final Value}}{\text{Cost} \times \text{Time spent.}}$$

$$(2) = \frac{\text{Value produced in one turn-over}}{\text{Costs of one turn-over} \times \text{Time of one turn-over.}}$$

$$(3) = \frac{\text{Value produced per Year.}}{\text{Costs per Year} \times \text{Time of one turn-over.}}$$

$$(4) = \frac{\text{Product per Year}}{\text{Working Capital.}}$$

In (1) and (2) the ratio of product to cost enters without either of them being reduced to per unit of time, that is, reduced to a time-rate. But reducing each of them to any unit of time, whether that unit be a year, a month, or a week, does not alter their ratio;

and, therefore, (3), in which both are so reduced, is identical with (1) and (2). In (3) the first factor of the divisor is a time-rate of total expenditure, and this is multiplied by the time occupied by one "turn-over," by which is meant the time elapsed from starting upon work until the realization of its finished value. The product of the time-rate and the time is the whole expenditure incurred during this period, which is the capital that has to be advanced in order to effect the realization of the value of the new product. This is the working capital devoted to this realization of new value, in which form the divisor appears in (4). As soon as this realization is effected the same working capital may be devoted to other production by repetition of the same process or otherwise.

If now it be thoroughly realized that the total expenditure incurred from starting any work to the realization of its finished value, is at once its "cost" and the "working capital" devoted to it, then the identity of (1) and (4) becomes obvious, because $\frac{\text{Final Value}}{\text{Time spent}}$ is clearly the time-rate at which that value is produced.

It must, moreover, be remembered that the working capital is not fully employed on the work throughout the whole time spent on production. On the assumption of uniform time-rate of expenditure, the full working capital is on the average employed half this time, or half the capital is employed the full time. With non-uniform rate of expenditure other ratios than one-half arise. But this non-uniformity does not affect the identity between the expressions (1) and (2). The fact that the working capital is sunk for various fractions of the full time spent on the work according to the method of its expenditure in respect to variable time-rate, does affect the question whether this coefficient as here given is a completely fair and scientific measure of economy. A more exactly scientific divisor would be the sum of all the daily expenditures, each day's expenditure being multiplied by the time elapsing from that day to the date of final realization. This would be mathematically expressed thus: C being the whole working capital spent, and d C the part spent at any time t' after starting, while T is the whole time spent, then $\int (T - t')\,dC$, integrated within the limits $t' = 0$ and $t' = T$, is the proper scientific divisor of the final realized value to give the true economy-coefficient.

With uniform daily expenditure throughout the whole time, this integral would give—Total Expenditure × ½ total time. If this divisor were used the economy-coefficient would become

$\frac{2 \text{ Final Value}}{\text{Cost} \times \text{Time spent}}$; and in other cases where the expenditure was not uniform a greater or less numerical factor than 2 would be substituted. It would be less than 2 if the expenditure were more rapid towards the beginning, and greater than 2 if this were more rapid near the end, of the whole period spent in producing the new value.

It may be useful to remember this more exact scientific method of measuring for important cases in which definite changes in the rate of expenditure during the progress of large works may be safely calculated beforehand. In any case it must be useful, and in many cases of paramount importance, to remember the principle that the economy is decreased if an unduly large proportion of the expenditure be incurred early in the period over which the work extends. The following Table III gives the numerical factors which would replace 2 in the coefficient with various ratios of initial to final time-rates of expenditure, assuming the time-rate to change uniformly between the start and the finish. With such uniform change, the extreme limits are seen to be 3 and $1\frac{1}{2}$.

TABLE III

Ratio of Initial to Final Rates of Expenditure per Day	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	Infinity or Final Rate Zero
Economy Coefficient = $\frac{\text{Product}}{\text{Cost} \times \text{Time}}$ multiplied by	3	2·4	2·25	2	1·8	1·7	1·5

On the other hand, if a certain proportion of the total expenditure be incurred immediately at the start, while the rest of it is uniformly distributed over the time spent on the work, then the numerical factor of the coefficient varies from 2 down towards 1, according to the following Table IV. The last ratio 1 is an impossible one, as the work cannot be carried out without some daily expenditure.

A large initial expenditure generally means the purchase of materials paid for on delivery. In large contracts it sometimes also arises from the purchase of special plant required for the execution of the work contracted for.

If an isolated personal view of the matter were to be taken, it might be desirable to consider the converse case, in which much

TABLE IV

Ratio of Initial to Total Expenditure with uniform expenditure after starting . . .	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1
Economy Coefficient $= \frac{\text{Product}}{\text{Cost} \times \text{Time}}$ multiplied by .	2	1·82	1·67	1·54	1·43	1·33	1·25	1·18	1·11	1·05	1

of the expenditure takes place only at the final stage when the value of the product is about to be realized. Because wages, salaries, and interest on capital outlay may not be paid by the management until after completion of the work, or at longer intervals than the period of a complete turn-over. Thus if the expenditure at the finish were $\frac{1}{4}$ of the whole, the remaining $\frac{3}{4}$ being uniformly distributed over the whole time, then the numerical coefficient taking the place of the normal 2, would be $2\frac{2}{3}$; if $\frac{1}{2}$ were spent at the finish and $\frac{1}{2}$ uniformly, it would become 4; if $\frac{3}{4}$ at the finish and only $\frac{1}{4}$ uniformly, it would be as much as 8; and if the final expenditure be much more than in this last proportion, the factor may become very large. But it will be seen below that this view is irrelevant to the real question of economy, which does not concern itself with individual and separate interests.

Tables III and IV together cover all the real distributions of expenditure over the time of working, and from them we see that the numerical factor in the coefficient varies from 3 for practically impossible conditions at one extreme down to 1 for wholly impossible conditions at the other extreme.

While these variations and their causes should be remembered as showing how special circumstances affect the real economy, it seems inadvisable to take account of them in framing the definition of a coefficient intended to measure economy for all-round purposes and to be applicable to general work in which are necessarily combined many classes whose individual economy differs over a large range. Leaving out the numerical factor altogether, the coefficient has a desirably simple form—

$$\frac{\text{Product}}{\text{Cost} \times \text{Time}} = \frac{\text{Product per unit Time.}}{\text{Working Capital.}}$$

Confusion and anomaly at once arises in the application of this formula to particular trades if its absolutely impersonal character is not remembered. If this be neglected, then the " time " divisor may sometimes appear to become zero, or even negative, reducing the coefficient to an absurdity. For example, commission agents sell goods and receive money for the sale before they pay for the goods, and considered in respect to the agent alone the time divisor becomes negative. So far as the goods themselves are concerned, it is not he, however, who really furnishes the working capital for this business, and its impersonal economy does not depend in the least upon who furnishes it. Besides, the agent himself cannot carry on his business without some working capital on which he draws for both living and office and warehouse expenses. Again, on the Stock Exchange certain persons often sell, and receive money for, stock for which they pay only afterwards ; although, in this case, it is perhaps difficult to define exactly the value produced by the transaction. Once more, a bootmaker may sell, and be paid for, boots and shoes before he has paid the wages due to his employés who have made them ; perhaps before he has paid for the leather and nails out of which they have been made. In respect of wages, it is here the artizan who furnishes the working capital, and in regard to the leather it is furnished by the leather merchant who gives the bootmaker credit. So far as regards the economy-coefficient, it does not matter who furnishes it. It must all be furnished by some one or some set of persons. The " cost " is the total final cost, and the " time " is the time elapsing from the instant at which the work is put in hand to that at which its final value is realized, no matter who has incurred the expenditure necessary to carry on the work through this period. According to ordinary industrial custom, it is clear that the artizan is a contributary working capitalist to the extent of his weekly wages ; and the higher employés to the extent of their monthly, quarterly, or yearly, salaries. Moreover the universal trade custom of giving credit does not affect the problem in hand. Working capital is contributed to the same amount as credit is given.

So long as the industry to be dealt with consists in the manufacture of articles, each of which has its distinct individuality, there can never be any difficulty in any orderly managed workshop in ascertaining the average time taken from first to last to complete each individual article. For example, if it be a special manufacture of steam engine governors, the average time occupied in making each individual governor of a given size can be readily

obtained if desired. This, of course, is *not* one week divided by the number of governors finished in one week.

Suppose that the output averages 8 per week, and that each takes, on the average, 3 weeks to put through. In 3 weeks the output is 24, and it is clear that on the average 24 are being worked upon simultaneously. Now it might, perhaps, be possible to finish the governors each in one week. If this could be done by concentrating the energies of the whole staff and plant upon only 8, instead of 24, at one time, the output per week would remain the same as before, namely 8, while the time of execution would become one, instead of three, weeks. *If* the cost of each were not changed by this concentration, the economy-coefficient would be increased threefold. Is this a just result? Otherwise, does this result confirm, or discredit, the rationality of this measure of economy? Undoubtedly it confirms it *under the conditions assumed*. This is most easily seen by supposing the latter productivity trebled by *broadening*, so as to produce $3 \times 8 = 24$ per week at 3 times the cost of 8 per week. This broadening, as already fully explained, changes in no wise the economy. We have now twenty-four governors produced per week at the same total cost as, in the first instance, twenty-four per three weeks, which means clearly a treble economy, because the same working capital is turned over in one-third the time.

The conditions assumed, namely, that the cost of each governor would remain unchanged by the intensified rapidity of manufacture, are so extremely improbable as to be practically impossible. It would not be a physical impossibility to finish the eight governors in one week. The writer has seen, in an emergency, engineering work started and finished in twelve hours, when the normal time spent on it would be three weeks. But such rapidity is accomplished only by putting aside all considerations of organization for cheap production. In the above case, although the factor $\frac{1}{\text{Time}}$ is reduced in the ratio $\frac{1}{3}$rd, the other factor $\frac{\text{Product}}{\text{Cost}}$ would be certainly increased, and possibly increased in a ratio much larger than three. In seeking the highest possible economy, the advantage gained from reduction of time of turn-over must be balanced against the disadvantage of increased ratio of cost to product due to speeding up the rate of production. At a certain limit this disadvantage is sure to begin to overbalance the advantage, because in each case there is a limit of speeding up which is *physically*

impossible, and any approach to physical impossibility is always necessarily excessively expensive. This limit is not shown on the diagram Fig. 2. Extending backwards into shorter times of completion the straight-line laws of Fig. 2, no such limit would be reached; but this simply means that these straight-line laws could not hold true for much shorter times than those in the diagram. With a quite probable backward extension of the curves of Fig. 1, such as is shown by fine dotted lines, the limit might be reached at a time of execution of the work equal to about eight months; but at a much higher limit, namely, about ten months, the total cost would be more than the value of the product, and thus all purpose in doing the work would be eliminated. This is shown by the crossing of the two dotted curves, cost and product; while it will be seen that at eight months the dotted curve "product per year ÷ by cost" has risen to its maximum, and falls sharply for shorter times than this.

In the example taken above, namely, steam engine governors, each product has so much individuality that it is easy for those in charge to follow the history of its making and to say with close approximation how long it takes to make it. This is not so in many industries; and for these we must, therefore, find some indirect way of estimating the "time" to be used in the divisor of the economy-coefficient. The manufactures of nails, of needles, of pins, of pens, are examples in which it is at least difficult to follow with certainty the time-history of each individual product. An example in which this becomes impossible is the water-supply to large cities. The water may be brought many miles by gravitation to a storage reservoir, and thence to a pumping station; thereafter it is pumped to filtering beds, and from these it is distributed through a vast ramification of water-mains and service-pipes. How long does any one individual cubic foot of water take to pass through these various stages of preparation and be finally delivered from the house-tap? It is impossible to say; partly because each 1,000 cubic feet of water spend entirely different times in getting through. Coming now closer to the special subject of this treatise, what length of time elapses between the date at which a particular cubic inch of water is forced into a boiler by the feed-pump or injector and the date at which it passes as exhaust steam out of the engine? No one can say. But an *average* time can be calculated without much difficulty.

This calculation of average time may, perhaps, be most simply grasped by thinking of the combustion of coal in the boiler furnace.

How long does a lump of coal take to burn and be finally used up before the gases resulting from its combustion pass up the chimney? Quite evidently this time will vary according to the size of the lump and according to the position in the fire into which the stoker throws it or subsequently rakes it. Many hundreds of true times, widely different from each other, will apply to different portions of the coal used. In fact, in each lump the outside layer passes through quickly, while the centre is held an immensely longer time in the furnace before being liberated as gas and sent through the flues and up the chimney. Nevertheless, the average time is very strictly definite, and is measured with extreme ease.

Suppose that there are at a given time 800 lbs. of coal on the grates, and that the condition of the fire is kept steady by stoking on 75 lbs. coal every six minutes. If the same steadiness of combustion could be maintained without any stoking, then evidently every six minutes 75 lbs. would be burnt off the grates, and the whole 800 lbs. would disappear by combustion in $\frac{800}{75} \times 6 = 64$ minutes. With the steady stoking and steady maintenance of condition of fire, each piece of coal takes, *on the average*, the same time to pass through, and this time is the above 64 minutes. Because the last pieces of coal put on to the fire, just before any period of 64 minutes, formed part of the 800 lbs. which has been wholly burnt at the end of the 64 minutes. This time is obtained by dividing the total quantity under combustion at one time, viz. 800, by the speed of stoking, or, what is the same thing, the speed of consumption, namely, 75 lbs. per six minutes, or $12\frac{1}{2}$ lbs. per minute. It does not matter whether we divide by the rate at which the material is fed in or by the rate at which it is taken out after being finished with. It does not matter whether we divide by $12\frac{1}{2}$ lbs. per minute, or by $60 \times 12\frac{1}{2} = 750$ lbs. per hour. The quotient $\frac{800}{750} = 1\frac{1}{15}$ hours is the same as 64 minutes.

As the readers of this book are presumably engineers, it may be remarked in passing that this simple relation does not appear to be commonly known among those in charge of boiler and other furnaces. Reducing the quantities to per square foot of grate surface, as is customary, and measuring them volumetrically, the relation may be stated in terms of thickness of coal on the grate. It may then take the form—

$$\left.\begin{array}{r}\text{Thickness of Fuel}\\ \text{required on Grate}\end{array}\right\} = \left\{\begin{array}{l}\text{Thickness of layer (taken as uniformly}\\ \quad\text{spread) stoked per unit of Time}\\ \quad\text{multiplied by Average Time taken}\\ \quad\text{to burn each piece of Fuel.}\end{array}\right.$$

The thickness of layer to be stoked per unit of time is proportional to the horse-power to be developed per square foot of grate, and for each class of work may be taken as a constant. The time taken to burn each piece of fuel depends on the nature of the fuel in respect of chemical quality and of size of lump and of caking or flaming quality; also upon the method of stoking.

If the fuel be oil, the relation may be stated in the form suited for estimating the time taken to burn, namely—

$$\left.\begin{array}{r}\text{Average time in minutes taken to}\\ \text{burn each small portion of oil}\end{array}\right\} = \frac{\text{Quantity of Oil in Furnace}}{\text{Feed of Oil per Minute.}}$$

Applied to the problem of heating and evaporating water in a boiler and passing it through a steam engine, it takes the form—

$$\left.\begin{array}{l}\text{Average time in minutes taken}\\ \text{to pass each portion of water}\\ \text{through boiler and engine}\end{array}\right\} = \frac{\text{Quantity of Water in Boiler}}{\text{Feed-Water per Minute.}}$$

The quantity of water in the boiler and the feed may be measured in lbs., or tons, or cubic feet, or cubic inches; but both must be measured in the same terms. Note, however, that if volumetric measurement be used only the volume of the water and not that of the steam in steam-space, steam-pipe and engine, is to be reckoned in the numerator. The steam in these latter spaces may be neglected in the reckoning, as may be recognized by measuring the quantities by weight, when it will be at once seen that the weight of the steam existing at any one instant throughout boiler and engine is a negligeably small fraction of the weight of water in the boiler; with a possible exception in the case of flash boilers.

Reverting now to the general terms of an economy-coefficient of universal application, and remembering that the time thus calculated enters as a divisor into this coefficient, the relation now obtained may be thus expressed—

$$\frac{1}{\text{Av. Time spent}} = \frac{\text{Quantity finished per unit Time}}{\text{Quantity in hand}}$$

where " time spent " means time spent from start to finish of any work, and " quantity in hand " means the total quantity being operated upon at any one date. If this be inserted in the formula

$$\frac{\text{Product per Unit Time}}{\text{Cost per Unit Time} \times \text{Time Spent}},$$

it will be noted at once that there is presented an opportunity of simplification by cancelling the " Cost per Unit Time " with the " Quantity finished per Unit Time." To effect such cancellation, all that is needed is to measure this last quantity by its cost when finished. If this be done, then the " Quantity in hand " must be measured in like terms, that is it must be evaluated at its final total cost.

Evaluating these quantities in this way and making the cancellation, we arrive at the simpler form of—

$$\text{Economy-Coefficient} = \frac{\text{Product per Unit Time}}{\text{Quantity in hand valued at final finished Cost.}}$$

Here the " Product " is not necessarily expressed in money. A little consideration will show that the divisor is, essentially, only another mode of estimation of the " working capital " employed.

It will now be convenient to collate the different forms in which we have found the coefficient to be expressible. These express only various aspects in which one identical measure of economy may be viewed.

Forms of Expression of the Economy-Coefficent.

1.	Product	divided by	Total Cost × Time spent on work.
2.	Product per unit Cost .	,, ,,	Time per Turn-over.
3.	,, ,, ,, Time .	,, ,,	Total cost per unit Time × Time spent on work.
4.	,, ,, ,, Time .	,, ,,	Total cost per Turn-over.
5.	,, ,, ,, Time .	,, ,,	Working Capital.
6.	,, ,, ,, Time .	,, ,,	Quantity in hand valued at Total Final Cost.

N.B.—A more strictly scientific measure would be the above multiplied by the numerical factor 2 *in the case of uniform distribu-*

tion of the expenditure of working capital over the period of each turn-over, and by numerical factors varying inside the limits 3 *and* 1 *in cases of non-uniform expenditure, as per Tables III and IV.*

When the "Time Spent on work" is variable, and cannot be directly estimated, then Average Time spent = Quantity in hand at one time divided by Quantity finished per unit Time.

In applying expression 6 to any case, such as the production of mechanical power by furnace-boiler-and-steam-engine, or by any other form of motive engine or plant, the "quantity in hand" is not simply the quantity of water and steam in the plant at any one instant, nor the quantity of coal or other fuel in the furnaces and in the bunkers : it is the total of all the things, including labour, which go towards the production of the mechanical power being produced, all these valued at their final total cost at the completion of the work of producing power. Thus if coal lies in the bunkers on an average for three weeks before being consumed, its original cost should be increased by three weeks' interest on the price paid for it. As this, however, would be only, say, one-third of 1 per cent., the increase is negligeable.

If the product and its cost be evaluated in the same terms, say, each in money, then this coefficient becomes a pure number divided by a time, Thus an economy-coefficient of 6 per year is the same as $\frac{1}{2}$ per month, or $\frac{6}{52}$ per week, or about $\frac{1}{60}$th per day, or $\frac{1}{500}$th per working hour with a year of 3,000 working hours.

But it should be distinctly realized that the product and the cost are not of necessity measured in the same terms. The product may be, for example, ft.-lbs. of energy, and the cost so many pence ; and, in order to make the economy-coefficient strictly definite and useful, there is no need to price the energy produced at so many pence per ft.-lb. The coefficient would, in this case, be so many ft.-lbs. energy produced, or work done, per penny of cost and per hour or per minute. If the product be filtered drinking water, the coefficient would be so many gallons of such water per £1 cost per day of turn-over of working capital. If it be coal mined and stacked at the pit-bank, the measure of economy would be so many tons of coal per year per £1 of working capital.

The coefficient may be raised, that is, improved, in three ways. The product per unit of time may be increased without increase of working capital. Or the ratio of cost to product may be lowered without change in the time elapsing in one turn-over of working capital. And again, this time of turn-over may be diminished

without alteration in ratio of cost to product. These are three perfectly distinct methods by which a manufacturer may attack the problem of improving his business. At any one time, one of the three methods may be feasible without either of the other two being so. But two, or all three, may, of course, be simultaneously practicable.

The economy may be raised by increasing either the quantity or the quality of the product. So far as increase of quantity is concerned, it remains a matter of indifference, as said above, in what terms the product may be measured. But when the quality is improved, it becomes difficult to obtain any exact measure of the improvement unless the product be measured by reducing it to its money or equivalent value. There may be no doubt as to the betterment in the quality; but, until it can be expressed in money or like terms, no correct judgment can be framed as to whether the extra quality is worth the extra cost at which it has been achieved, or worth the extra time involved in the completion of each single turn-over.

For example, if boilers can be made to produce more steam of unaltered quality, there is no need to consider the realizable money value of the steam. But if the output be raised by improving the quality of the steam, either by raising its pressure or by superheating it to a higher temperature above the saturation temperature, then the improved value of the steam must be measured either in money or in equivalent capacity for developing cylinder horse-power.

In the production of mechanical power, the work done is the useful product, and the quantity of work done per unit of time is termed *horse-power*. In this case the economy-coefficient is, therefore, horse-power divided by working capital kept employed, or horse-power per £1 of working capital. Careful distinction must be drawn between this and the more commonly quoted figure, "horse-power per £1 of annual cost." The reciprocal of this, or "annual cost of 1 horse-power," is a measure much used. It is simply the ratio of so much money to so much work done, and includes no real reference to any unit of time: it includes no time-element in its composition.

An engineer who knows the annual cost of his power plant, should be able to estimate without much difficulty and with fair accuracy, what fraction of that annual cost is the money that is continuously held up and devoted to the running of this plant. Exactly expressed, it is the maximum cash reserve ever required to be held in hand or at the bank for this particular purpose of power generation. It is

this reserve which is the "working capital" involved in the measure of economy. In the case of a manufacturer generating the power and using it in his own works, the estimate may be made on the hypothesis that the value of the power is realized immediately on its delivery to the driven machines, or to the shop main-shaft. In the case of a company generating power and selling it to outside workshops, the company does not realize the value of the power it delivers until it receives payment of its quarterly accounts. This is the simplest case, the time of turn-over being definitely three months, and the working capital being the three months' expenditure, inclusive of interest on both fixed capital and on the three months' expenditure itself.

No improvements in economy-coefficient can be effected without overcoming extra mechanical and other difficulties ; and to overcome these difficulties an increase of costs is nearly always inevitable. If the costs go up in the same proportion as the time-rate of valuable production, there is no change in the coefficient. If the time-rate of production of value be increased in less ratio than the costs, disadvantage results. There is industrial gain only when the incurred cost is raised in less proportion than the time-rate of produced value. Perhaps the more correct expression would be to say that only then are the conditions under which industrial gain is won improved.

These results may be effected by some complete and abrupt change in design or method. Such change is called in mathematical language "discontinuous." It represents a complete break in practice. In such cases the desirability of the change can only be considered in relation to the completely different results obtainable by the old and the new methods. There is here no question of more or less change : either the whole or none must be adopted.

It is more common, however, that change is possible gradually in one or the other direction. For instance, if the policy of increasing or decreasing piston speed, or of increasing or decreasing boiler pressure, were under investigation, experiments or calculations in gradual change of these elements of practice may be made. Such gradual change is called "continuous," in contradistinction to the abrupt discontinuous alteration. If in such continuous change we find a resulting improvement in the economy coefficient, that is, if the numerator increase in a higher ratio than does the denominator, it will, of course, be advantageous to push the change further and further until the limit is reached beyond which the opposite effect arises. As the changes are gradual, the limit is found at the place where both numerator and divisor increase in equal ratio. This

limit is that at which this particular sort of development gives maximum economy, as measured by the coefficient. Further development in the same direction is no longer improvement but the reverse. The first effort towards further development may show that the limit has already been passed, and then the economic problem becomes to find out how much change in the opposite or backward direction is needed to arrive at the conditions of greatest economy. For instance, let us suppose that certain engine tests have shown that the economy of a particular set of engines has been improved by increasing piston-speed, and that, in consequence, high piston speed has become popular among mechanical engineers, resulting in great efforts being successfully made to overcome the mechanical difficulties in the way of very high speed. The results may not be watched scientifically, and pride in difficult mechanical achievement may make engineers blind for a time to the true test of economy. Piston speeds may thus become extravagant, and when scientific method and investigation is once more directed to the question of economy, the problem may become how far it is profitable to reduce piston speeds. If scientific method and regard to true economy stood guard upon all such development, mistakes would not be made in overstepping the true limits of improvement.

Similarly mechanical enthusiasm in overcoming the difficulties connected with very high steam pressures may lead to the use of pressures higher than are commercially economical. The use of superheated steam is proved to be advantageous, and it becomes popular among engineers who are unable to apply scientific method to their work. The result is that superheating up to temperatures long past the true economic limit becomes common, and has to be corrected by further experience. Another obvious illustration may be taken from the tendency towards increase of railway speeds; and another again from the rapid growth of voltage in the electrical transmission of power; and yet another from the recent increase of cutting speed in machine tools.

In all such cases there can be hardly any doubt that a profitable limit actually exists. As technical improvement is pushed towards extreme limits, the costs of further gain always begin to rise with more and more excessive rapidity; each additional measure of gain is obtained at a cost that leaps up in geometrical (or even worse than geometrical, namely, logarithmic) progression.

The Economy-Coefficient, $\frac{P}{CT}$, involves the three factors, product,

cost, and time. If any one of these varies without change in either of the other two, such variation cannot, of course, lead to a limit of maximum economy. Increase of P alone necessarily increases the coefficient: it does not first increase and then decrease it. Increase of either C or T alone necessarily always decreases the coefficient. A maximum limit of economy, such as has been spoken of above, can only occur when at least two of the three vary. It may be the result of simultaneous and essentially inter-dependent variation of all three. Each of them may be affected by any one of a great variety of physical or social elements influential in the industry involved. Let s indicate any such influential element. Numerous examples have been already mentioned, viz. steam pressure, steam superheating, electric tension, etc., etc. A commonly used and very simple way to express the rate at which any quantity influenced by s varies with s, is to add the super-affix $'$ and the sub-affix s. Thus P_s' indicates the increase of P per unit increase of s, and P_s' is implicitly negative if *increase* of s makes P *decrease*. C_s' is the increase of C, and T_s' the increase of T, per unit increase of s. The elements of the differential calculus show that the increase of $\frac{P}{CT}$ per unit increase of s equals—

$$\frac{P}{CT}\left\{\frac{P_s'}{P} - \frac{C_s'}{C} - \frac{T_s'}{T}\right\}.$$

P, C, and T being essentially always positive, this rate of increase of the economy-coefficient is positive, that is, the economy becomes better with the increase of s, so long as $\frac{P_s'}{P}$ is greater than $\left\{\frac{C_s'}{C} + \frac{T_s'}{T}\right\}$. If, however, $\frac{P_s'}{P}$ be less than this last, then increase of s produces worsening of economy. If gradual change of s produces *continuous* change in the economy coefficient, at first bettering it and afterwards worsening it; then the limiting value of s, below which the bettering occurs and above which the worsening occurs, gives maximum economy, and at this point the rate of change of $\frac{P}{CT}$ is zero, or—

$$\frac{P_s'}{P} = \frac{C_s'}{C} + \frac{T_s'}{T}.$$

This is the perfectly general criterion of maximum economy: the test indicating how far it is economical to increase the influential

element s. Note that it is not to be assumed that change of s necessarily affects all three quantities P, C, and T. For example, if it affects P and C without affecting T, then T_s' simply becomes zero, and the criterion reduces to $\frac{P_s'}{P} = \frac{C_s'}{C}$.

If r indicate any other element influencing continuously these quantities, the criterion indicating the value of r giving maximum economy is similarly written—

$$\frac{P_r'}{P} = \frac{C_r'}{C} + \frac{T_r'}{T}.$$

The best *combination* of s and r in respect of economy is obtained by combining these two equations and solving them *as simultaneous* equations for s and r. Similarly each element influential upon the economy will, if it cause *continuous* variation, give a similar critical equation. There will be the same number of such equations as there are influential elements in the problem; and the combination of all these as simultaneous equations affords, theoretically, the means of fixing the best values to give to each and all such elements. Increase of the number of such elements and corresponding equations, however, causes very rapid increase in the practical difficulty and laboriousness of the complete solution. The geometrical ratio in which this difficulty increases is so serious that it is not practically desirable to attempt a complete solution for more than, say, half a dozen variables; and if the really important variables can be reduced to three or four, there will be a vastly greater probability of practical industrial use being made of such theoretical investigations. In fact, it is only in the somewhat unlikely case of the above equations being of the first degree that they would be found to be practically soluble by ordinary algebraic means.

When difficulties of this kind arise, graphic methods, if skilfully used, very often offer comparatively easy methods of surmounting the algebraic obstacles. We give here only one graphic method of solving the problem of finding the maximum when it depends upon two independent factors. In the diagrams S and R of Fig. 5, these two factors are referred to as s and r, while the co-efficient $\frac{P}{CT}$ is, for shortness sake, called ρ.

In diagram S the horizontal ordinates are values of s; while in R they are values of r. In both the plotted heights of the curves

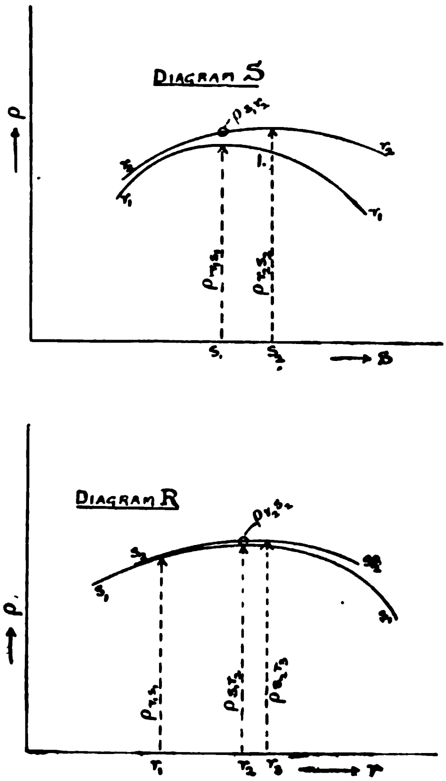


FIG. 5.—General Graphic Determination of Maximum Economy.

are various values which ρ may take. The graphic process for finding the maximum for combined variation of s and r is as follows—

(1) Choose a value of r, namely r_1, considered to be not far from that giving the maximum. No advantage is derived from making this first choice a close approximation to the value finally found to be correct for maximum ρ, and therefore it should be chosen rounded off to a number making the calculations easy. With this value r_1 and five values of s calculate five corresponding values of ρ. These are plotted as points on the curve marked $r_1\ r_1$ in diagram S. The points should each be plotted as soon as obtained, and a rough hand-drawn curve passing through the first three points so plotted should guide the choice of the remaining two values of s taken so that the fourth and fifth points may be near the highest part of the curve; but these two last points should, nevertheless, not be close together. Through these five points the curve is to be drawn fairly by help of a suitable drawing-office curve-template. The highest point of this curve is to be found and marked, and the corresponding horizontal ordinate s_1 measured. Call the greatest height of this r_1 curve by the name $\rho_{r_1\ s_1}$.

(2) In diagram R at the horizontal ordinate r_1 plot upwards the same height $\rho_{r_1\ s_1}$ as just found to be the greatest on the curve drawn in S. With the same s_1 and four other values of r calculate four other values of the coefficient ρ, and plot them on the diagram R as heights to the curve marked $s_1\,s_1$. As in the first operation, these heights should be plotted each as soon as found, and the fourth and fifth values of r should be chosen by guidance from a rough curve drawn through the first three so as to define well the top of this curve. This curve s_1 is to be drawn in carefully through the five points by help of a suitable curve-template—which is unlikely to be the same template as served well for curve r_1 in diagram S. Next find the highest point of this s_1 curve, and measure its height and horizontal ordinate at this maximum. Call these $\rho_{s_1\ r_2}$ and r_2. Since $\rho_{s_1\ r_2}$ is the greatest height of this curve, it is greater than $\rho_{s_1\ r_1}$, unless this latter turns out to be itself the highest point.

(3) In diagram S, at the horizontal distance s_1, plot upwards the height $\rho_{s_1\ r_2}$, which will be above the curve $r_1\,r_1$. Next with this value r_2 calculate for two other values of s, one greater and one less than s_1, two other values of ρ and plot them as points upon the curve marked $r_2\,r_2$. These three points are sufficient guidance for the accurate drawing of the portion of the curve $r_2\,r_2$ required;

because this curve must have the same general shape as curve $r_1 r_1$ and must be drawn in by help of the same curve-template, although not using precisely the same part of this template as was used for $r_1 r_1$. Three points will, at any rate, be found ample if the author's "reversible homogeneous scaled" curves be used for the purpose. The curve $r_2 r_2$ is thus to be drawn in, and its highest point found, the height $\rho r_2 s_2$ and the horizontal ordinate s_2 of this highest point being measured. $\rho r_2 s_2$ will be greater than $\rho s_1 r_2$, unless this first point plotted be found to be itself the highest.

(4) In diagram R at horizontal distance r_2 is next to be plotted this height $\rho r_2 s_2$, which will lie above the curve $s_1 s_1$. Then with the value s_2 and two other values of r, one greater and one less than r_2, are calculated and plotted two other points of the curve marked $s_2 s_2$; and through these three points this curve is to be carefully drawn by help of the same curve-template as served for curve $s_1 s_1$, although not exactly the same part of this template. On this curve $s_2 s_2$ is to be found its highest point, and its height $\rho s_2 r_3$ and horizontal ordinate r_3 measured. $\rho s_2 r_3$ is greater than $\rho r_2 s_2$, unless the points are found to coincide.

By repeating this process alternately in the S and R diagrams, there will very soon be reached a stage at which no further increase of height can be found, the first point plotted in each curve—the height being transferred from the last obtained maximum on the other diagram—being discovered to be the maximum of the new curve. As soon as increase of height from initial point to top of curve ceases in both diagrams, the problem of finding the desired maximum is solved. If skilfully carried through, the operation does not entail drawing more than three curves on each diagram, and often two on each will be found to give sufficient approximation to the desired solution. The first curve involves five calculations; the second four calculations; and each subsequent curve only two calculations. Thus two pairs of curves involve thirteen; and three pairs, seventeen calculations.

This process may appear clearer to some minds if thought of in the manner adopted in the author's book, *The Calculus for Engineers*. The two variables r and s are graphically represented by the longitudes and latitudes of spots upon a hill, while ρ is the height of each corresponding spot above sea-level. The problem is to find the value of ρ at the top of the hill by help of east-west and north-south vertical sections. The curve $r_1 r_1$ is the first east-west section taken, guessed to run across not far south or north of the top. Through the highest point of this section $r_1 r_1$, there is to be run a

second section due north and south. This is the $s_1 s_1$ section. From the highest point of $s_1 s_1$ there is now to be run another due east-west section $r_2 r_2$; and through its highest point a second due north-south section $s_2 s_2$ is run. Its highest point will be found to be fairly close to the desired summit.

This chapter will be concluded by stating two general propositions, one regarding the fundamental necessary relation between "efficiency," costs of production, and realizable value of product; the other concerning the adjustment of output and efficiency that yields maximum commercial profit.

A single machine, or a collection of machinery and apparatus constituting a complete plant, is capable of more or less output according to the manner in which it is worked. At very low outputs, much below that for which the plant has been designed the efficiency is generally, although not always, small. Increasing the output up to a certain limit, the efficiency also increases, and beyond this limit it begins to fall off again. This limit gives the output corresponding to maximum efficiency. It is a very common, but wholly erroneous, belief that this is the *best* rate at which to work the plant, and, therefore, its output at this rate is very commonly called its *normal capacity.*

Its output may, however, be increased beyond this limit, generally by a variety of alternative methods. Whatever means be adopted to effect such increase, their adoption generally involves a decrease of efficiency. The output may increase, and the efficiency decrease, according to more or less complex laws, which might be described graphically by more or less simple or irregular curves.

To take only one illustration, if there be burnt on the grate of a boiler-furnace a quantity of fuel much below that needed for the so-called normal capacity of the boiler, the boiler efficiency is naturally low because of the large heat-losses due to a variety of cooling causes. On the other hand, if the fires be forced beyond this normal capacity, greater horse-power is developed by the boiler at the cost of efficiency lower than the maximum; and the higher the horse-power rate at which the furnace and boiler is worked, the lower does the efficiency fall. But even an elementary common-sense consideration of the matter leads one to recognize that for the sake of utilizing the plant to a fuller extent it is worth while to sacrifice efficiency in some degree. The real commercial problem is to find in what degree it ought to be sacrificed in the direction of securing the advantage of increased output.

Call the output per unit of time P. The realizable value of this

output may not increase in strictly constant proportion to P. It generally increases in a less proportion than P. Starting from zero value for zero output, it increases at first at a more rapid rate, and afterwards not so rapidly. The consideration of the values of the very small outputs are not of commercial interest. Throughout a considerable range—throughout such a range as that over which it is interesting to investigate the above problem—a sufficient approximation to its actual mode of increase may be taken as represented by a "straight-line" law. Thus if p be the price obtainable per unit *increase* of output, the total value of the output P may be expressed by $(P_0 + p P)$, where P_0 is a constant. On a diagram this gives a straight line which lies above that part of the actual true curve corresponding to very small outputs, and which cuts through the curve at two points not far distant on either side of the normal capacity and is parallel to the tangent of the curve at a point somewhat higher than the normal capacity.

This output is the result of working up a certain quantity of what may be called "raw material," on the understanding that such "material" is not necessarily material in the physical sense but may sometimes be, for example, energy or any other quantity of value. This material may be of various kinds, a combination of which goes to the making of the completed product. The quantity used is proportional to the output, in a proportion which varies with the efficiency, decreasing as the efficiency increases. It may be measured as $\frac{P}{e}$, where e is an efficiency factor. This factor is not necessarily a pure numerical fraction, although it is so in many cases.

It is so in the case of production of one kind of power by utilization of another kind of power. For instance, the efficiency of a dynamo is the ratio of electrical power taken out of it to the mechanical power put into it; and that of an accumulator storage battery is the ratio of the electrical powers taken out of and put into it. In the case of power production by turbines working with a given constant head of water, the efficiency is the ratio of the mechanical power delivered by the turbine to the quantity of water used multiplied by the water "head." But so long as the head remains constant in all the comparisons, the divisor might be taken simply as the quantity of water. Similarly in steam engines, the efficiency is often measured as the ratio of the power developed to the weight of steam used, and no error arises so long as the pressure and quality of the steam remains the same through-

out the comparisons made. In these latter cases the efficiency factor e has physical dimensions.

The cost of the raw material and of working it up into the finished product varies not in proportion to the quantity used but according to some law which, throughout a sufficient range of comparison, may be taken as a straight-line law. This cost may thus be written $\left(C_2 + \frac{wP}{e}\right)$, where C_2 is a constant and w is the cost per unit of *increase* of raw material used, *including* the cost of working it up.

These quantities being taken per unit of time, the resulting profit per unit of time is—

$$(P_0 + pP) - \left(C_2 + \frac{wP}{e}\right) = \frac{wP}{e}\left(\frac{p}{w}e - 1\right) - (C_2 - P_0)$$

In nearly all cases where real profit is possible C_2 is greater than P_0,—usually very considerably greater—so that $(C_2 - P_0)$ is a positive quantity to be subtracted from the first term. Therefore in order that profit, and not loss, may result, the first term must not only be positive but considerably greater than zero, or $\frac{p}{w}e$ must be considerably greater than unity. This gives a limit above which the efficiency must be kept in order to avoid loss. This limit is—

$$\text{Efficiency } e \text{ greater than } \frac{w}{p}$$

$$\text{or greater than } \frac{\text{Cost per unit of Increase of Raw Material}}{\text{Price per unit of Increase of Output.}}$$

Here the cost in the numerator includes both the purchase price and the total cost of working up. In case both the constants P_0 and C_2 be known, we may say more exactly that the efficiency e must be greater than $\frac{w}{p}$ in a ratio greater than 1 to $\left(1 - \frac{C_2 - P_0}{Pp}\right)$.

Now if by any means the output P be increased, the means adopted involving decrease of efficiency, the constant part $(C_2 - P_0)$ of the profit does not affect the change in the profit. The term pP increases, but so also does $\frac{wP}{e}$, which latter term is to be subtracted. If P' express the rate at which P increases, and e' that

at which e *increases*—by supposition this latter is generally negative—then the ordinary rules of differentiation give—

$$\text{Rate of Increase of Profit} = \frac{w\,\mathrm{P}}{e}\left\{\frac{\mathrm{P}'}{\mathrm{P}}\left(\frac{p}{w}e - 1\right) + \frac{e'}{e}\right\}.$$

As e' is negative in the problem investigated, the term $+\frac{e'}{e}$ is negative, and in what follows it should be remembered that wherever $(-e')$ appears, this means the rate of *positive decrease* of e.

If the means adopted to increase output be profitable, then this last must be positive, and its being positive is the true criterion of the development being profitable or the reverse. The factor $\frac{w\,\mathrm{P}}{e}$ is, of course, always essentally positive. The factor $\left(\frac{p}{w}e - 1\right)$ is also positive as has just been shown. The whole is thus undoubtedly positive if e' be so ; that is, if the efficiency increase along with the output : but this is self-evident without mathematical demonstration. When, however, as in the problem now under consideration, e' is negative, then the criterion by which the profitableness of the development must be judged is—

$$\frac{\mathrm{P}'}{\mathrm{P}} \text{ greater than} - \frac{e'}{e\left(\frac{p}{w}e - 1\right)}$$

$\frac{\mathrm{P}'}{\mathrm{P}}$ is the ratio of increase of output, while $-\frac{e'}{e}$ is the simultaneous ratio of decrease of efficiency. The criterion may, therefore, also be expressed thus : The percentage decrease of efficiency divided by the percentage increase of output must be *less* than $\left(\frac{p}{w}e - 1\right)$, this latter being the excess over unity of the product of the efficiency by the ratio of price of unit of increased output to cost of unit increase of material ; or—

$$-\frac{e'/e}{\mathrm{P}'/\mathrm{P}} \text{ less than } \left(\frac{p}{w}e - 1\right).$$

Further development in this direction will increase profit per unit of time so long as this holds, that is, up to the limit at which equality, instead of inequality, holds between these quantities ;

beyond this limit the inequality being in the reverse direction and loss resulting from the development.

Thus the Maximum Profit per unit of Time is obtained when the development is pushed to the limit at which

$$-\frac{e'/e}{P'/P}=\left(\frac{p}{w}e-1\right).$$

This maximum-profit ratio of—e'/e to P'/P follows the straight-line law in respect of increase of efficiency e and also of increase of ratio $\frac{p}{w}$ of price to cost per unit of raw material used. To illustrate further the concrete meaning of this general law of maximum profit, and for the sake of facilitating its application by those readers who have followed the argument, its numerical results are given in Table VI and plotted in diagram (Fig. 6). In this six straight lines are drawn corresponding to the six values of the efficiency—

$$e = \cdot4,\ \cdot5,\ \cdot6,\ \cdot7,\ \cdot8,\ \text{and}\ \cdot9.$$

TABLE VI

Value of ratio $-\dfrac{\frac{e'}{e}}{\frac{P'}{P}}$ giving efficiency yielding maximum profit from given size of plant.

$\frac{p}{w}$	$e=\cdot4$	$e=\cdot5$.	$e=\cdot6$	$e=\cdot7$	$e=\cdot8$.	$e=\cdot9$
2	—	—	0·2	0·4	0·6	0·8
4	0·6	1	1·4	1·8	2·2	2·6
6	1·4	2	2·6	3·2	3·8	4·4
8	2·2	3	3·8	4·6	5·4	6·2
10	3·0	4	5·0	6·0	7·0	8·0
12	3·8	5	6·2	7·4	8·6	9·8
14	4·6	6	7·4	8·8	10·2	11·6
16	5·4	7	8·6	10·2	11·8	13·4
18	6·2	8	9·8	11·6	13·4	15·2
20	7·0	9	11·0	13·0	15·0	17·0

The horizontal ordinate is $\frac{p}{w}$, running from the value 1 up to the value 20. The oblique lines themselves do not run back so far as 1 on the horizontal scale because in every case 1 would give

a negative value to the height. In special cases the value of $\frac{p}{w}$ goes higher than 20.

In the diagram the height of the straight line for any one e gives the maximum-profit ratio of $-e'/e$ to P'/P.

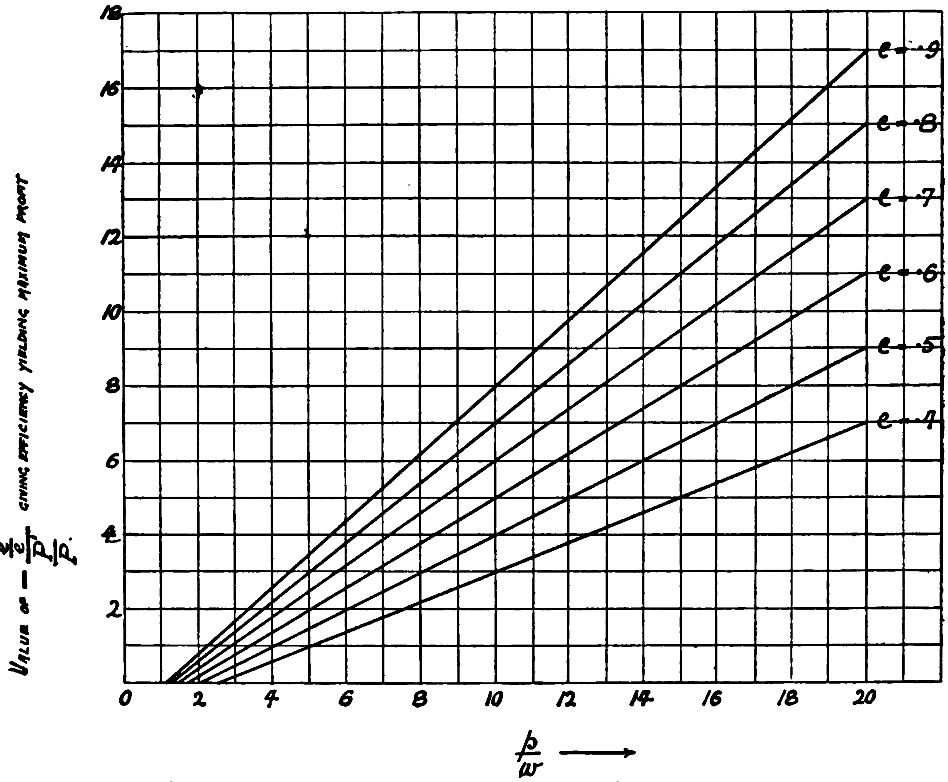


FIG. 6.—Values of Efficiency, giving Maximum Profit from given size of Plant.

In regard to the rates of increase written P' and e', it must be understood that the above rules hold by whatever means the product P and the efficiency e be changed, and that they may be applied as criteria for the investigation of the profitable limits to any number of developments in different directions. In each case P' measures the ratio of the increase of P to the increase of element operated on to effect the development, while $-e'$ is the ratio to this last of the decrease of e. For instance, if development of boiler horse-power by increase of lbs. of fuel burnt per hour per square foot of grate be considered, then P' means $\frac{dP}{df}$ and e'

means $\frac{de}{df}$, where f is the lbs. fuel burnt per hour per square foot of grate : that is, P′ is the differential coefficient of P with respect to f, while e' is that of e with respect to f. By using the non-particularized symbols P′ and e', it is intended to make clear that P and e may be here differentiated with respect to *any* element having a real effective influence on the working of the plant.

In the problem just solved in general terms, the size of the plant was assumed as fixed, and the question was at what rate to work it to make it most profitable. In another economic problem very important to designers of plant, the required output is definitely specified, and the question is what size of plant it is most economic to use—what size makes the total cost of the specified output least. Here P is not variable, but the working expenses increase as the efficiency e is lessened in proportion to $\frac{wP}{e}$. But with smaller efficiency a smaller size of plant will yield the required output P. The capital charges, which in the above are included in the constant C_2, are now variable, increasing with the efficiency. They will in most cases be found to equal an initial constant plus an amount proportionate to P and to a function of the efficiency which may be called f(e). The form of this function varies greatly with the kind of plant in question; but it always increases with e. Call the proportion of its increase to that of e by the symbol f′(e); so that, when any change is made which makes e vary at the rate e', then f(e) varies at the rate e' f′(e). The total cost of the perscribed output P is thus—

$$C_w = \text{Constant} + k\,P\,f(e) + \frac{wP}{e};$$

and any change affecting the efficiency and consequently the necessary size will make C_w vary at the rate—

$$C'_w = P\,e'\left(k\,f'(e) - \frac{w}{e^2}\right).$$

When the efficiency is very small. the negative part of this rate C'_w is very great, which means that C_w, the total cost, goes down as the efficiency is increased. But very often, although not necessarily in all circumstances, the positive part becomes greater than the negative part long before e has come near the perfect value unity. The reason is that after moderate efficiency has been reached the straining after higher degrees of efficiency becomes more and

more seriously expensive in construction-costs. Thus there is always a limit giving least cost for the prescribed output. This limit is found where the rate C'_w is zero. Therefore, for prescribed output P, the criterion of least total cost is—

$$e^2 f'(e) = \frac{w}{k}.$$

This equation gives the size of plant and also the corresponding efficiency which it is commercially most economic to employ under the specified conditions.

Stated in this general form, this very important and practical rule is beautiful in its extreme simplicity; but when applied to particular technical problems, $f'(e)$ is usually found to be a somewhat complex function, so that both technical and mathematical skill are needed for its practical interpretation and solution.

Chapter II

Dynamic and Thermal Action.

Physical Data.

Furnace, Boiler and Engine Efficiences.

It is very remarkable evidence, which seems to be never or little heeded, of the importance of the time-element in all industrial concerns, that in common engineering phraseology quantities of energy are most frequently measured and named by the product of a time-rate by a time. The rate is the time-rate of production or of transmission of energy, such as so many horse-power. One horse-power means 1,980,000 ft.-lbs. of energy per hour, so that say 5 horse-power hours mean 9,900,000 ft.-lbs. of energy. From the purely theoretical point of view nothing could be more wanting in mathematical elegance or apparent rationality than this process of first reducing the energy to a time-rate by dividing by a time and then multiplying once more by the same time. On the face of it the method appears inane. Yet "horse-power hours," "kilowatt hours," "watt-hours," etc., etc., are the technical terms in which quantities of energy are now most frequently stated by engineers. In a similar way quantities of current electricity are measured in "ampère-hours" instead of in "coulombs," the "ampère" being nothing more than the time-rate of passage of electric quantity. In this latter case one reason for using "ampère-hours" is that the ampères, or current strength, is easily read directly from a commonly used instrument, whereas the quantity is not easily measured by a direct method. Again, in using the phrase "watt-hours," the watts may be read direct from an instrument, or may be calculated as the product of the two readings from the volt-meter and the ammeter; whereas the more expensive and less common direct-reading "energy meters" are merely mechanical devices for integrating the product of the watts and the time.

On the other hand in engines and other dynamic machines, the horse-power is never read direct from the dial of an instrument; it

is always calculated by making two measurements of force and of speed by help of two instruments and combining these by multiplication. The mean driving force on the piston is measured and this is multiplied by the stroke; the product giving the energy delivered in one stroke. To get the horse-power the number of strokes per minute is to be measured, and this applied to the above product as a multiplier.

Thus it appears that energy is never a direct measurement, but it also appears that, in at least very many cases, it is a measurement that can be made without help of a time measurement—as in force multiplied by distance, weight multiplied by height. Yet horse-power-hours are the most used terms in which industrial and commercial men state quantities of energy: and this arises from the fact that the horse-power or time-rate is the thing of greatest commercial importance; is, therefore, the thing that becomes most familiar to the commercial mind; and thus becomes the base quantity in terms of which all other things concerning energetic action are thought of.

But although horse-power-hours is not the simplest definition of energy in terms of what pure science takes as its base units, viz. length, mass and time, it may very easily be shown to be not one whit less rational than the commonly used scientific definition (of mechanical energy), namely force multiplied by distance. It is just as true to say that force is the space-rate of exertion of power as that horse-power is its time-rate; so that the definition "energy equals force multiplied by distance" is equivalent to saying that it is—

$$\frac{\text{Energy}}{\text{Distance}} \times \text{Distance}.$$

The fact is that there is nothing *intrinsically* fundamental about length, mass, and time. Any physical quantities may be taken as the base elements of physical science, and the other kinds of quantities "derived" from them. According to the method by which physical science was first attacked, the expressions of what was then exactly known of the physical universe came out most simply in terms of length, mass and time. These methods have become rather stereotyped while scientific knowledge of physics has been so enormously enlarged that it is now quite legitimate to doubt whether, if the theory of science were now to be studied *de novo* and for the first time in the light of all our existing knowledge of physical fact, the fundamental dominance of the established sacred three elements would be maintained, or would even, perhaps, suggest itself.

Force itself is a time-rate, namely, that of transference of momentum ; and thus work done, which is of the same dimensions as energy, is—

$$\frac{\text{Momentum}}{\text{Time}} \times \text{Distance},$$

while horse-power is this divided by a time or—

$$\frac{\text{Momentum}}{\text{Time}} \times \text{Velocity}, \quad \text{or—}$$

$$\text{Momentum time-rate} \times \text{Motion time-rate},$$

that is, the product of two t'me-rates ; and in the first of these the quantity (momentum) whose time-rate is taken, is itself a time-rate, namely that at which mass is carried through linear space.

Horse-power being of dominating everyday importance in industry, it may thus be recognized that there is nothing essentially irrational in expressing energy as the product of horse-power and time.

One horse-power is a perfectly definite time-rate of production, or of transmission, of energy ; but as energy is unfortunately measured in Britain by a very large variety of units, and as, moreover, various time-units are found convenient to use in different circumstances, one horse-power is represented by correspondingly numerous numerical values. Table VII gives examples of such of these numerical values as are most useful in practical calculations. All these refer to one or other of three energy time-rates ; that is, the table gives only three really different energy time-rates, which are distinguished by three different fonts of type. The British Horse-power is printed in ordinary Roman type ; the Metric Horse-power, which is about $1\frac{4}{10}$ per cent. less than the British Horse-power, is printed in Italics ; and the Kilowatt, which is 13·4 British horse-power and 1·36 Metric horse-power, is printed in Capitals.

The table is divided into four vertical columns, these giving the four numbers of units of energy of the kinds mentioned in the first column which must be produced or transmitted in the four time-units, Second, Minute, Hour, and Working Year of 2,700 Hours, to constitute one horse-power or one Kilowatt.

It is again divided into four sections. The first is marked Dynamic and General. It might be called Mechanical, except that the second section is also equally mechanic in the kind of units employed. This first section employs the fundamental mechanical units of force, distance, and time ; and its second title " general " is justified by the

TABLE VII

Numerical Values of ONE BRITISH HORSE-POWER, or ONE METRIC HORSE-POWER, or ONE KILOWATT.

British Horse-power printed in Roman type.
Metric Horse-Power „ *Italic* type.
KILOWATT „ CAPITAL type.

	Energy Unit.	Per Second.	Per Minute.	Per Hour.	Per Working Year of 2,700 Hours.
Dynamic and General	Mile-tons	$46{\cdot}5 \div 10^6$	·00279	0·1674	452
	Mile-pounds	0·104	6·25	375	$1{\cdot}012 \times 10^6$
	Foot-tons	0·2455	14·73	884	$2{\cdot}386 \times 10^6$
	Foot-pounds	550	33×10^3	$1{\cdot}98 \times 10^6$	$5{\cdot}346 \times 10^9$
	Inch-tons	2·946	176·8	$10{\cdot}61 \times 10^3$	$28{\cdot}647 \times 10^6$
	Inch-pounds	6600	$0{\cdot}396 \times 10^6$	$23{\cdot}76 \times 10^6$	64152×10^6
	Kilogram-metres	*75*	*4500*	270×10^3	729×10^6
Hydraulic or Fluid.	Water. Cubic feet × feet fall	8·813	528·8	31731	$85{\cdot}67 \times 10^6$
	Water. Cubic metres × millimetres fall	*75*	*4500*	270×10^3	729×10^6
	Fluid. Gallons flow × lbs. sq. inch pressure	23·83	1430	85,800	$231{\cdot}66 \times 10^6$
	Fluid. Cubic feet flow × lbs. sq. inch pressure	3·82	226	13,750	$12{\cdot}375 \times 10^6$
	Fluid. Cubic feet flow × ins. pressure water gauge	105·75	6346	$0{\cdot}38 \times 10^6$	$1{\cdot}026 \times 10^6$
Thermal	Pound Fah. degree heat units	0·707	42·42	2545	$6{\cdot}8715 \times 10^6$
	Kilogram Cent. degree heat units	*0·1757*	*10·54*	*632·55*	$1{\cdot}7078 \times 10^6$
	Steam pounds evap. from and at 212° F.	$0{\cdot}73 \div 10^3$	0·0438	2·63	$0{\cdot}71 \times 10^6$
	Fuel burnt, pounds at 14,000 heat units per lb.	$50{\cdot}5 \div 10^6$	$3{\cdot}03 \div 10^3$	0·1818	491
	Fuel burnt, pounds at 12,725 heat units per lb.	$5{\cdot}55 \div 10^6$	$3{\cdot}33 \div 10^3$	0·2	540
	Oil burnt, pounds at 20,000 heat units per lb.	$35{\cdot}3 \div 10^6$	$2{\cdot}12 \div 10^3$	0·1272	343·6
	Lighting gas burnt, cubic feet at 636 heat units per cub. ft.	$\frac{1}{900}$	$\frac{1}{15}$	4	10800
	Producer gas burnt, cub. ft. at 127 heat units per cubic foot	$\frac{1}{180}$	$\frac{1}{3}$	20	54000
Electrical Horse-power	Joules Watt. seconds = 10^7 ergs = 10 meg. ergs	746	44760	$2{\cdot}685 \times 10^6$	7251×10^6
KILOWATT	JOULES	10^3	6×10^4	$3{\cdot}6 \times 10^6$	$9{\cdot}72 \times 10^9$
	FOOT-POUNDS	737	$4{\cdot}423 \times 10^4$	$2{\cdot}654 \times 10^6$	$7{\cdot}166 \div 10^9$

fact that it has become the common habit of scientists to use the mechanical unit as the common measure in which to express the "equivalents" of all other kinds of energy.

The second section is marked Hydraulic, and the measures here given include those most conveniently used in pneumatic work. A more strictly accurate and inclusive title would, therefore, be Fluid.

The third is entitled Thermal, and relates to the passage of energy in the shape of heat by conduction, and all other means of transmitting heat without changing the heat energy into other forms of energy; and also to the generation of heat energy by combustion of fuel, or by any other chemical or like means, as well as to its generation by frictional, viscous, magnetic, and electric actions. It should be understood that when heat is transmitted by conduction between two masses, other energetic interactions between the same masses very commonly accompany the heat transmission. One such important accompanying action is often that of mechanical work. This mechanical work is usually accompanied by disappearance of heat, but this disappearance of heat due to the mechanical work is not to be included in the reckoning of the *conductive heat* horse-power. This latter is reckoned exclusively from the quantity of heat transmitted per unit of time as heat without transformation to other energetic forms.

The fourth section of the Table VII refers to Electrical horse-power and Kilowatt. One Kilowatt-hour is called in Britain a Board of Trade Unit, written B.T.U., and is equal to 3·6 million Joules, or $3{\cdot}6 \times 10^{13}$ Ergs., or 2,654,000 ft.-lbs.

The term horse-power has not been so far commonly adopted in connection with pure conductive transmission of heat-energy. Its use was recommended by the present writer many years ago in order to clarify ideas in regard to the horse-power of boilers. It is, indeed, strange that physical scientists have not introduced any name either for time-rate of transmission of heat or for the unit time-rate of such transmission. In the absence of any other adopted name and unit, and in view of the undesirab'lity of adding to the already excessive variety of our units, it seems proper to adhere to the writer's original proposal to use "Heat horse-power," or "Thermal horse-power," and to apply the same unit as used in dynamics, namely, 33,000 ft.-lbs. per minute, which means the same as 42·42 Brit. Heat Units per minute, or 2,545 of the same units per hour.

The horse-power of boilers is generally rated by reference to the average indicated engine cylinder horse-power which they may be

expected to supply with steam. This is extremely vague and indefinite, depending to a very large degree upon the character of the engine apart from that of the boiler. Thus boiler makers got into the habit of rating their productions at so many square feet of heating surface per horse-power; 9 to 12 sq. ft. per h.p. being needed. But as this measure is also indefinite, being a mere measure of the geometrical size of the boiler and taking no account of the disposition and efficiency of its heating surface, nor of the activity of the water circulation and of the hot-gas lateral circulation in the flues, the newer rating is by evaporation, allowing 30 lbs. evaporation per hour per horse-power. Both these ratings, viz. by extent of heating surface and by water evaporation, are evidently rough, and somewhat barbaric, efforts at measurement of the time-rate of passage of heat into the water. Why not simply take this heat entering the water per hour measured in ordinary dynamic or thermal units and divide by the ordinary standard measure of a horse-power, namely, 1,980,000 ft.-lbs. per hour?

There are, however, really four actual horse-powers concerned in the activity of a boiler and its furnace. There is (1) the rate at which heat is produced in the furnace by the combustion of the fuel; and this may be termed Furnace Heat Horse-power. There is (2) the smaller rate at which the larger portion of this heat, so generated, is got into the water through the heating surfaces; and this may be called the Effective Boiler Heat Horse-power. It might be called the Conductive Boiler H.H.P., except that much of the heat is transferred from the incandescent fuel and the hot flue gases to the heating surface by radiation and not by conduction. It is, however, transmitted through the heating surface plates and from them to the water, wholly, or almost wholly, by conduction.

The ratio between this Effective Boiler Heat Horse-power and the Furnace Heat Horse-power is the Heating Surface Efficiency. Both of them are purely thermal.

In the expansion of the water into steam by evaporation actual mechanical work is done, measured by the product of the intensity of pressure and the volume of steam generated. The exact volume to be taken in this measure is the excess of that of the steam over that of the water; but this differs from that of the steam alone by a percentage smaller than that of the errors in the best boiler tests. The time-rate at which this mechanical work is done—it is very important to note that the work is done inside the boiler—is a mechanical horse-power, and is a proper measure of the boiler horse-power. This measure is the absolute steam pressure per square inch multi-

plied by the volume of steam in cubic inches produced per hour and divided by 23·76 million, this being the number of inch-lbs. of work per hour that makes one horse-power. This is the proper scientific measure of the Boiler Mechanical Horse-power. It is the third horse-power involved in the boiler activity.

Lastly, in this same evaporation there is involved the production of a further amount of steam resilience, which is not spent within the limits of the boiler in doing mechanical work. The work included in the horse-power of the previous paragraph is equally due to the creation of resilience; but the resilience involved in it is immediately spent in doing mechanical work inside the boiler. The work so done, however, does not exhaust the resilience. A further portion remains which is, or may be, spent in mechanical work in the cylinder of an engine. That done primarily inside the boiler is transmitted forwards through the column of steam lying in the steam-pipe and is delivered to the cylinder-piston. The remainder is done primarily in the cylinder upon the piston by the further exhaustion of the steam resilience during the "expansion," that is, out of contact with the steam in the boiler. Both are spent finally on the piston, and both are due to resilience generated by the entrance of heat energy into the water. The total resilience thus generated is the sum of the work done inside the boiler and that which may be subsequently done outside the boiler by the *complete exhaustion* of the resilience gained inside it. This subsequent work is that measured under the *adiabatic* expansion curve from the absolute pressure zero with the expansion carried out down to zero pressure. The expansion curve must be taken adiabatic because the matter in hand is the measure of the boiler activity, and, therefore, only that work is to be measured that is possible on account of the energy supplied inside the boiler. Thus inside the boiler the mechanical result is not only the mechanical work done in the boiler, but, in addition to this, the development of resilience which, when subsequently spent, also gives mechanical work. The rate at which the two resilient effects together are produced may be called the Boiler Resilience Horse-power. It is greater than the Boiler Mechanical Horse-power as defined above, and includes it: the Mechanical is one part of the Resilience Horse-power.

The ratio between the two results from the character of the adiabatic curve. If dry saturated steam expanded adiabatically down to zero pressure always along a curve such as Rankine prescribed, namely, with the pressure decreasing always in inverse proportion to the $\frac{100}{88}$ths power of the volume, then the ratio of the total resili-

ence to the Mechanical Boiler power would be $8\frac{1}{2}$. It is, however, now known that the index of the expansion curve of steam does not remain constant throughout any long range of expansion.

The following Table VIII shows how rapidly the ratio changes with the index.

TABLE VIII

Ratio of Total Resilience to Internal Mechanical Horse-Power in Boilers or values of $\frac{a}{a-1}$.

Index of Expansion Curve assumed constant down to Zero Pressure . . . . =	1·1	1·15	1·2	1·25	1·3	1·35	1·4
Ratio of Total Resilience to Initial Product of Pressure by Volume	11	$7\frac{2}{3}$	6	5	$4\frac{1}{3}$	$3\frac{6}{7}$	$3\frac{1}{2}$

Diagram (Fig. 8) shows graphically the same variation of ratio. Starting from highly superheated steam in the boiler, the index is usually taken a little over 1·3, and the ratio would be $4\frac{1}{3}$. Incomplete expansion, back pressure, clearance volumes, and the steam-heating of the cylinder walls, prevent much of this resilient power being possibly developed in the engine cylinders; and various economical reasons, explained fully in subsequent chapters, make it undesirable to develop so large a proportion as is mechanically possible. This incomplete development is, however, the fault of the engine, and is not to be debited against the boiler, which has its own burden of inherent inefficiencies to bear.

The physical properties of water and saturated steam most involved in thermodynamic action are given in the diagrams in Figs. 9A, 9B and 10. In Fig. 9A they are coordinated with a temperature (Fahr.) scale, and in Fig. 9B with the temperatures in Cent. degrees. In Fig. 10 the same information is given coordinated with an (absolute) pressure scale in lbs. per square inch.

Our knowledge of these properties, so far as it is accurate, is chiefly due to Regnault's experiments. The direct and calculated results of these and later experiments have been set forth in the form of numerical tables. These tables are deficient in two respects;

(1) Although they contain the elements from which all quantities appearing in steam calculations can be reckoned, still this reckoning often entails the loss of considerable valuable time. For practical purposes we want a larger number of tables giving these elements combined in a larger variety of useful forms. (2) The accurate use of the numerical tables involves troublesome interpolation.

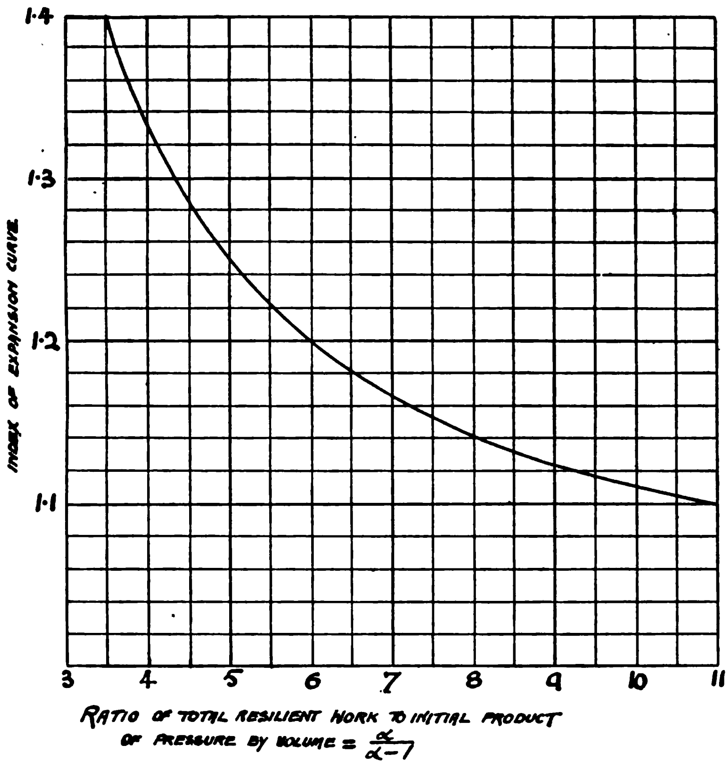


Fig. 8.

The interpolation actually always adopted is linear (i.e. by simple proportion); any other would use up too much time. But for very few of the tables is linear interpolation at all exact; indeed, when the interpolation is to be only linear, it is little less than absurd to tabulate the quantities and their differences with five or six significant figures, when the difference is from $\frac{1}{80}$ to $\frac{1}{10}$ even of the quantity itself. By reference to the curves on the diagrams (Figs. 9 and 10) it is easy to obtain any number of illustrations of the degree of error resulting from linear interpolation. To do so one has only to lay a

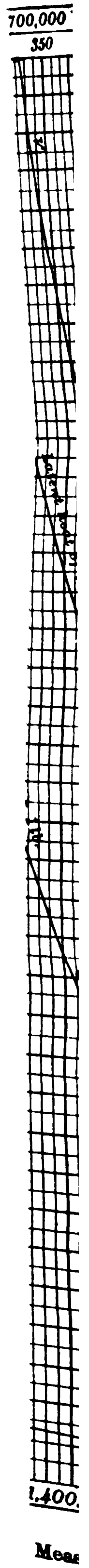
straight edge between two points on the curve corresponding to two successive values in the numerical tables, and note the deviation of the straight line from the curve. The interpolation in English tables is peculiarly inconvenient, because these are usually tabulated for successive temperatures, 9°F., 18°F., or 27°F. apart. This results from the tables being simple translations from the French, the 9°F., 18°F., and 27°F. corresponding to the much more convenient intervals 5°C., 10°C., and 15°C. Again, it is not with a knowledge of the temperature, but much more commonly of the pressure, that one refers to the tables; and the tabulated pressure having quite odd fractional parts, the interpolation becomes still more awkward.

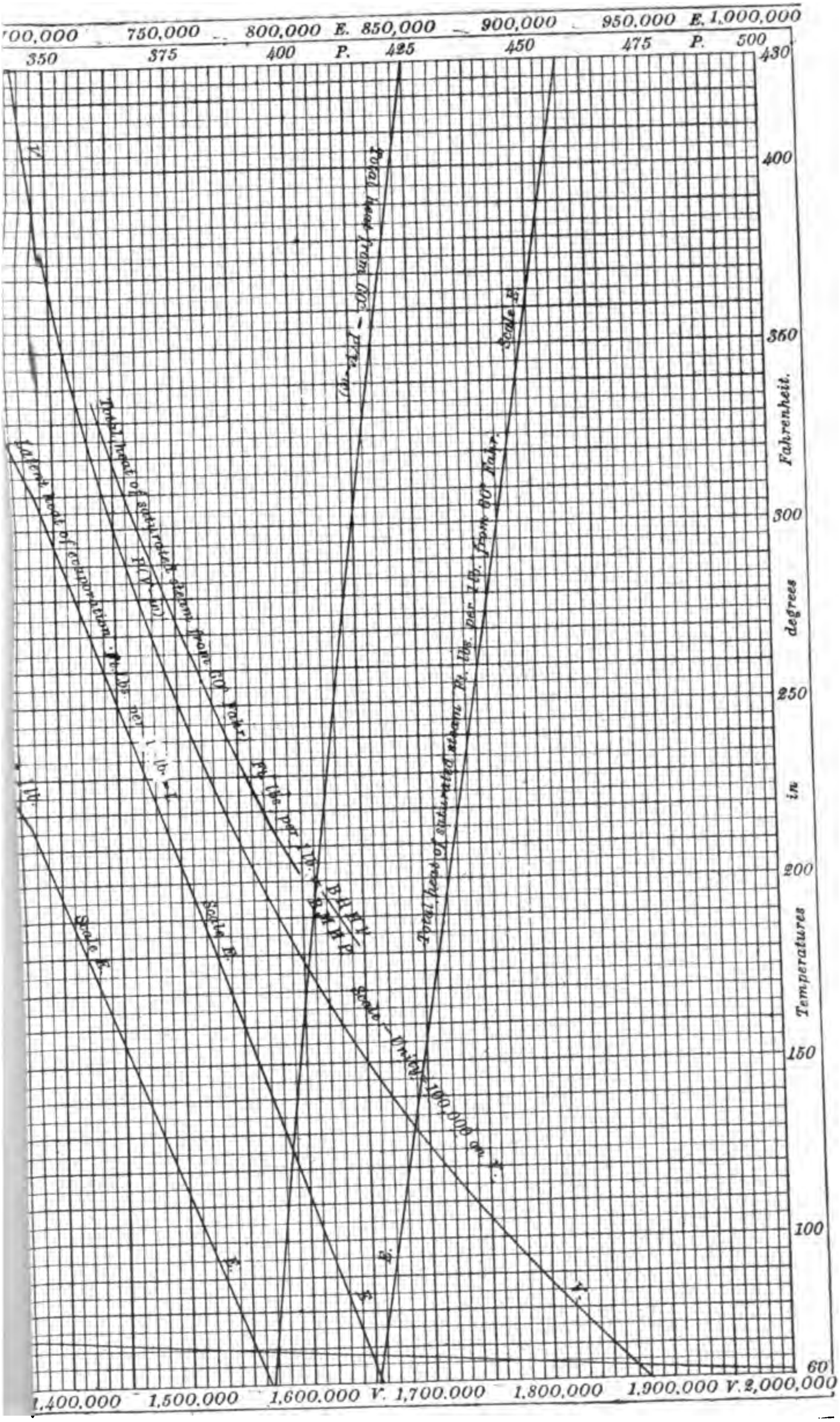
The employment of fairly drawn curves instead of numerical tables does away at once with all the inconvenience and inaccuracy of interpolation, and in other ways saves a world of trouble. The saving of trouble in these matters is of great scientific importance; because those acquainted with practical office work will readily acknowledge that the necessity of employing intricate and very tedious calculations simply prohibits the use of the method involving them, from sheer lack of time and weariness of spirit. The curve is continuous, and the result desired is read off directly, without error, and without calculation.

The advantage of a curve over a table of numbers in enabling one to grasp the general law of variation is well and widely understood. The present writer considers it a further important advantage that the impossibility of reading from curves with excessive minuteness prevents those engineers who use them from making ill use of their time by dealing with absurdly long strings of figures. The curves shown on the diagrams have been accurately reduced from others of larger size, but the scale of the engraving is sufficient for all ordinary purposes. A very few of these curves have been published in such books as Rankine's *Steam Engine*, but to so small a scale as to render them rather illustrations of the general law, than accurate tables of reference for the various exact quantities.

In the first diagram twelve distinct curves are given, corresponding to twelve complete and continuous tables ranging from 60°F. to 430°F.

There is only one vertical scale, viz., that of temperatures, in Fig. 9A. There are, however, three horizontal scales for the various quantities tabulated. One of these is marked P, and is used for the pressure curve. A second scale is marked V, and is




700,000
750,000
800,000 E. 850,000
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950,000 E. 1,000,000
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375
400 P. 425
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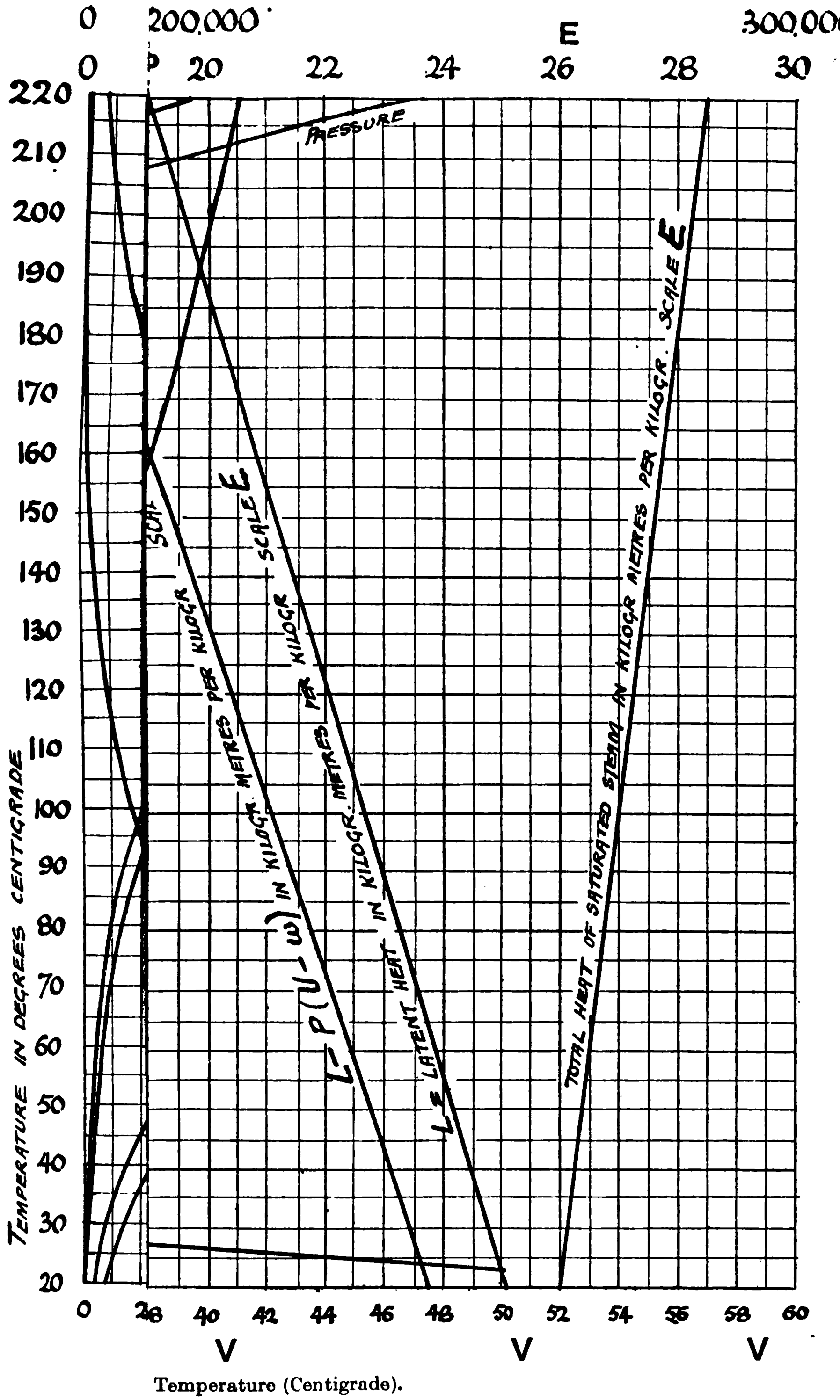
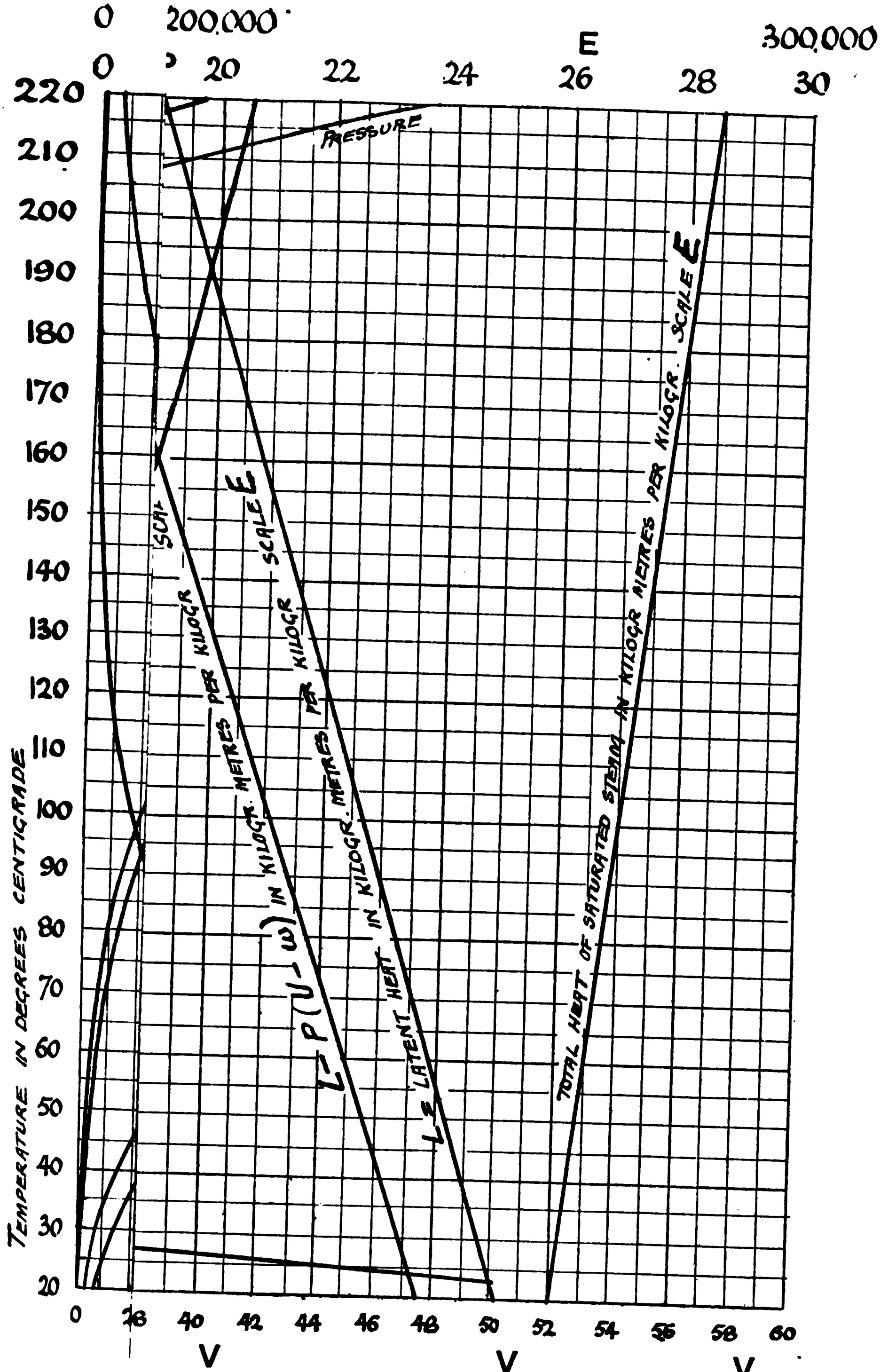
475 P. 500
430
400
350
300
250
200
150
100
60
Temperatures in degrees Fahrenheit.
Total heat from 60° Fahr.
Scale E.
Total heat of saturated steam Ft. lbs. per 1 lb. from 60° Fahr.
Total heat of saturated steam from 60° Fahr.
Scale E.
Scale E.
Scale V. Unit = 100,000 on V.
V.
E.
E.
E.
1,400,000
1,500,000
1,600,000 V. 1,700,000
1,800,000
1,900,000 V. 2,000,000

Measures.


220
210
TEMPERATURE IN DEGREES CENTIGRADE



Temperature (Centigrade).



Temperature (Centigrade).

used chiefly for the volume curve. Most of the other curves represent quantities of energy—heat energy and mechanical energy spent in doing mechanical work. These quantities are all given in foot-pounds, the foot-pound unit being more convenient for engineering calculations than the "British heat unit." The third scale, marked E, is that to which these various quantities of energy are plotted. Each curve has printed along its length, at several points, the letter indicating the horizontal scale to which it is to be read. In Fig. 9B the same quantities are expressed in metric measures.

The twelve curves or graphic tables are the following:—

1. Total Heat of Water from 60°F.—This is the heat needed to raise 1 lb. of water in temperature from 60°F. to any other temperature read off on the vertical scale. It is reckoned from 60°F. because one may always without serious, or almost without appreciable, error take this as the average feed water temperature, unless the feed be from a hot well. The curve is nearly, but not quite, a straight line; the deviation being due to the slight variation of the specific heat of water.

2. Pressure of Saturated Steam.—This is given in pounds per square inch, and is to be read to Scale P. The diagram runs down to 60°F. for the sake of condenser pressures. At 60°F. the pressure is only $\frac{1}{4}$ lb. per square inch. Below 200°F. the pressure curve to Scale P slopes so very little, and the ordinates are so small, that in order to enable the pressures corresponding to different low temperatures to be accurately distinguished, this part of the pressure curve is drawn out a second time, with the horizontal scale increased ten times. Of course, the Scale P can still be used; but, for example, 25 on scale P must be read as meaning 2·5. The full-line pressure curve is plotted directly from Regnault's experiments. The dotted curve represents the author's approximate formula $p=\{\cdot 005644\,t+\cdot 4360\}^{\frac{100}{18}}$ where t is the temperature on the ordinary Fahrenheit scale. The three constants of this formula are so chosen as to make the curve coincide with Regnault's curve at $\frac{6}{10}$ lb., 45 lbs. and 300 lbs. per square inch, and to give at 212°F. the pressure about $\frac{3}{8}$ lb. per square inch too high; it being considered that for engineering purposes it is more important to get a coincidence at low condenser pressures, at medium and very high boiler pressures, than at 212°. In using these graphic tables, however, readings should always be taken from the full in preference to the dotted curve. The pressures tabulated here are "absolute," i.e. from zero (not in excess above atmospheric pressure).

3. Specific Volumes of Saturated Steam.—These are given in cubic inches per pound of steam; and the upper part (from 212°F. to 430°F.) is drawn out a second time with horizontal ordinates increased tenfold to make it more readable. For this enlarged curve 100,000 on Scale V must be read to mean 10,000 cubic inches per pound.

4. Latent Heat of Evaporation of Steam.—Scale E gives this in foot-pounds of heat energy per pound of steam. It is nearly but not quite a straight line.

5. Total Heat of Saturated Steam from 60°F.—This is read to Scale E in foot-pounds per pound of steam. It is reckoned from 60°F., as this is the average feed temperature. Its horizontal ordinate is the sum of those of 1 and 4, and it is, therefore, also very nearly a straight line.

6. $P(V - w)$.—The so-called latent heat tabulated in 4 is the total heat that has to be supplied to effect the evaporation; but not the whole of this is absorbed as heat, because during the evaporation the steam does mechanical work in expanding against resistant surrounding bodies, and part of the heat energy supplied is immediately spent in doing this work. The amount of this work in foot-pounds per pound of steam evaporated is tabulated by the $P(V - w)$ curve. It is plotted out twice: first, to Scale E, in order that its amount may be readily comparable by eye with the latent and total heats; and secondly, to ten times that scale, in order that its amount may be accurately read off, this being, perhaps, the most important of all the curves from a mechanical point of view. It runs from about 45,000 foot-pounds when the steam pressure is about ½ lb. per square inch, up to about 66,000 foot-pounds at 300 lbs. per square inch pressure. Its rate of increase decreases only slightly towards the higher temperatures and pressures. 66,000 being just double 33,000, it is useful to remember that one boiler mechanical horse-power is afforded by the evaporation of ½ lb. of water per minute, or 30 lbs. per hour, to steam at 300 lbs. per square inch absolute pressure. This quantity is found by multiplying P, the pressure, by the excess of the volume of steam (V) over the volume of water (w).

7. $\{L - P(V - w)\}$.—This is called the *real latent heat.* It is the apparent latent heat (L) minus the quantity of heat not really absorbed, but spent in doing mechanical work during evaporation. Scale E gives it in foot-pounds per pound of steam evaporated. It is the difference between Curves 4 and 6.

8. $\left\{\dfrac{L - P(V - w)}{V}\right\}$.—This is the last quantity calculated

per cubic foot of steam evaporated instead of per pound, as in Curve 7. It is often found more convenient to have it per unit of volume.

9. TOTAL HEAT FROM 60°F. *less* P (V − w).—This, also to scale E, is that part of the total heat which is really absorbed, the rest being spent in doing the evaporative mechanical work. It is the difference between Curves 5 and 6.

10. RATIO OF P (V − w) TO TOTAL HEAT FROM 60°F.—This is the proportion of the whole heat spent in raising the steam from feed temperature 60°F. that is developed in the form of mechanical work done during evaporation. It is the $\frac{\text{B.M.H.P.}}{\text{B.H.H.P.}}$, or boiler mechanical horse-power divided by boiler heat horse-power. This is a numerical ratio. To avoid multiplying scales, the volume scale is utilized for this curve, unity being represented by 10,000,000 on Scale V. Thus the ratio runs from ·053 at very low pressures up to ·073 at the top of the diagram.

11. RATIO OF TOTAL HEAT TO P (V − w), OR $\frac{\text{B.H.H.P.}}{\text{B.M.H.P.}}$—This is simply the reciprocal of the last quantity. It is often convenient to be able to find this reciprocal by reference to a table such as this; indeed, this quantity is of more frequent practical use than 10. It is to be read by help of Scale V, 100,000 on that scale representing unity. Its value runs from 18·8 to 13·8.

12. TOTAL HEAT FROM 60°F. DIVIDED BY LATENT HEAT AT 212°F. —In tests of boilers the actual evaporation is usually reduced to an equivalent evaporation of feed water taken at 212°F. into steam at same temperature. This reduction is performed by finding the ratio of the actual heat supplied in the boiler to each pound of water (taken here at 60°F. feed temperature) to convert it into steam to the latent heat of evaporation of 212°F., and multiplying the actual evaporation in pounds of water per pound of fuel by this ratio. For convenience in making this reduction this ratio is tabulated by this last curve. It is read on Scale V, 1,000,000 on that scale representing unity. The ratio runs from 1·11 to nearly 1·23.

In Fig. 10 the same properties of water and steam are plotted to a pressure scale. No temperature curve appears on this diagram. The coordination of temperature with pressure is given both upon Fig. 9 and on the three sections of Fig. 12. No further explanation of the diagram is needed; except to note that for most practical work this diagram will be more frequently read from than Fig. 9. Later in this book will be found many other diagrams giving useful

quantities in respect of steam and gas action calculated in accordance with economic laws to be demonstrated.

In Fig. 11 is given the so-called Entropy Diagram of Water and Steam, both in saturated and superheated condition. The vertical ordinate is temperature, and the horizontal ordinate entropy. No use of entropy calculations is made in this volume ; but as many engineers prefer to work some questions in terms of entropy, and as no other accurate entropy diagram has been published elsewhere, the author has considered this a useful addition to this book.

Many years ago Mr. MacFarlane Gray gave the Institute of Mechanical Engineers a small-sized unscaled entropy diagram. More recently Mr. Sankey has published a carefully prepared diagram, which is, however, to too small a scale to be readily and accurately readable, and in the preparation of which no account is taken of the variation of the specific heat of water. This variation has an appreciable influence upon the results of calculation. It was taken into account in preparing diagram (Fig. 11), the main lines of which were plotted by the author fifteen years ago. For the present publication a few extra lines have been added. The diagram gives (1) the water line ; (2) the saturated steam line ; (3) equal pressure lines, which are horizontal straight lines coincident with the equal temperature lines inside (or to the left of) the saturated steam curve, but are rising curves outside this limit ; and (4) equal specific volume curves for mixtures of water and steam, and for superheated steam. The lines of equal temperature, and those of equal entropy are, of course, the horizontal and vertical scale lines of the squared paper.

The three diagrams in Fig. 12 give the adiabatic, or equal entropy, curves on the ordinary pressure-volume steam-diagram. As the expansion at the lower pressures is very great, the diagram, in order to be readable, is divided in three parts with three different volume scales. It does not carry the curves beyond the saturated steam line. The horizontal straight lines are lines of equal temperature, as well as of equal pressure ; and a temperature curve has been added to the diagram to enable one to " enter " the diagram with any specified temperature. The temperature scale for this pressure-temperature curve is printed along the top edge of the diagram, while the volume scales for the adiabatic curves are printed along the bottom edge.

The first section of this diagram runs through the pressure range 300 down to 20 lbs. per square inch absolute ; the second section from 20 down to 5 ; and the third from 5 down to 1 lb. per sq. inch.

The entropy difference between the successive adiabatic curves is $\frac{1}{10}$th of a Brit. Thermal unit divided by 1°Fahr.

Inside the saturated region, by help of Fig. 12, one can make exactly the same entropy calculations as those made with Fig. 11, while the scales are those with which engineers are more familiar. For practical work inside the saturation curve it will be found considerably more convenient than the much belauded $\theta\ \phi$ diagram of Fig. 11.

The diagrams in Figs. 13 and 14 are perhaps the most useful of any of these diagrams in respect of facilitating estimates of the power that may be expected from steam, gas and oil engines of any proposed designs. In each diagram there are three groups of curves. In each the horizontal ordinate is the ratio of expansion. It must be borne in mind that all three sets of curves must be used only in conjunction with "absolute" pressures, and not with pressures measured above any standard datum such as atmospheric pressure. Those of the first group are marked F, and give the ratio of final or "terminal" pressure at the end of each grade of expansion to the initial pressure. The ratio is plotted as a percentage, the vertical scale giving 100 as the initial pressure. These curves can be used either to find the terminal pressure for a given ratio of expansion, or to find the ratio of expansion required to reduce the pressure to a given terminal amount. They can also be used to find the results of compression, either the terminal pressure after a given ratio of compression, or the ratio of compression required to effect a given ratio of terminal to initial pressure.

The second set of curves is marked M, and from these are read the mean pressures (absolute) for each ratio of expansion as percentages of the initial pressure. The mean pressure here plotted is *not* that of the area under the expansion curve alone, but that of this area added to the area under the horizontal line showing the initial volume and pressure.

In both these sets of curves the first portions lie so close together, and cut through the vertical lines so obliquely, that it is difficult to read them accurately. They are, therefore, plotted out again to a horizontal scale ten times more open. This more open scale is figured along the line marked A A. Throughout their range these curves should be used in preference to the others. The vertical scale for both these two sets is the same, and is marked F M at each end of the diagram.

The third set of five curves in Fig. 13 is marked W. This gives the area referred to above, including that under the horizontal

line ; or rather it gives the ratio of this area to the area under the horizontal line alone, this latter being the value of the area without any expansion. The vertical scale for this set is different from that of the others, and is marked W at the two ends of the diagram.

These curves W rise rapidly at first and then more and more slowly, thereby showing clearly how greatly the advantage derived from further expansion falls off after very moderate expansion-grades have been passed. Each curve gradually becomes almost horizontal, rising asymptotically towards the extreme limiting height $\frac{i}{i-1}$, where i is the index of the expansion curve. This limit $\frac{i}{i-1}$ has already been plotted in Fig. 8 co-ordinated with i through the range 1·1 to 1·4. In Fig. 8 the index is called a because the diagram is constructed specially for adiabatic expansion, but, of course, the calculation of $\frac{i}{i-1}$ is the same whether for adiabatic or any other expansion. The formula $\frac{i}{i-1}$ for the limiting height applies only when i is greater or less than unity ; but the curve W on Fig. 14 for $i=1$ shows that for this intermediate index the general character of the law is precisely the same as for the higher and lower indices.

To find, by the help of these curves, the area of the "effective indicator diagram," or the "mean effective pressure," with given clearance and given back pressure, let

$c =$ ratio of clearance volume at one end of cylinder to volume swept through by piston in one stroke ;

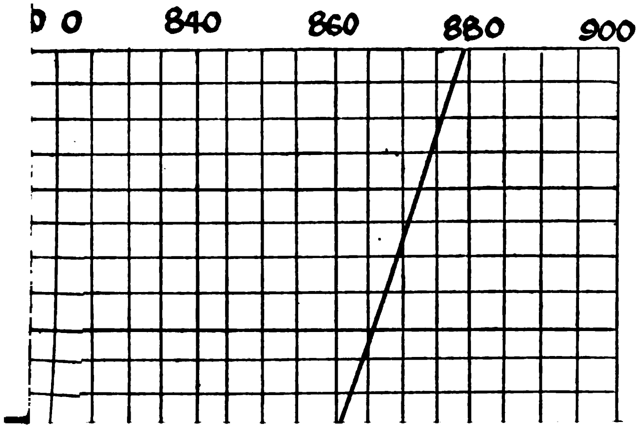
$\frac{1}{r'} =$ cut-off, i.e. ratio of volume swept by piston up to point of cut-off to whole volume swept through ;

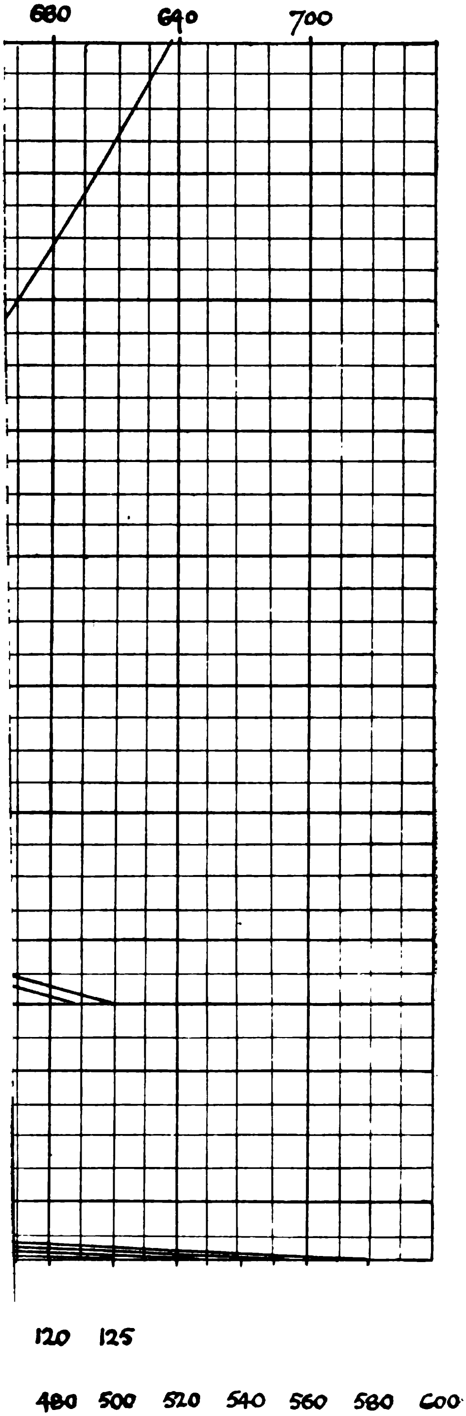
$b =$ ratio of mean back pressure to initial absolute pressure.

Then the real ratio of expansion is—

$$r'\frac{1+c}{1+r'c} = \text{say } r.$$

On the proper curve W find the ratio corresponding to r and call it w. Then the ratio of the area of the effective diagram to the rectangular area between the admission line and the line of zero pressure is—


0
840
860
880
900



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$$w\,(1+r'\,c) - r'\,(b+c),$$

and the ratio of mean effective pressure to initial absolute pressure is this divided by r' or

$$w\left(\frac{1}{r'}+c\right) - (b+c),$$

which may be calculated rather more simply by the form

$$m\,(1+c) - (b+c),$$

where m is the mean absolute pressure for the real ratio of expansion r obtained from the proper curve M divided by 100 so as to give it as an ordinary fractional ratio and not as a percentage.

Each of the expansion curves is drawn out to the formula—

Final pressure = Initial pressure $\div$ i^{th} power of ratio of expansion,

where i is different for each curve, its value being marked in the diagram.

If f and m represent the ratios of final and of mean to initial pressure, this gives,
for the (F) curves—

$$f = r^{-i}$$

and for the (M) curves—

$$m \quad \frac{1+\log r}{r} \text{ when } i=1$$

and

$$m = \frac{i - r^{1-i}}{(i-1)\,r} = \frac{1}{i-1}\left(\frac{i}{r} - f\right),$$

when i is either greater or less than 1.

When $i=1$, the logarithm to be used is the hyperbolic or Naperian logarithm.

Again, if w represent the ratio of the area of the diagram, including the initial horizontal line, up to the ratio of expansion r to the area under the initial horizontal line, we have for the (W) curves

$$w = m\,r = \frac{1}{i-1}\,(i - f\,r) = \frac{i - r^{1-i}}{i-1}$$

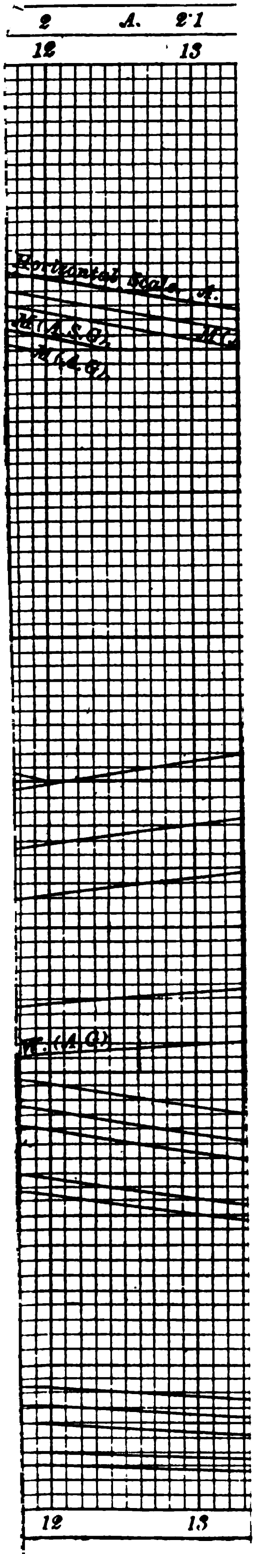
In Fig. 13 there are five curves in each of the three groups, F, M, and W. These are for particular values of the index, the highest and

lowest being for isothermal and adiabatic "perfect dry gas" expansion, and the three intermediate curves for steam supposed to expand along the Rankine curves, that is, with the indices stated by Rankine to be approximately correct. These are as follows—

1. Hyperbolic or isothermal expansion of a perfect gas, marked H . . . . $i=1$
2. Expansion of initially dry saturated steam with conduction of heat during expansion just sufficient to prevent condensation, marked S . . . . $i=1{\cdot}064$ $=\text{nearly } \frac{100}{94}$
3. Expansion of initially dry saturated steam without conduction of heat to or from the expanding steam, i.e. adiabatic expansion, marked A S . . . . $a=1{\cdot}135$ $=\text{nearly } \frac{100}{88}$
4. Adiabatic expansion of initially superheated steam, or steam gas, marked A S G . . . . . $a=1{\cdot}304$ $=\text{nearly } \frac{100}{77}$
5. Adiabatic expansion of perfect gas, marked A G . . $a=1{\cdot}409$ $=\text{nearly } \frac{100}{71}$

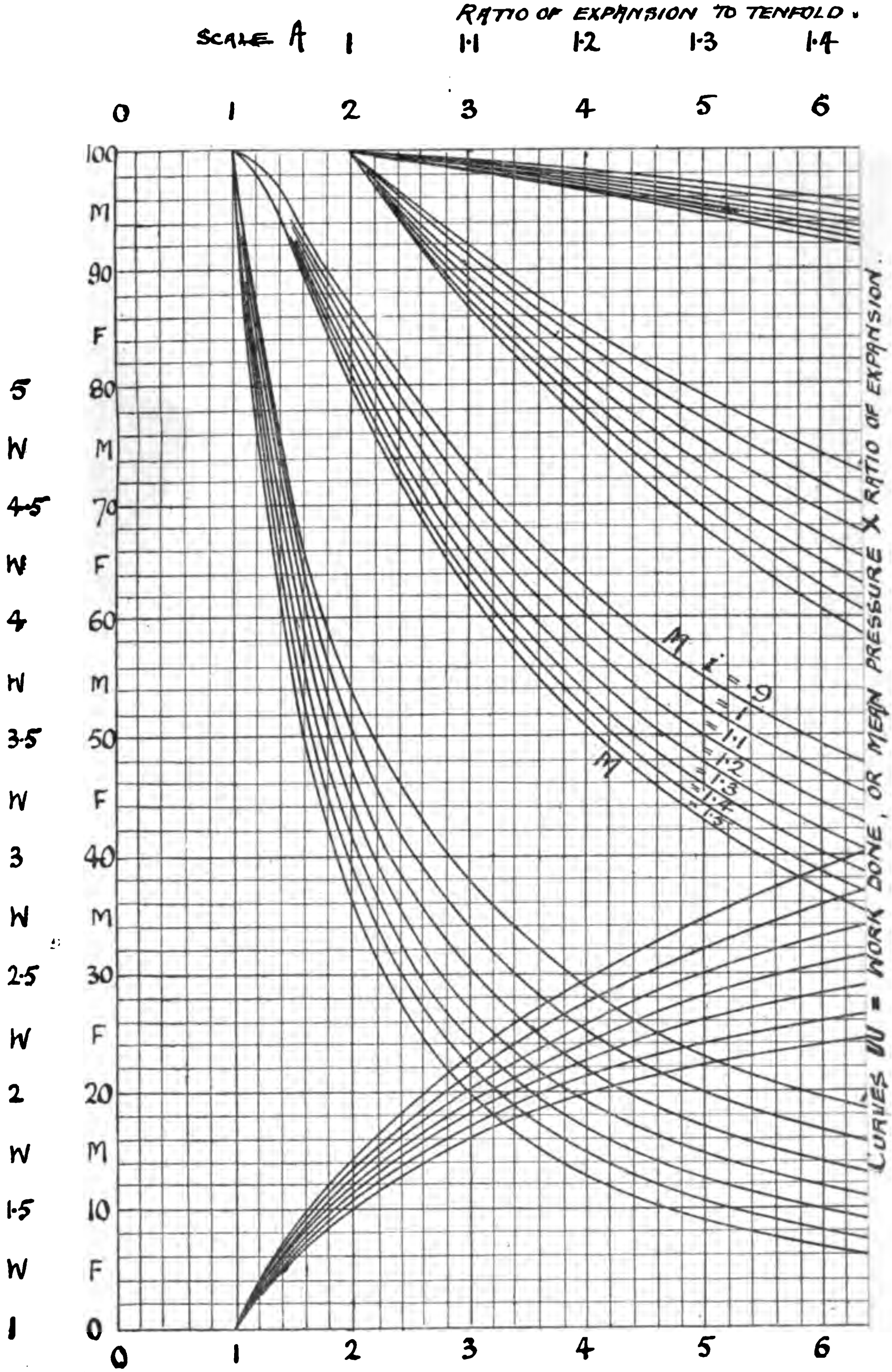
Regarding No. 3, it is known that mixtures of steam and water with various proportions of wetness start adiabatic expansion along curves with different indices, while it is also common knowledge that initially dry saturated steam, as also all mixtures with less than a certain degree of wetness, suffer partial condensation as the result of adiabatic expansion. It follows that Rankine's index 1·135 cannot be true throughout any such curve. The adiabatic curves for mixtures of water and steam plotted out in Fig. 12 are obtained directly from the experimental physical data without help of any assumption as to the algebraic character of the curves. The author has examined them somewhat exhaustively, and finds that they cannot be represented by any formula of the nature "pressure falls in inverse proportion to a constant power of the volume"; that is, for each curve, no such constancy of the index holds throughout any long range of the curve. Other investigators of this question have found the same result. Wherever possible, therefore, it is preferable to follow the curves of Fig. 12. It does not follow, however, that the above formula, which is very convenient in many ways for calculation, is worthless. In the first place gases follow such curves much more accurately than water and

A S=Adi[illegible]
A S G=Adia
A G=Adia



of Steam and Gases.

CURVES W = WORK DONE, OR MEAN PRESSURE X RATIO OF EXPANSION.


RATIO OF EXPANSION TO TENFOLD.
SCALE A
1 1·1 1·2 1·3 1·4
0 1 2 3 4 5 6
100 M 90 F 80 M 70 F 60 M 50 F 40 M 30 F 20 M 10 F 0
5 W 4·5 W 4 W 3·5 W 3 W 2·5 W 2 W 1·5 W 1
M
M
·9
=1
=1·1
=1·2
=1·3
=1·4
=1·5
CURVES W = WORK DONE, OR MEAN PRESSURE X RATIO OF EXPANSION.

steam mixtures; and in the second place if only the proper index be chosen in accordance with the given physical conditions of expansion, the curve may be taken as applying with sufficient accuracy to the expansion of almost any fluid whether liquid, vapour, or gas, through moderate ranges.

For this reason, in Fig. 14 the three groups of curves are drawn out for what may be looked upon as the standard, or leading, indices, ·9, 1, 1·1, 1·2, 1·3, 1·4 and 1·5. The curves in each group lie so well together that when any index lying between any two of the above is known to be the proper index, it is quite easy to read off the desired results from Fig. 14 by eye-interpolation. All the results similar to those already explained for Fig. 13 can be similarly read from Fig. 14, both for expansions and for compressions. For the compression curves in air-compressor cylinders, if water-spraying and piston water-jacketing be used to keep the rise of the curve as low as possible, the index has been found to be about 1·2. With cooling by simple cylinder water-jacketing, the index is generally somewhat above 1·3. In the expansion curve in compressed-air motor-engine cylinders the index will lie above 1·4 in consequence of the cooling by the cylinder walls. The index for the higher parts of the curve will be smaller than for the lower parts in the case of compression, and vice versa for expansion. Similarly during the period of expansion in steam engine cylinders, the index is higher for the first part of the curve immediately after cut-off than for the tail-part coming before the point of exhaust. During the first part there is more or less rapid cooling to the cylinder walls—less rapid with than without steam-jacketing—and during the last part there is very frequently recovery of heat from the cylinder walls. The result is that the last part has sometimes an index less than 1. For this reason the curve for $i = \cdot 9$ has been added to Fig. 14.

In an expansion curve the first parts of which fall according to higher indices than suit the lower parts, the ratio of final to initial pressure may, with fair approximation, be taken from a curve with uniform index judged to be a fair mean between the initial and final indices; but evidently the mean pressure under the compound curve is less than that under the mean uniform curve. Again in compression curves, where the initial index is also generally higher than the final index, the ratio of final high pressure to initial low pressure may be taken from a mean index curve, but the mean pressure will in this case be greater than that corresponding to this mean index.

Thus the variation of index always tells against mechanical

efficiency both in the expansion-curves of motor-engines, and in the compression curves of compressors, lessening the work done in the former case, and increasing the work to be done in the latter.

It is a fairly simple calculation to find the mean index that will give, by one uniform curve throughout the total range of expansion, the same final pressure as results from a succession of expansions by curves of given indices differing from each other. For instance, if the total ratio of expansion r be divided into five successive stages r_1 r_2, etc., so that $r = r_1 r_2 r_3 r_4 r_5$; and the indices in these stages be $i_1 i_2$, etc.; then the pressure at the end of the first stage in terms of the initial pressure is $r_1^{-i_1}$, and from this the pressure at the end of the second stage is $r_1^{-i_1} r_2^{-i_2}$. At the end of the fifth stage its ratio to the initial pressure is $r_1^{-i_1} r_2^{-i_2} r_3^{-i_3} r_4^{-i_4} r_5^{-i_5}$. Now if the same pressure were reached by a uniform curve of index i carried through the whole range of expansion r, this final pressure would be represented by r^{-i}. Equating this to the above; taking logarithms; and dividing by *log* r; we have—

$$i = \frac{1}{log\ r}\left(i_1\ log\ r_1 + i_2\ log\ r_2 + i_3\ log\ r_3 + i_4\ log\ r_4 + i_5\ log\ r_5\right)$$

where *log* r is the sum of the *logs* of r_1 r_2 r_3 r_4 and r_5.

If now the successive stages of expansion have equal ratios, that is, if

$$r_1 = r_2 = r_3 = r_4 = r_5,$$

then their *logs* are equal, and *log* r is 5 times the *log* of each. So that in this case

$$i = \frac{1}{5}(i_1 + i_2 + i_3 + i_4 + i_5),$$

or the index for the uniform curve is the arithmetic mean of the indices in the successive stages. Thus if the whole be divided into two equal stages, in each of which the ratio of expansion is the square root of the total ratio ($r_1 = r_2 = \sqrt{r}$), then i is half the sum of i_1 and i_2. If there be 3 equal stages, the ratio of each being the cube root of the total ratio, the uniform-curve index is ⅓rd the sum of the 3 stage indices. If there be 4 equal stages, each of ratio equal to the fourth root of the total, then the uniform-curve index is ¼ the sum of the 4 indices in the four stages. This simple rule holds good in whatever way the successive indices differ among themselves. It is a rule finding useful application in compound, triple, and quadruple, expansion engines, where the expansion curves

in the successive cylinders hardly ever follow the same index. For its strict application to such engines, however, the successive expansion curves should follow each other without break, or "sudden drop," of pressure. In such engines the expansions in successive cylinders are not in equal ratios; but, for example, if in a compound engine the expansion in the second cylinder is double that in the first, this second expansion may be divided into two equal stages, each equal to that in the small cylinder, the whole expansion being thus considered in three equal parts.

There is no similar simple rule whereby the mean pressure under the diagram formed by the successive expansion curves, may be directly calculated.

In Table XV are given the physical and chemical data, and the calculated results therefrom, concerning the combustion of fuel.

This table is divided into six horizontal sections. In the first of these are shown the results of the separate combustion of each of the important constituents of fuel, taken chemically pure, and burnt in that quantity of pure oxygen needed for complete chemical union. The second section is devoted to the combustion of the same fuel constituents in that quantity of dry air furnishing the same quantity of oxygen as just stated. The third gives the chemical analysis of a considerable number of solid fuels, including different varieties of coal, wood, and peat; and the results of burning these in air with no excess of oxygen. The fourth section treats liquid fuels similarly; and the fifth the commonest gaseous fuels. The sixth section shows the similar results when double the chemically necessary quantity of air is admitted to the furnace. For other ratios than 2 of air excess a comparison of this last section with the previous sections of the table will enable easy interpolations to be made.

The table is divided vertically in sections as follows: The first gives the main items of the chemical analysis of the fuel. The second gives the number of pounds of oxygen or of air used in the combustion. The third gives the volume in cubic feet occupied by this same weight of oxygen or air at atmospheric pressure and at 500° Fahr. absolute temperature, that is at 39° above the ordinary Fahr. zero of temperature, or 7° above freezing. At any other temperature the volume is greater in proportion to the absolute temperature, and this calculation is easy because 500 is a simple divisor. The fourth vertical section gives the chemical analysis by weight of the gases produced by the combustion. The total weight here is less than that of the fuel plus that of the air by the weight

of ash in the fuel, this yielding no gas. The fifth section is a column giving the total volume in cubic feet of these gases at atmospheric pressure and 500° Fahr. absolute temperature. The sixth is a column of total heat in lb. Fahr. heat units, developed by the perfect and complete combustion of 1 lb. of the fuel. The next column gives the specific heat per 1° Fahr. rise of temperature of the total gaseous product of combustion of 1 lb. of fuel; this being the average specific heat per lb. multiplied by the total weight of these gases. The average specific heat per lb. varies little from 0·25. In the next column is stated the quotient obtained by dividing the heat of combustion by this total specific heat, this number being well rounded off as the accuracy of the data does not justify a close calculation. These last-mentioned three columns are headed h, H, and T.

The quotient $\frac{H}{h}$ T would be the rise of temperature of the furnace gases above atmospheric temperature, if (1) the combustion were perfect and complete; (2) all the heat H were spent in uniformly heating these gases; (3) the average specific heat of the gases were constant up to furnace temperature; and (4) there were no dissociation of the gases at very high temperatures. The actual rise of temperature, however, always falls far short of this for many reasons. Among these the most important are the following. The combustion is never perfect and complete, although in skilfully managed gaseous-fuel furnaces it does not fall far short of completeness. The most influential item of incompleteness is the burning of some of the carbon to CO instead of to CO_2; and the proportion of CO in the chimney gases is often taken as a measure of the incompleteness. Nextly the water, or H_2O, produced by the burning of the hydrogen is evaporated, and this absorbs a corresponding amount of "latent" heat of evaporation, leaving the "sensible," or thermometric heat by so much the less. This deduction has, however, been already made in the figure H in Table XV. Nextly a large proportion of the heat generated is spent by radiation from the solid incandescent fuel, passing as radiant heat into the walls of the furnace without affecting the temperature of the gases. Lastly at very high temperatures some of the gases dissociate and the heat represented by their chemical union before dissociation is again absorbed as potential energy of chemical separation. Such dissociation does not take place if the furnace-gas temperature be kept down below certain limits by the other above-mentioned causes, or by a large excess of air.

Fortunately for tests, or estimates, of boiler efficiency, it is unnecessary to know the temperature of the furnace gases. It is difficult to obtain a reliable measurement of it, because any pyrometer, however trustworthy or the reverse, when introduced into the furnace, must be exposed to, and heated by, the radiant heat, and is thus not heated by conduction from the gases alone.

If the gases be delivered into the base of the chimney at $t°$ Fahr. above atmospheric temperature, then for each 1 lb. of fuel burnt, an amount of heat equal to $h\,t$ is carried into the chimney. If ϵ be a factor making allowance for incompleteness of combustion, so that the heat actually generated per 1 lb. of fuel is $\epsilon\,H$; then $(\epsilon\,H - h\,t)$ is the heat actually left behind in the boiler and its surroundings. This divided by H is the furnace and flue efficiency. This efficiency-coefficient is, therefore—

$$\epsilon - \frac{h}{H}\,t = \epsilon - \frac{t}{T}.$$

Besides $h\,t$, a further deduction should in strictness be made for work done by the gases flowing up the chimney. The volume that flows into the chimney is greater than that flowing into the furnace, chiefly on account of the greater temperature. This excess of volume multiplied by atmospheric pressure measures mechanical work done by the gases in the furnace and flues, and an amount of heat equal to this is spent in doing this work and does not appear as thermometric heat in the furnace-gases. This correction is, however, negligibly small. Atmospheric pressure is about 14·7 lbs. per sq. inch, or 2,100 lbs. per sq. foot, and each cubic foot of the above *excess* of volume therefore represents 2,100 ft.-lbs. $= \frac{2{,}100}{772} = 2{\cdot}7$ Brit. heat units of energy. The excess of volume per lb. of fuel is small, so that this energy, which is usefully spent in helping the chimney draught, is only a fractionally minute percentage of the whole.

The diagram in Fig. 16 puts the general average results of the two formulas $(H - h\,t)$ and $\left(\epsilon - \frac{t}{T}\right)$ into an easily readable form very convenient for reference. It is applicable only to coal fuel or to any fuel which gives a ratio $T = \frac{H}{h}$ not very different from that for coal. Reference to Table XV shows that 14,000 Br. Th. Units is a fair rounded-off average for British coals; while 3 is a corresponding average for h if the coal be burnt in just that quantity of air

needed for complete chemical union. This quantity of air would be 11·6 lbs. if the fuel were pure carbon; but as a considerable proportion of hydrogen exists in all fuels, while 1 lb. of hydrogen needs 34·8 lbs. of air to burn it completely, we may take $11\frac{3}{4}$ as the average ratio by weight between the air needed and the coal burnt. The specific heat of air is ·238, and $11\frac{3}{4} \times \cdot 238 = 2\cdot 8$. Thus in the lowest section of the Table XV, where double quantity of air is admitted, the average of h for coal-fuel is about 2·8 greater than in the section lying above it, where no excess of air is admitted. In the following diagrams and tables n expresses the ratio of air actually admitted to that chemically required; and, as $3 - 2\cdot 8 = \cdot 2$, a general average value of h for coal burnt in any quantity of air. is—

$$\text{Coal-average } h = (\cdot 2 + 2\cdot 8\, n) \text{ per lb. of fuel.}$$

With perfect combustion, the available heat, i.e. the whole heat generated less that carried to the chimney is, for average coal—

$$\text{Available Heat} = \text{H} - h\,t = 14{,}000 - (\cdot 2 + 2\cdot 8\, n)\, t\,;$$

while the "efficiency," or ratio of this to H, is—

$$\left.\begin{array}{r}\text{Available Heat Efficiency, or}\\ \text{Furnace and Flue Efficiency}\end{array}\right\} = 1 - \frac{\cdot 2 + 2\cdot 8\, n}{14{,}000}\, t.$$

These two—heat and efficiency—differ only in that the one is 14,000 times greater than the other; and therefore they are both represented by the heights of the same straight lines in Fig. 16, the scale for the available heat being the left hand vertical scale, while that for the efficiency is on the right hand of the diagram. The horizontal ordinate is t, or the excess of chimney over air temperature. The successive straight lines are for different values of n, that for $n = 1\frac{1}{2}$ being dotted. For any chimney temperature and any n, the values of $(\text{H} - h\,t)$ and of $\left(1 - \frac{t}{\text{T}}\right)$ are read off the respective scales as the height of the given n line at the given horizontal distance t.

So long as the fuel is coal, and so long as the quantity of air admitted per lb. of coal burnt remains unchanged, the value of h alters very slightly with different heats of combustion, so that

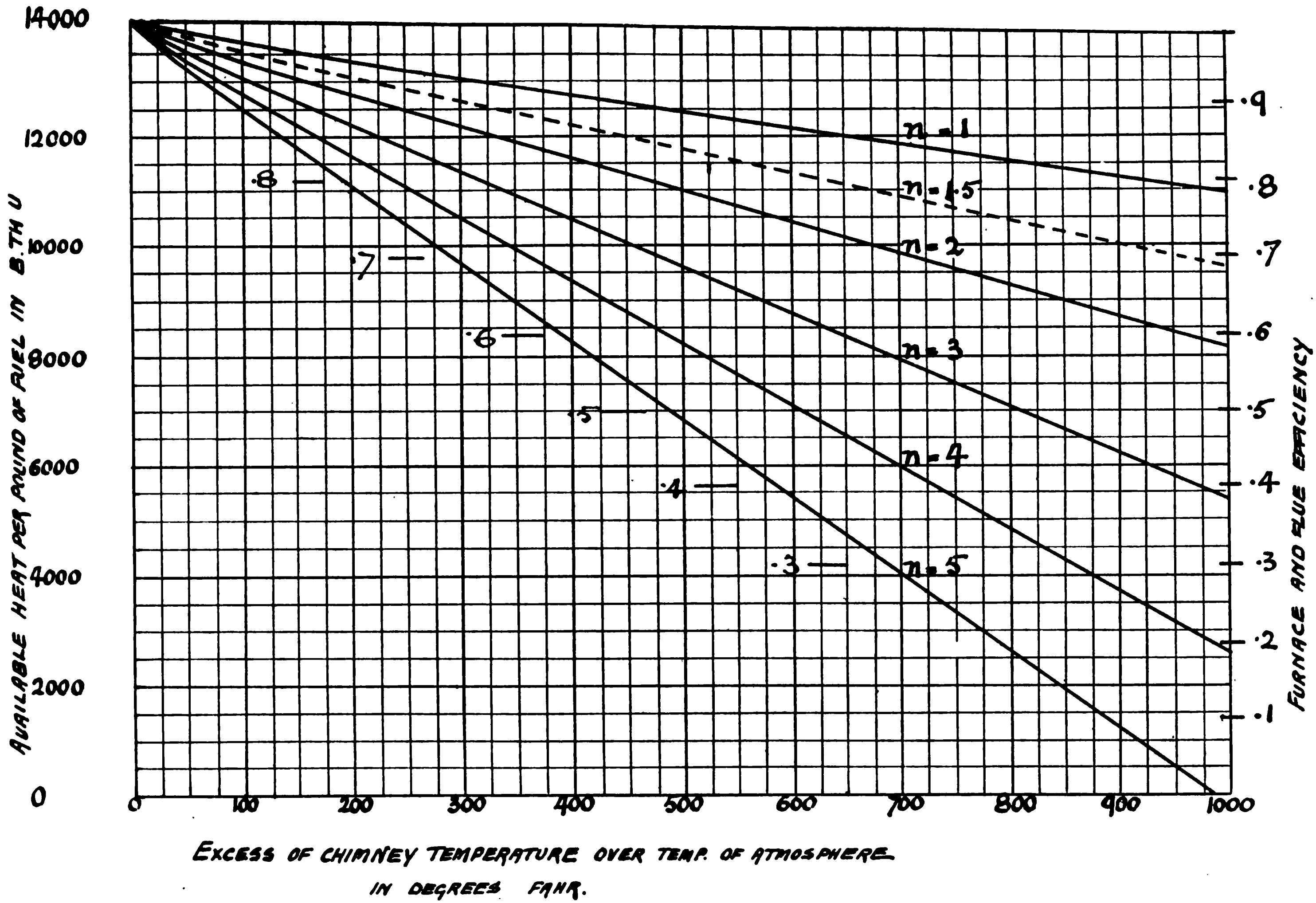


Fig. 16.—Available Heat per pound of Coal (H − ht) and Furnace and Flue Efficiency $\left(1-\frac{it}{T}\right)$

Fig. 16 may still be used when H is either less or greater than 14,000 by simply subtracting from, or adding to, the diagram reading, the deficiency or excess of H below or above 14,000.

Similarly, for the average 14,000, if through imperfect combustion only $\epsilon \times 14{,}000$ heat units are generated per lb. of fuel, one may still use the right hand scale of Fig. 16 to obtain $\left(\epsilon - \frac{t}{T}\right)$ by simply subtracting from the diagram reading the deficiency $(1 - \epsilon)$.

To make an estimate of the amount of heat transmitted to the water, a further deduction must be made by multiplying by a boiler efficiency-factor. This factor is the ratio of the heat actually passed into the water (and into the steam, if there be superheating arrangements) to the total heat left by radiation and conduction combined in the walls surrounding the furnace and flues. Part of this heat is led by conduction to the outer surfaces of the boiler casing or "setting," and is thence lost to the outside air, earth and buildings by radiation and by conduction assisted by air convection-currents. This loss is the principal one in determining the boiler efficiency, and it depends upon the relative disposition of the heating surfaces and of the setting and outside clothing. The area and efficient disposition of the heating surfaces, and the effective water circulation and cross-circulation of the hot flue-gases, are, of course, influential in determining the factor $\frac{t}{T}$ as well as this last factor.

t is the temperature to which the hot gases are reduced by passing through the furnace and flues during which passage they give up heat to an extent mainly dependent on the design and size of the heating surface. The time-rate at which they give heat to each sq. ft. of this surface is greater the higher the average excess of their temperature over that of the water, and this average excess increases with the furnace temperature. This furnace temperature is greatly affected by the amount of air admitted to burn each 1 lb. of fuel, as may be seen from column T in Table XV. Thus T is over 4,700° Fahr. for bituminous coal burnt in no excess of air, while it is reduced to nearly 2,500° Fahr. if burnt in double the chemically necessary quantity of air. The area of heating surface required per horse-power thus increases rapidly with the quantity of air admitted per lb. of fuel stoked. It also increases along with the efficiency demanded of the boiler. One form of stating this efficiency is the ratio of Effective Heat Horse-Power to the quantity

of fuel burnt per hour. The set of three formulas in Table XVII give simple approximate relations between heating surface, effective heat horse-power, furnace air per lb. of fuel, and horse-power per lb. of fuel burnt per hour. These formulas are deduced from Peclet's experiments on heat conduction through metal plates from hot gas to water, and from Rankine's theoretical investigation of the conductive efficiency as dependent upon the area of conducting surface. They were the result of the present author's earliest efforts to put the calculation of boiler power on a scientific basis. For extreme sizes, they give only rough approximations to the proper values, as the heating surface required does not really vary as the square of the quantity of air admitted, which was Rankine's approximation.

TABLE XVII

Formulas for Effective Boiler Heat Horse-Power, Area of Heating Surface, and Fuel Burnt per hour per Horse-power.

H.HP. = Effective Boiler Heat Horse-power (into water).
F = Lbs. of Fuel burnt per Hour.
S = Square feet of Heating Surface.
n = Ratio of Air passing through furnace to that chemically required for complete combustion.
γ = a factor dependent upon design of heating surface, upon cleanliness from soot and scale on heating plates and tubes, upon the water circulation, and upon the cross-circulation of the hot gases in the flues and tubes. Its lowest value for best conditions and boiler at full power is about ·3.

Heat Horse-Power	$\text{H.HP.} = S\dfrac{6}{\gamma n^2 + S/F}$	$= F\dfrac{6}{1+\gamma n^2\, F/S}$
Fuel Burnt, Lbs. per Hour	$F = \text{H.HP.}\dfrac{1}{6 - \gamma n^2\,\text{H.HP.}/S}$	$= S\dfrac{1}{6\,S/\text{H.HP.} - \gamma n^2}$
Heating Surface Sq. Ft.	$S = \text{H.HP.}\dfrac{\gamma n^2}{6 - \text{H.HP.}/F}$	$= F\dfrac{\gamma n^2}{6\,F/\text{H.HP.} - 1}$

Since 1 Furnace Heat Horse-Power with perfect combustion requires the burning of from ·18 to ·2 lbs. of coal per hour, the fraction $\dfrac{\text{H.HP.}}{F}$ used in the formulas is from 5 to 5½ times the furnace

and boiler efficiency; and the formulas might be rewritten in terms of this efficiency. The numerical factor is intended to decrease slightly with the steam temperature and with the amount of air admitted to the furnace; but the rounded-off average value 6 only is here inserted, more complete calculations being given in Table XVIII. The factor γ varies greatly with the way in which the furnace is stoked and the manner in which air is admitted above or beyond the grate, with the quantity of scale and soot upon the heating surfaces, with the cross-circulation of hot gas in flues and tubes produced by baffle-plates or otherwise, and with the rapid or sluggish circulation of water in the boiler and the free or obstructed escape of steam from the heating surfaces. Under best conditions it lies between ·25 and ·3, and under the worst is probably as much as $1\frac{1}{2}$ or even 2 in a badly scaled boiler.

When a boiler is being worked at half or quarter normal power, these formulas will still represent its action without large error if due account be taken of alteration in n and of γ. If the stoking be specially skilful, there will be only a small increase of air admitted per lb. of fuel burnt: in one case n increased only 5 per cent. when the boiler was being worked at quarter power. But γ increases largely as the horse-power developed falls below the normal for which the boiler is designed, although not in the same proportion. An example from test measurement is γ increased from ·28 to ·87, or in ratio 3·1, when the rate of working was decreased to one-sixth its maximum. When the boiler is forced above normal power γ still decreases.

At a more recent date, when water-tube boilers had become largely used, the author framed a formula for heating surface required per boiler heat horse-power, which takes much closer account of the variations of design and of condition. In Table XVIII and diagram, Fig. 18, are given the results of this formula. In the formula itself the following are the meanings of the symbols:—

$\rho =$ a factor to allow for loss by radiation and conduction from outside shell of boiler and by incomplete combustion in the furnace.

This factor ρ may vary from 1·02 to 1·20 with good firing and good covering, and up to as much as 2 or 3 with careless, unskilful stoking and much outside heat loss from lack of non-conducting clothing.

γ = a factor of heat conductivity per sq. ft. of heating surface, allowing for the effect of sludge, scaling and sooting of the plates, and for the activity of the water circulation and freedom for escape of steam into steam-space.

d = mean hydraulic depth, in inches, of the spaces filled with hot gases and surrounded by heating plates or tubes; that is, the whole of such volume in furnace or fire-box, tubes and flues, divided by the whole heating surface. In any round fire tube, small or large, the mean hydraulic depth is ¼ the inside diameter. For water tubes, the fire and hot-gas volume surrounding them (exclusive of volume occupied by the tubes themselves) must be divided by the total outside tube surface.

n = ratio of actual air supply to that needed for complete chemical combustion.

t = excess of steam temperature over outside air temperature in Fahr. degrees.

ϵ = furnace and boiler efficiency, that is, the ratio of the heat received by the water and steam to the heat generated by combustion in the furnace.

Note that here the factor γ covers generally much the same variation of condition as the γ of the previous formulas of Table XVII, but it has a larger numerical value. Under the best conditions it may be as low as 1·2, and under the worst lies probably between 3 and 4. In tests of a Thorneycroft water-tube boiler, it varied from 1·2 at maximum power to 2·8 at less than ¼ full power. This factor varies over a much smaller range than does the γ of Table XVII.

The formula is—

$$\frac{\text{Sq. Ft. of Heating Surface}}{\text{Boiler Heat Horse-Power}} = \rho\,\gamma\,(1 + {\cdot}08\,d'')\,\phi$$

where ϕ is a factor tabulated below in Table XVIII and Fig. 18, and which depends on the air admitted, the efficiency, and the steam temperature. This factor involves a rather complicated calculation; but as its value can be read from the table or the diagram with amply sufficient accuracy, its complex character should not deter any one from using the complete formula.

The factor ϕ is calculated thus :—

$$\phi = \frac{1 - \cdot 02\, n}{1 - \cdot 0002\, n\, t} \cdot \frac{n^{\,1+2\epsilon}}{50\,(1 - \epsilon)}$$

TABLE XVIII

Values of ϕ in the formula.

$$\frac{\text{Sq. Ft. Heating Surface}}{\text{Boiler Heat Horse-power}} = \rho\, \gamma\, (\,1 + \cdot 08\, d''\,)\, \phi.$$

n	ϵ	t								ϵ	n
		250	300	350	400	250	300	350	400		
1·0	0·5	0·0412	0·0417	0·0421	0·0426	0·398	0·414	0·428	0·445	0·5	3·0
	0·6	0·0514	0·0521	0·0527	0·0532	0·619	0·643	0·667	0·693	0·6	
	0·7	0·0688	0·0695	0·0702	0·0710	1·029	1·067	1·108	1·152	0·7	
	0·8	0·103	0·105	0·105	0·1065	1·924	1·994	2·070	2·152	0·8	
	0·9	0·206	0·2085	0·211	0·213	4·79	4·97	5·16	5·36	0·9	
	0·95	0·413	0·417	0·421	0·426	10·72	11·08	11·50	11·97	0·95	
1·5	0·5	0·094	0·096	0·097	0·099	0·553	0·577	0·604	0·634	0·5	3·5
	0·6	0·128	0·130	0·132	0·1345	0·888	0·926	0·969	1·017	0·6	
	0·7	0·184	0·187	0·190	0·1944	1·518	1·587	1·660	1·737	0·7	
	0·8	0·301	0·306	0·311	0·316	2·93	3·06	3·20	3·35	0·8	
	0·9	0·653	0·662	0·673	0·686	7·52	7·86	8·22	8·60	0·9	
	0·95	1·360	1·382	1·405	1·428	17·06	17·81	18·60	19·46	0·95	
2·0	0·5	0·171	0·174	0·178	0·183	0·736	0·775	0·817	0·866	0·5	4·0
	0·6	0·245	0·250	0·256	0·263	1·214	1·277	1·348	1·428	0·6	
	0·7	0·375	0·383	0·392	0·402	2·14	2·25	2·37	2·51	0·7	
	0·8	0·647	0·662	0·677	0·693	4·23	4·45	4·70	4·97	0·8	
	0·9	1·485	1·519	1·555	1·592	11·16	11·76	12·41	13·12	0·9	
	0·95	3·185	3·257	3·333	3·413	25·63	26·98	28·47	30·15	0·95	
2·5	0·5	0·271	0·279	0·288	0·298	1·20	1·29	1·39	1·50	0·5	5·0
	0·6	0·407	0·419	0·432	0·446	2·07	2·22	2·39	2·59	0·6	
	0·7	0·653	0·672	0·692	0·713	3·81	4·08	4·39	4·76	0·7	
	0·8	1·176	1·209	1·247	1·290	7·88	8·44	9·09	9·85	0·8	
	0·9	2·824	2·907	2·996	3·090	21·7	23·3	26·1	27·2	0·9	
	0·95	6·192	6·374	6·566	6·773	51·1	54·7	58·9	63·8	0·95	

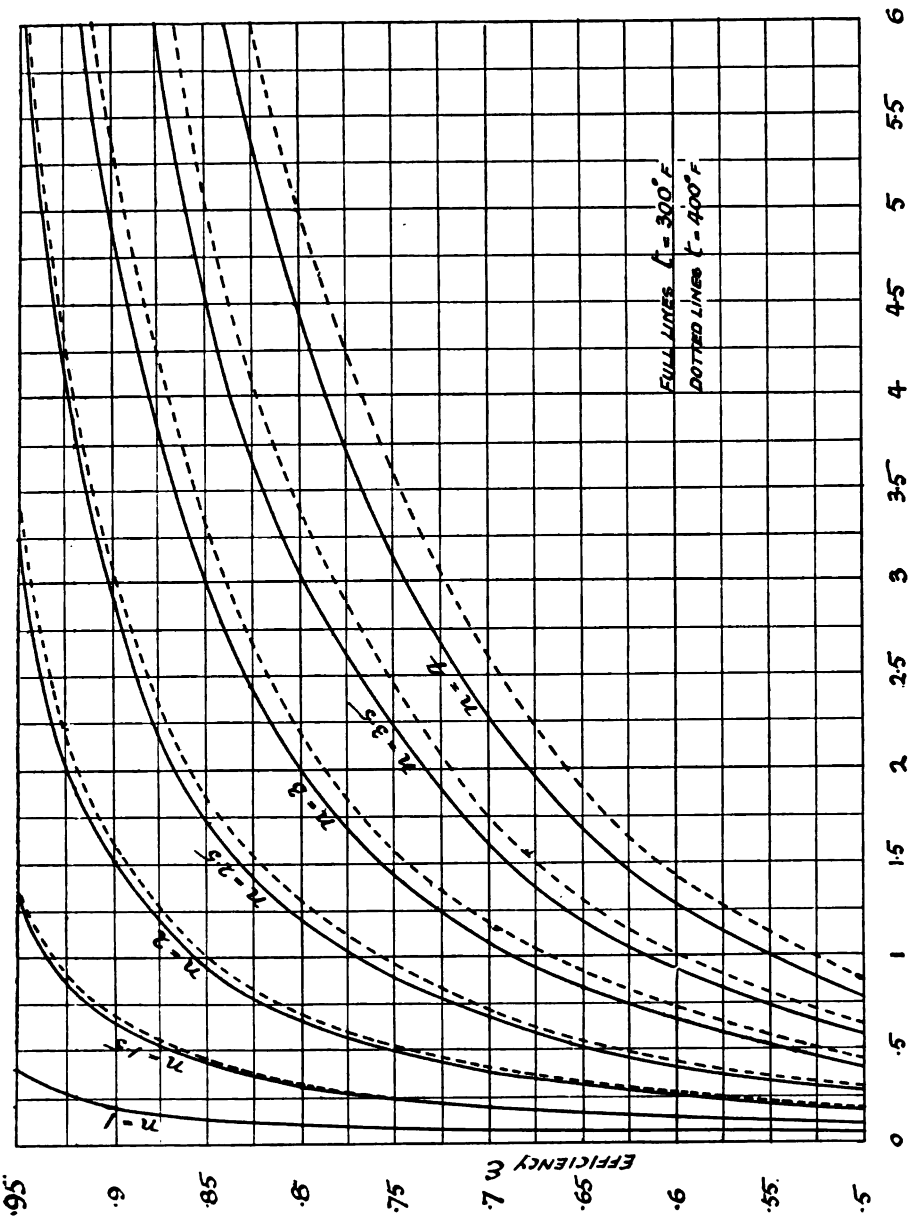


FIG. 18.—Main factor ϕ in Heating Surface per Boiler Heat Horse Power $\phi = \frac{1-\cdot02n}{1-\cdot0002nt} \cdot \frac{n^{1+2\epsilon}}{50(1-\epsilon)}$

In this table there are entered many values of ϕ too large to be used in any rational boiler design. These correspond with high efficiency-rates combined with large excess of air admitted to the furnace and high steam temperature. Under such conditions it is impossible to get high efficiency without an extravagant extent of heating surface, and extravagance in this direction defeats its own object by increasing the ρ losses by outside radiation and convection currents by amounts not compensated by the gain in ϵ; and also by involving useless increase of construction and maintenance costs. But these high values of ϕ are still useful in the table, because the formula indicates what can be done with a boiler when worked much below its normal designed power, when, of course, the heating surface per heat horse-power developed is "extravagantly" large.

These high values of ϕ are, however, not plotted in diagram Fig. 18, which is confined to the limits that should not be exceeded in any normal design. It will be noted how all the curves turn with sharp curvature to the right on reaching near certain limits of efficiency which the quantity of air admitted make very difficult of attainment. Thus, an efficiency of over 95 per cent. is impossible to attain except with an extremely small excess of air, and even with no such excess not much higher than 95 per cent. can be reached.

The steam temperature is the limiting factor which makes more than a certain efficiency absolutely impossible. The hot gases will give heat to the water and steam, that is, they will do useful conductive work, only so long as they are hotter than the steam. If at the tail-end of the heating surface they are cooled to near steam temperature, that part of the heating surface is inoperative and therefore useless. Taking the simpler formula $\left(\epsilon - \frac{t}{T}\right)$, even with complete combustion or $\epsilon = 1$, and without losses by radiation, etc., etc., since t in the chimney gases cannot be operatively reduced to t in the steam, therefore a boiler efficiency of $\left(1 - \frac{\text{Steam Temperature}}{T}\right)$ is unattainable. T, which is $\frac{H}{h}$, depends on the quantity of air admitted, and is roughly $\frac{14{,}000}{2 + 2{\cdot}8\,n}$; so that the curves in Fig. 18 for large n's turn over horizontally at low practicable limits of efficiency.

The ratio between the Boiler Mechanical Horse-power and the Boiler Heat Horse-power is shown on the curves marked $\frac{P\,(V - w)}{\text{Total Heat}}$ in Figs. 9 and 10. On the temperature diagram, Fig. 9, the curve

is flat, varying little from a straight line, and the value of the fraction runs from ·065 at atmospheric pressure to ·073 at 300 lbs. per square inch absolute steam pressure. These are the ratios when dry saturated steam is produced; but priming greatly reduces the ratio, because much heat is spent in heating the priming water without any resulting mechanical work done inside the boiler by evaporation of that water. Taking $\frac{1}{4}$ as an average value of the ratio of the heat spent in heating the water to the total heat of the saturated steam, if x be the percentage of wetness in the steam, then the ratio of Mechanical to Heat horse-power in the boiler is reduced from that read from the curve in Fig. 9 or 10 in the ratio $\frac{1-x}{1-\frac{3}{4}x}$.

On the other hand, if the steam be superheated beyond the saturation density, the superheater being legitimately counted as part of the boiler, the volume is increased at the expense of some additional heat conduction. In considering the commercial economy derivable from superheating, it should not be overlooked that the conduction of heat to saturated and superheated steam is a very much more difficult process than its conduction to water or to wet steam. Thus, so far as boiler horse-power is concerned, the first part of a "superheater," which acts really as a drier of wet steam, and not as a superheater, gives greater economical advantage than the portions of the "superheater" through which the steam passes after being dried, and which portions act as real superheaters. To make a superheater effective as a superheater requires the excess of hot-gas temperature over steam temperature to be greater than at the tail-end of the heating surface where the conduction is still to water. If not separately fired, the superheater should, properly, be heated by the hottest gases available; that is, it should be placed in or near the furnace. Such placing is objectionable, because the high temperature would burn the superheater-tubes and wrack their joints so as to start leaks. This very fact is the clearest evidence of the difficulty in conducting heat into superheated steam. It is only because the steam takes in the heat so unreadily and so slowly—that is, takes it out of the metal of the tubes so slowly—that these tubes get overheated when placed as above.

Superheating is thus a difficult and expensive process, and the fact that great advantage is derived from it in the production of steam for engines is a forcible illustration of the bad character of saturated steam as a substance for use in such engines.

Its bad character results mainly from its partial condensation when coming in contact with the cooler metal surfaces in the engine admission ports and cylinder, and in a much smaller degree from its condensing when expanded adiabatically. The latter action is very clearly seen on the entropy diagram, Fig. 11, page 60. Here the saturated curve slopes downwards to the right hand, so that any adiabatic parting from it towards lower pressure, which means lower temperature, is a vertical straight line falling inside the saturation curve, and thus indicating admixture with water. Similar inspection of the left hand side of the same diagram shows that adiabatic lowering of pressure from the condition of water at evaporation temperature, results in partial evaporation of the water: part of the water at once bursts into steam. All wet steam up to a limiting degree of wetness condenses partially in adiabatic expansion. But when there is a great excess of water with the steam, the reverse, or evaporation, results from such expansion. At each temperature and pressure, therefore, there is one certain proportion of admixture of water and steam which yields neither evaporation nor condensation when the pressure drops adiabatically. In Fig. 11 this limit is marked at each temperature, and a line joining all such points is drawn down the diagram. The line is marked "Equilibrium Curve—Adiabatic Expansion gives No change of Wetness." It is a curved line sloping downwards slightly to the left. Starting from any point of it, it will be found that any *small* movement up or down on the diagram does not change the proportion between the intercepts on either side of it of the horizontal line between the water and the saturated steam curves. But at the same time it must be noted that a continuation of this vertical, or adiabatic, motion leads away from this "equilibrium curve"; so that any adiabatic expansion more than a small one at once leads into the region in which continued adiabatic expansion causes condensation. Starting adiabatic expansion from any point a little to the left of the equilibrium curve, partial evaporation or drying takes place until the drop of temperature brings the stuff down to this curve; but immediately this curve has been crossed downwards, this process is reversed to partial condensation.

Superheating, in all the apparatus at present used in connection with boilers, is carried out by expansion of volume at constant pressure, the pressure not rising above that of the saturated steam in the boiler. The specific heat of superheated steam has not yet been well determined by experiment at many different pressures

and densities. It is believed to vary from ·4 to ·8.[1] In the absence of certain information as to this variation, the mean value ·48 may be used at present for calculation. If the amount of superheat in Fahr. degrees be t, the extra heat conduction per lb. of steam is thus ·48 t, which has to be added to the "total heat" of Fig. 9 or Fig. 10. If t_s be the saturated steam temperature measured from absolute zero, i.e. the Fahr. reading plus 461°, then the superheating expands the volume in a ratio probably somewhat less than $\left(1 + \frac{t}{t_s}\right)$. The P V measuring the boiler mechanical horse-power is increased in the same ratio. Calling the saturated "total heat" H, the heat supply per lb. has been increased in the ratio $\left(1 + \cdot 48 \frac{t}{\mathrm{H}}\right)$.

Thus superheating increases the ratio of Boiler Mechanical to Boiler Heat Horse-power in the ratio—

$$\frac{1 + \frac{t}{t_s}}{1 + \cdot 48 \frac{t}{\mathrm{H}}}$$

To illustrate this ratio, take $t = 100°$ of superheat,

and	t_s =	700	800	850	Fahr. absol.
corresponding to Pressure	=	$24\frac{1}{2}$	117	218	lbs. per sq. in. absol.
and giving	H =	1,127	1,157	1,174	Br. Th. U. from 60°.

Then—

$$\frac{1 + \frac{t}{t_s}}{1 + \cdot 48 \frac{t}{\mathrm{H}}} = 1{\cdot}097 \quad 1{\cdot}080 \quad 1{\cdot}073.$$

This ratio of increase is seen to be small and to vary very little. It means per 10° Fahr. superheat a gain of from 1 per cent. for very low pressures to $\frac{3}{4}$ per cent. for very high pressures.

On the other hand, the adiabatic curve of expansion of superheated steam is much steeper, its index being about 1·3, so that the work done under it is less than that of saturated steam, in proportion to their respective initial P V.

[1] See the author's chart of the specific heat of superheated steam in *The Engineer* of July 8, 1904.

The advantage derivable from the superheating of steam appears therefore to be limited to securing absence of water in the cylinder. This is a very great advantage, resulting not from directly mechanical causes, but from the cure it effects of the bad thermal qualities of iron cylinders. Wet steam parts with its heat to a wet iron wall very quickly, while dry steam conducts to a dry iron wall only very slowly.

The ratio of the indicated horse-power in the engine cylinder to the boiler mechanical horse-power depends mainly on the character of the engine. If there were no thermal action in the engine, if there were no leakage past valves and piston, if there were complete expansion and no back pressure, and if there were no losses due to "clearance" at each end of the cylinder, then this ratio would be what has already been tabulated in Table VIII and Fig. 8, page 55, for different indices of adiabatic expansion curves, the index for saturated steam being according to Rankine 1·13, that for superheated steam about 1·3, and that for gas 1·4; and the corresponding ratios between the two horse-powers would be 8·7, 4¼ and 3½, in these three cases.

The thermal action of the cylinder is mainly by initial condensation of the steam on entrance into the cooler clearance space in the cylinder. This condensation destroys a proportionate amount of the P V delivered from the boiler without work being done on the piston. The amount of this injurious action is proportionate to the area of the metal surfaces of the clearance space —including ports and cover and piston surfaces. Much has been written on the subject which need not be summarized here. The only proper method of dealing with the evil is now well known to be superheating to such a degree as prevents cylinder condensation altogether. The cylinder condensation losses without superheating are larger than all other losses put together, and in bad cases have been known to be as much as 60 per cent., while in ordinary cases they run to 20 and 30 per cent.

The leakage losses are also very large except in engines of high-class workmanship and design. They are not amenable to calculation. The best fitting of the valves to their seats, of the glands, and of the piston, is ineffective unless the mechanical design of these parts is such as to avoid their being distorted by the high and varying temperatures to which they are exposed, and unless the metal of which the bearing surfaces are made be suitably selected.

The back pressure and clearance losses are more susceptible of calculation, and at the same time they are unavoidable. In

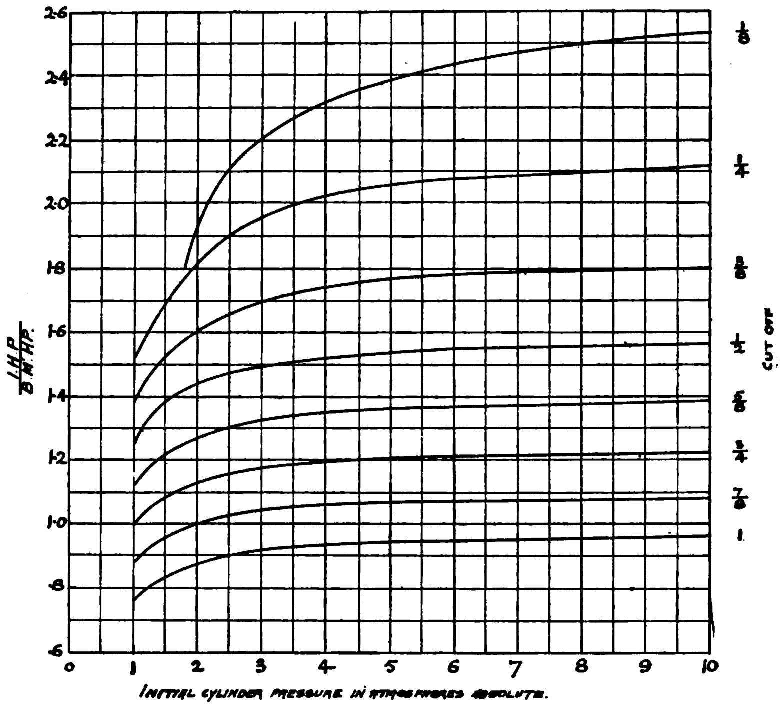


FIG. 19.—Ratios of Cylinder Indicated Horse-power to Boiler Mechanical Horse-power in Condensing Engines with Back Pressure ⅕th atmos. and Clearance Volume 5 per cent. Expansion Index 1·1.

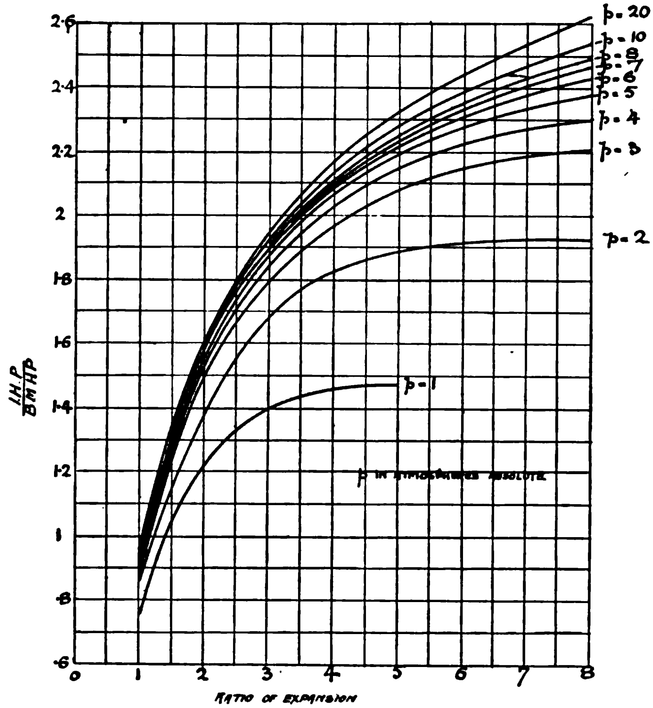


FIG. 20.—Ratios of Cylinder Indicated Horse-power to Boiler Mechanical Horse-power in Condensing Engines with Back Pressure ⅕th atmos. and Clearance Volume 5 per cent. Expansion Index 1·1.

Figs. 19, 20, 21 and 22 are plotted the ratios of cylinder indicated horse-power to the boiler mechanical horse-power for different initial pressures and different ratios of expansion, with back pressure and clearance volume losses taken into account. No leakage losses and no cylinder thermal action are considered in the calculation of these curves. They are based on the assumption that the expansion curves have the index 1·1.

In all cases the clearance volume is taken as 5 per cent. of that swept through by the piston in one stroke. In well made engines it is often less than this, and it does not need to be so large in large cylinders as in small ones. But the diagrams well illustrate the effect of clearance in limiting the efficient ratio of expansion. The clearance increases with the volume of the cylinder, and this latter per lb. of steam used must increase with the ratio of expansion. The proportion of clearance loss to indicator diagram area thus increases so rapidly that beyond certain limits of expansion the advantage obtained by further expansion is outbalanced by the increasing clearance loss.

The same general law applies to the back-pressure loss, which increases in direct proportion to cylinder bulk. It is naturally more serious in non-condensing engines than in condensing; and its importance diminishes as the initial pressure is taken higher.

Figs. 19 and 20 refer to condensing engines, and for these the back pressure is taken as $\frac{1}{5}$th of atmospheric pressure; i.e. the "vacuum" is taken as $\frac{4}{5}$ atmospheric pressure. Figs. 21 and 22 refer to non-condensing engines, for which the back pressure is taken as $1\frac{1}{5}$ atmospheres, or $\frac{1}{5}$ atmosphere gauge pressure.

In all the four diagrams the heights of the curves indicate the ratio of cylinder indicated to boiler mechanical horse-power. In Figs. 19 and 21 the horizontal ordinate is initial pressure, and the different curves are for different ratios of expansion. In Figs. 20 and 22 the horizontal ordinate is the ratio of expansion, while the different curves are drawn for different initial pressures.

In Fig. 22 for non-condensing engines, it is seen that for each initial pressure the curve rises to a maximum height and then falls again. Beyond the maximum height limit the increase of loss from clearance and back pressure overbalances the gain from further expansion. This limit, therefore, gives the ratio of expansion yielding the maximum mechanical efficiency, independently of cylinder thermal losses and independently of consideration of increase of construction and maintenance costs involved in the larger size of engine.

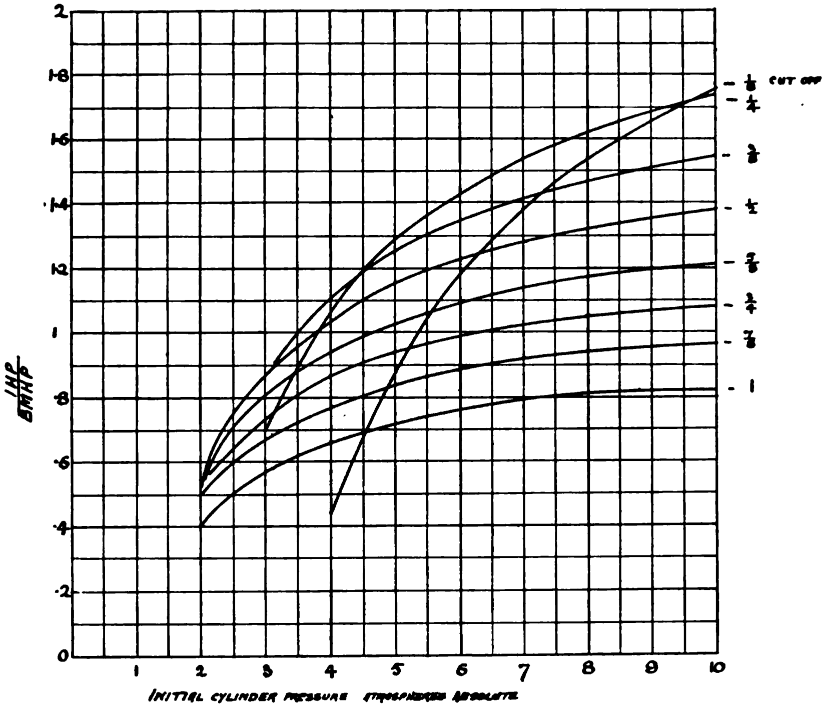


FIG. 21.—Ratios of Cylinder Indicated Horse-power to Boiler Mechanical Horse-power in Non-Condensing Engines with Back Pressure 1⅛ atmos. Clearance Volume 5 per cent. Expansion Index 1·1.

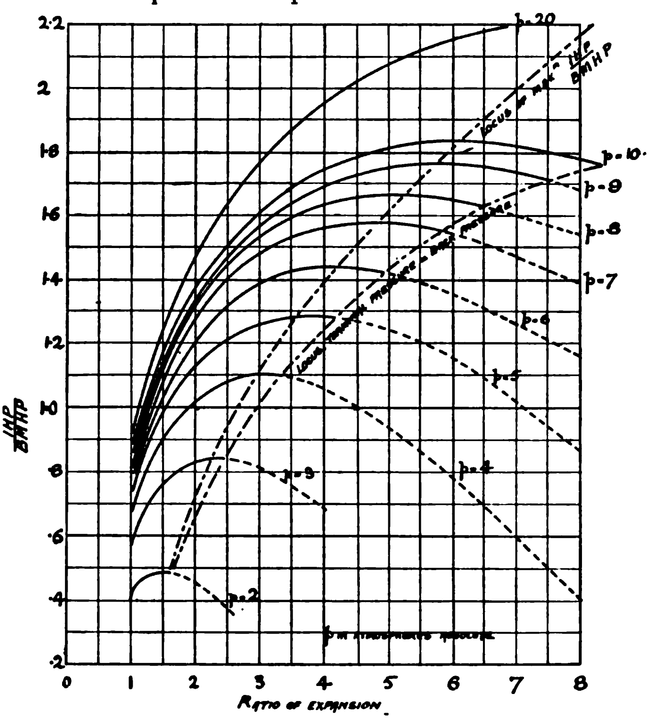


FIG. 22.—Ratios of Cylinder Indicated Horse-power to Boiler Mechanical Horse-power in Non-Condensing Engines with Back Pressure 1⅛th atmos., and Clearance Volume 5 per cent. Expansion Index 1·1.

It should be noticed how low these grades of expansion for maximum mechanical efficiency are. For 6 atmospheres initial pressure the efficient ratio of expansion is only 4, and for 10 atmospheres only 6. A dotted curve is passed through these maximum points; and a second dotted curve passes through the points of greater expansion which bring the terminal pressure down to equality with the back-pressure. Still lower grades are found to be the most economic when the costs of construction, maintenance and working are taken into account. The problem of maximum commercial economy is explained in full detail in the later chapters of this book.

In Fig. 20 for condensing engines these limits of maximum efficiency are not reached within the diagram, because of the smallness of the back-pressure losses; but this diagram shows that even with such small back-pressure loss, the curves rise very slowly indeed beyond quite moderate grades of expansion. The method whereby these diagrams have been calculated has already been explained in connection with the W curves of Figs. 13 and 14.

In Fig. 21 for non-condensing engines, except for very early cuts-off, the ratio $\frac{\text{I.H.P.}}{\text{B.M.H.B.}}$ rises very little with greater initial pressure beyond from 4 to 7 or 8 atmospheres. This feature is still more marked in Fig. 19 for condensing engines, where the curves become nearly horizontal straight lines beyond 3 to 5 atmospheres initial pressure, except for the early cut-off $\frac{1}{8}$. These results should be considered along with those shown by the curve $\frac{\text{B.M.H.P.}}{\text{B.H.H.P.}}$ on Fig. 10. This is the ratio of the boiler mechanical H.P. to the boiler heat H.P., and it is seen in Fig. 10 to rise very little indeed after 3 or 4 atmospheres pressure, the curve becoming nearly a vertical straight line beyond this limit. The conclusion is that the commercial utility of very high pressure results largely from small prime cost of plant, and in a less degree than is usually supposed from great thermodynamic efficiency.

Chapter III

Costs.

Capital Outlay and Working Expenses.

Steam, Gas and Oil Plants. Electric Power Stations.

Depreciation.

The whole problem of Economy viewed in all its completeness must take account of four great aims. These are perfection or excellence of the finished product, rapidity of production, mechanical efficiency of the means of production, and saving in the costs of production. The present chapter deals with the last of these four aims. Increased excellence of product and increased efficiency are often achieved at the expense of greater costs; but the economic endeavour must be to minimize the increase of cost thus involved, and also to lessen costs without affecting, or affecting as little as may be, either the excellence of the product or the mechanical efficiency of the production. But greater mechanical efficiency means saving of certain kinds of costs, namely costs of material and sometimes costs in wages; and, although the means adopted to increase efficiency may be in themselves expensive, they are adopted in the hope that saving in cost will result more than enough to recoup these extra expenses. Thus the question of the desirability of any item of increased mechanical efficiency must be judged by, and is really included in, the problem of costs.

Costs are broadly divided into two classes, capital outlay and running or annual expenditure. The first is a slump sum of money or other value: the other is a time-rate of exhaustion, sacrifice, destruction of value. The essential distinction lies in the one being a time-rate while the other is not so, and that the one is spent, the other only lent.

The Capital Outlay in Heat Power-Plants consists of the following parts.

NOTE.—In the titles of all the diagrams, Figs. 23A *to* 23O, *the phrase "per cub. inch of final vol." is used as short for "in terms of final volume expressed in cubic inches."*

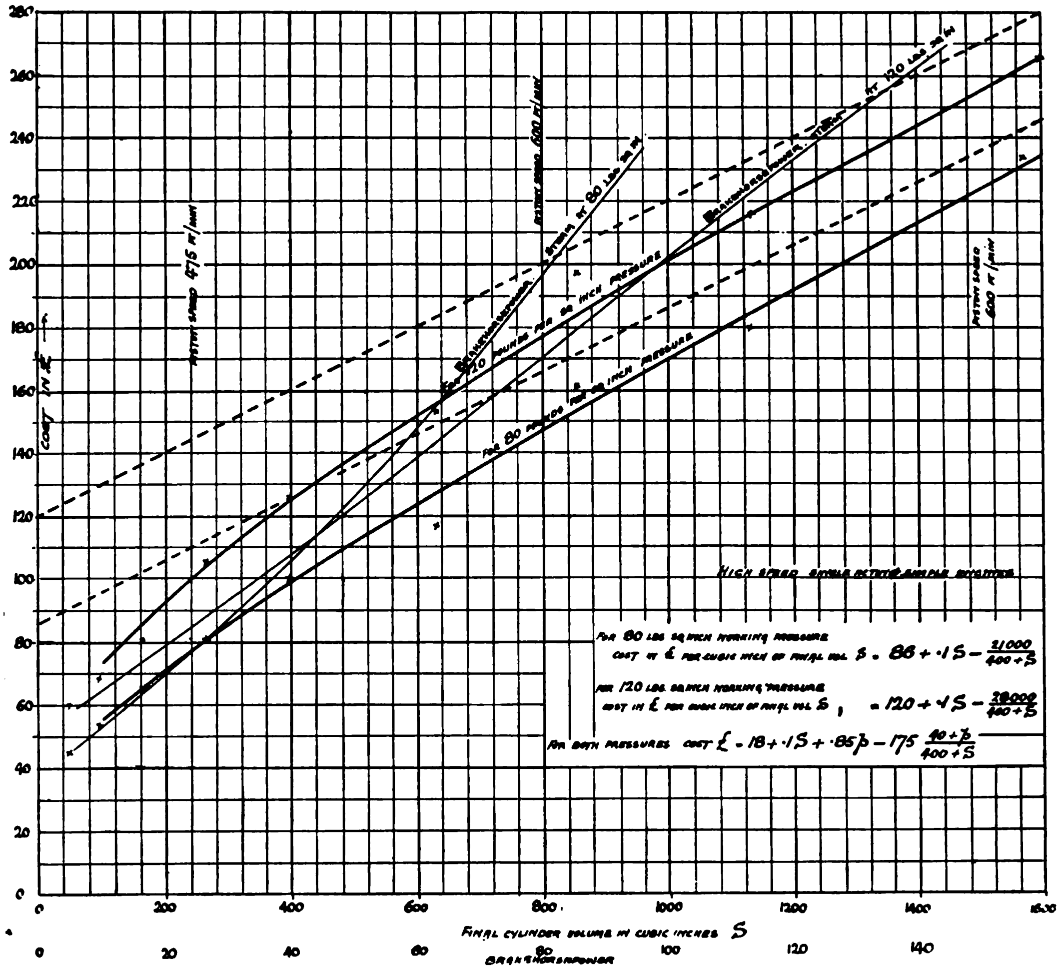


Fig. 23A.—Engine Costs. High-Speed, Simple, Single-acting, Vertical Steam Engines.

N.B.—The piston speed marked on the diagram is based on double stroke not single stroke movement.

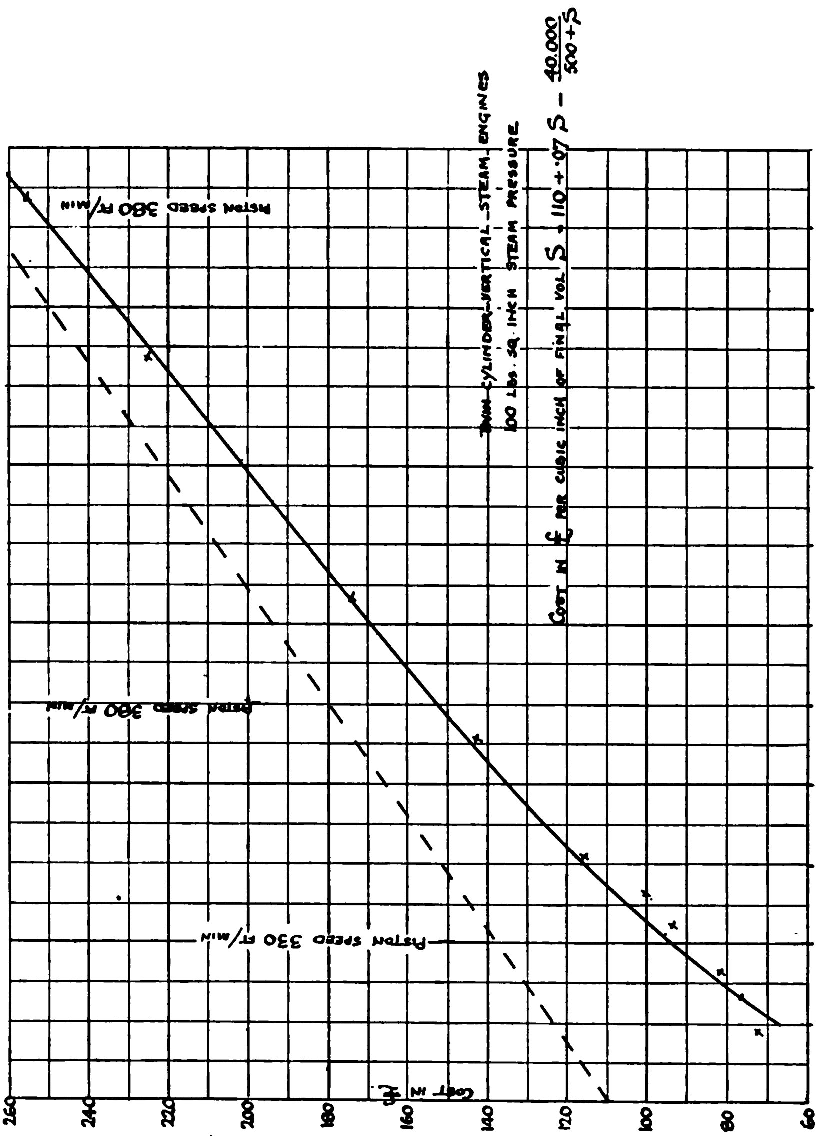


FIG. 23B.—Engine Costs. Twin Cylinder, Vertical, Double-acting Steam Engines.

In the case of Steam Power, it includes—

Fuel Conveyors and Mechanical Stokers;
Boilers and Feed Pumps and Steam Piping;
Engines, including Valves and Governors;
Condensers, Air and Circulating Pumps;
Hot-Well, and Condenser Cooling Tank or Tower;
Buildings, including chimney.

Except in large installations, the first item very generally disappears; and the fourth and fifth do not exist in non-condensing plants.

In the case of Gas Engine Power, the items are—

Fuel Conveyors and Hoppers;
Gas Producers;
Engines, including Gas Meters, Valves, Igniters and Governors;
Exhaust Silencer, and Cooling Water-Jacket Tanks;
Building without chimney.

Gas may be taken from the public supply-mains, in which case the first two items fall out.

In the case of Internal Combustion Oil Engines, they are—

Oil Tanks;
Engines, including Valves, Igniters and Governors;
Exhaust Silencer, and Cooling Water-Jacket Tanks;
Buildings without chimney;

to which may be added Special Oil Wagons usually required for the conveyance of the oil fuel to the site of the plant, except when the installation is quite small.

Each of these three kinds of engine needs to have mounted on its crankshaft a fly-wheel, which is a rather heavy item of cost. The need of the fly-wheel arises partly from periodic variation of driving effort and partly from fluctuation of driven resistance. In so far as the resistance is concerned, all three classes of engine are on an equality, the fluctuation of resistance having nothing to do with the character of the engine. In respect of fluctuations of driving effort, this is most violent in oil engines and least so in steam engines; so that steam engines require less heavy and costly fly-wheels than either of the other two classes. This difference between the three is, however, due to the special forms of design, and the special means of governing, which it is at present the fashion to adopt for gas and oil engines; and is not by any means inherent in the characters of the three working substances, or of the fuels, employed. It is inherent in the principle of "internal combustion"

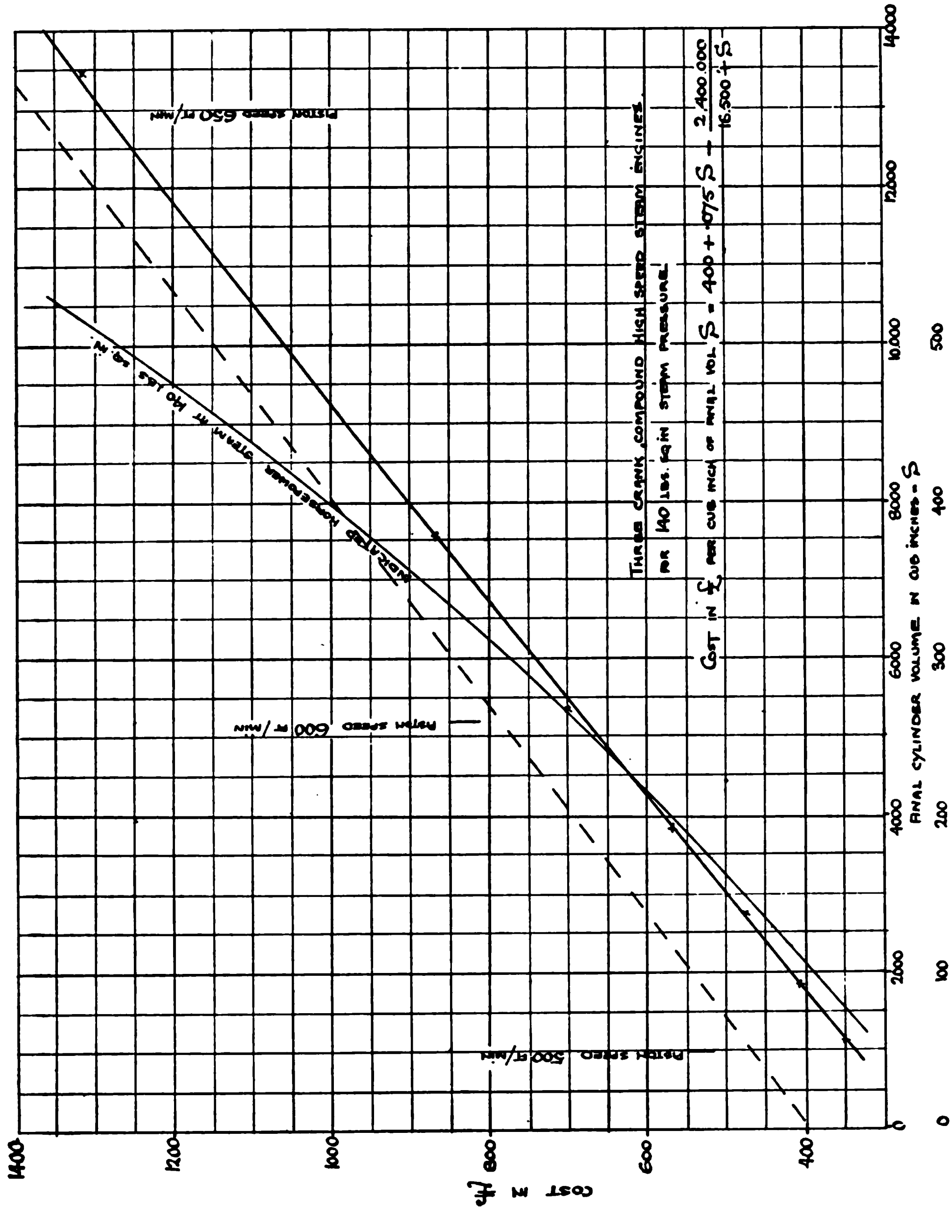


FIG. 23C.—Engine Costs. Three-crank, Compound, Vertical, High-speed, Single-acting Steam Engines.
N.B.—The piston speed marked on diagram is based on double-acting not single-acting movement.

of the fuel inside the engine cylinder as contrasted with that of burning the fuel in a furnace entirely separated from the engine. Thus in respect of fly-wheel cost it would be more rational to place engines in only two classes, external and internal combustion engines. Two-and three-cylinder gas engines with cranks at 90° and 120° apart are beginning to appear on the market, and in these new designs the difference between gas and steam engines in regard to fly-wheel weight is diminished; but these improvements only mitigate the adverse influence of internal combustion.

The steam turbine, which is the only successful form of "rotary" steam engine, is the sole example of a driving engine with perfectly uniform driving effort, not only in regard to the moment exerted directly by the steam pressure, but also in regard to the modification of this moment by mass accelerations of "reciprocating parts." It is thus the only form of driving engine whose fly-wheel weight is determined by fluctuation of driven resistance alone. It therefore frequently requires no fly-wheel at all. It is only fair, however, to point out that the high rotary speed at which turbines necessarily run, and, in the Parsons' turbine, the great weight of large shaft and blades rotating inside the engine casing, give a very large fly-wheel steadying effect without the addition of an actual fly-wheel.

The whole plant, in each case, centres about the engine, and the size of each other part is proportioned to that of the engine. This is natural, because it is in the engine that the power is finally developed in directly applicable form for delivery to the machinery to be driven. Thus the sizes of other parts of the whole plant are most commonly rated by reference to the size of the engine to which they are proportioned.

This rating generally goes by horse-power. But the engine horse-power varies with so many conditions which affect the cost little or not at all, that it is not a rating on which strict scientific estimates of economy or cost can be based. It is familiar knowledge that every steam engine can be worked considerably above or below its horse-power as stated in the maker's list, without any change in mechanical efficiency, by altering the speed and the boiler pressure from those stated in the list.

The capital cost to the buyer of any engine of a given style depends mostly upon its cylinder size. After size the element in the design most influential on the cost is the working pressure; but this influence is distinctly inferior in magnitude, and each engine maker will sell the same identical engine as suited for any working

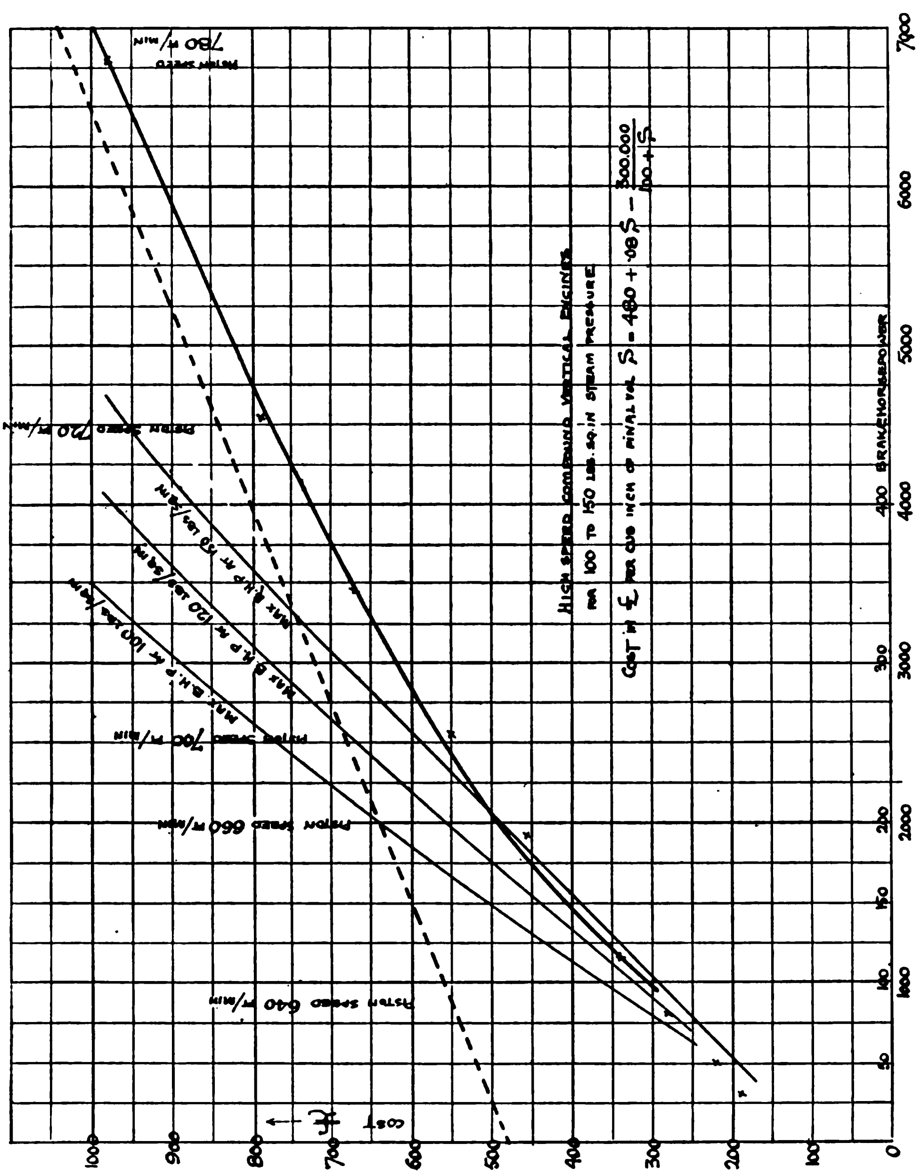


Fig. 23D.—Engine Costs. High-speed, Compound, Double-acting, Vertical Steam Engines.

pressure desired by his customer within a very considerable range of pressnres.

For instance, within the range from 80 to 110 lbs./in.^{2} or within that from 120 to 160, the makers will not propose to make any alteration in the design, and the cost of construction to them will not be altered. The maker may possibly ask a slightly higher price for the somewhat higher pressure; but only on the principle that he believes he can obtain a better price in accordance with the greater usefulness of the engine to his customer, and this difference of price will not appear in any price-lists that he may publish. But for great differences of working pressure modifications of design are needed to give equal security against leakage at joints—at fixed bolted joints and at moving joints such as glands, valve-seats, etc. Greater strength and stiffness are also required in many parts; and, especially when superheating of steam accompanies the higher pressure, special forms and special metals have to be employed to overcome mechanical difficulties due to extra high temperature. All these modifications involve extra cost of construction, and increase of price to the purchaser. If such influences on price were continuous all through the range of actually used working pressures from 50 to 250 lbs. per square inch, then in the scientific formula giving the costs of engines the working pressure would enter in very much the same way as does the cylinder size. Owing to these influences not being continuous, and since each maker designs his engines for say two pressure-classes only, which he calls "high" and "low" pressure—although the "low" of one maker may be the "high" of another—the author has found great difficulty in obtaining much authentic data from which to obtain proper factors for the pressure terms in the formula for cost. The few pressure factors given in this chapter are deduced from comparisons between data which he considers are too few to justify any generalizations. They serve, however, to illustrate how the pressure may be expected to influence cost when it becomes common usage to allow for pressure in designing and pricing engines. The absence of more ample evidence from existing commercial practice is the more to be regretted because the designing and pricing of boilers in somewhat strict accordance with the maximum pressure to be used in them has for long been the common usage of engineers. The discrepancy between these two usages results somewhat grotesquely in the mating of boilers carefully calculated and selected for an exact pressure with engines designed, selected, and bought for no particular pressure within a range of at least 60 or 70 per cent.

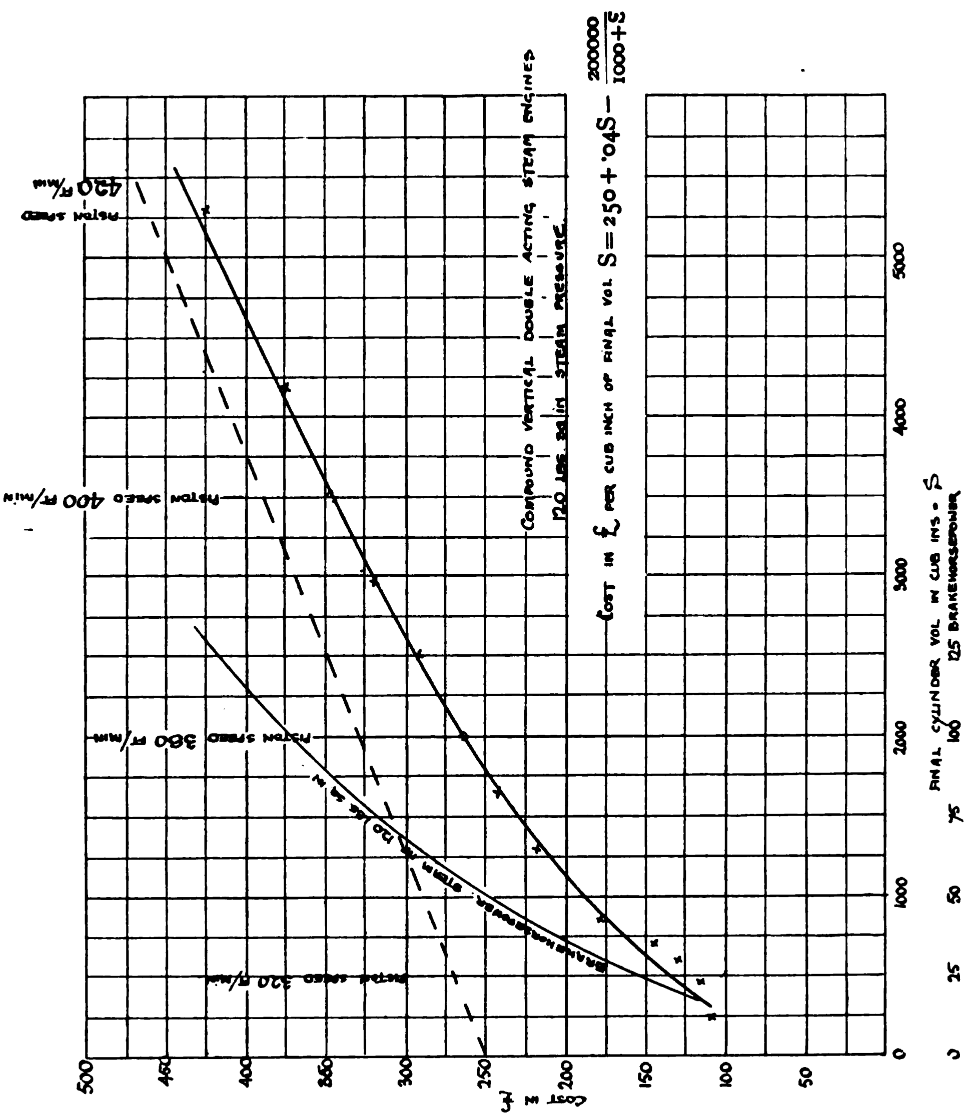


FIG. 23E.—Engine Costs. Compound, Vertical, Double-acting Steam Engines.

Working speed has a rational, and a practical, influence upon cost of much the same kind as pressure. It is little taken account of in design, except for extremes of speed. For such extremes the mechanical difficulties arising through high speed force themselves into consideration. None the less is it a physical truth that these difficulties develop gradually and continuously as the speed rises, just as do the requirements due to rising pressure. Makers, however, construct only for "high speeds" and "low speeds," and know nothing of intermediate speeds, while either "high speed' or "low speed" may mean anything within 50 per cent. of the mean up or down, at least so far as details of constructive design and prices are concerned.

It is much more rotary speed than linear piston speed that creates the difficulties that have to be overcome by the mechanical designer by more or less expensive methods. It is due to this fact that what are commercially termed "high-speed" engines are really those of high rotative speed, not those of high piston speed. They are, in fact, for the most part small-sized engines in which very high speed of rotation is adopted in the generally vain endeavour to attain a very moderate piston speed with a short stroke. High rotative speed is in itself a good thing in two respects: it diminishes the oscillation of temperature on the metal inside faces of the cylinders, and it reduces the stroke and therefore the bulk of the machine per horse-power. It is the reduction of bulk per horse-power which is the main motive in using high rotative speed. Its evil is the hammering of the brasses at all the bearings by very rapid reversal of pressure, the intensity of each hammer-blow becoming intolerably destructive if high linear piston speed be maintained with very short stroke. These hammer-blows are partly due to the reversal of the direction of steam pressure and partly to the reversal of the mass accelerations. Even perfect "balancing" does not in any degree eliminate these latter effects, because the balancing is effected by simultaneous reversals through the working joints. The blow due to acceleration, per lb. of accelerated mass, is proportioned to (Linear Speed ^{2} $\div$ Stroke), which means the same as proportional to (Rotary Speed ^{2} $\times$ Stroke), or to (Linear Speed $\times$ Rotary Speed). This last formula is the one yielding the easiest and clearest understanding of the bearings of the question. It shows that, for given linear piston speed, the difficulty increases in direct proportion to the rotary speed.

As regards the reversal of steam pressure on the joints, the case stands much on the same footing. For given horse-power if the

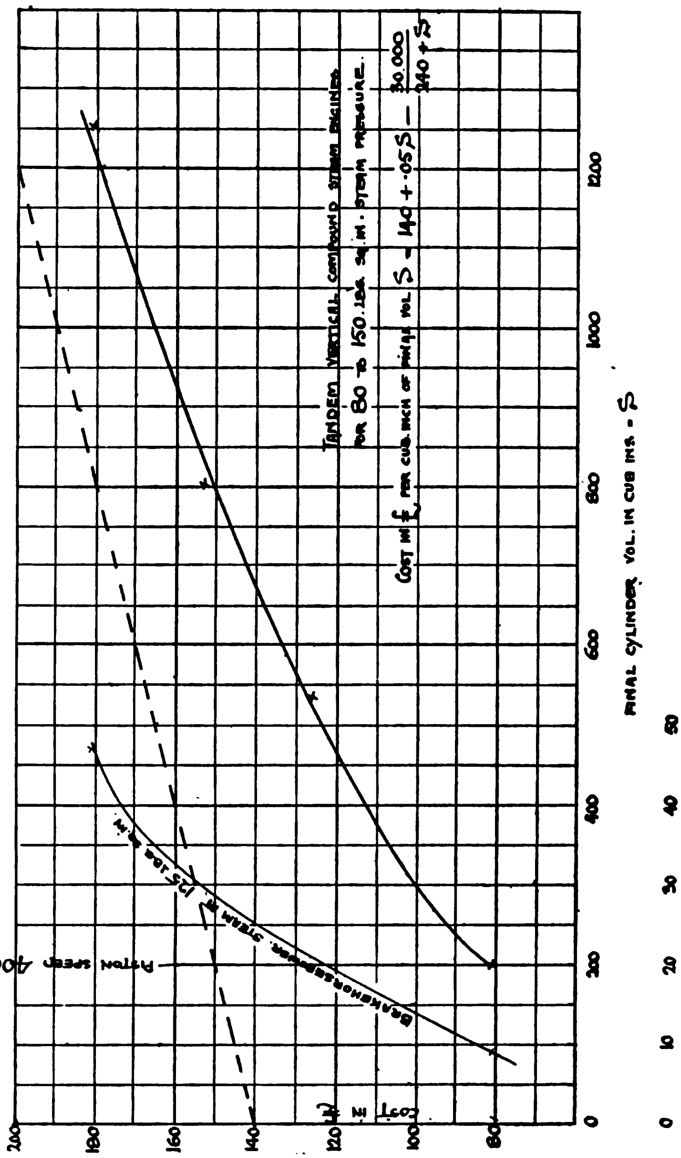


FIG. 23F.—Engine Costs. Tandem, Vertical, Double-acting, Compound Steam Engines.

H

piston speed be reduced, the steam pressure on the whole piston area must be increased in like proportion. Thus if in small engines using high rotative speed it be found impossible to keep up the linear piston speed, then either the piston area or the boiler pressure must be increased. This being the case, the makers of these engines have been forced into the practice of providing a permanent steam or pneumatic cushion on an auxiliary piston which provides such an over balance of dead pressure on the one side of the series of bearings that there is no absolute reversal of pressure from side to side. Thus they prefer an oscillation on one side of the bearing between moderate and very high pressure to an oscillation between zero and moderate pressure on each half of the bearing. The former does not constitute so destructive a blow as does the latter.

The price of an engine of given style varies, therefore, mainly with three things, cylinder size, working pressure, and speed. The variation of actual market price with size is "continuous"; but as regards pressure and speed there are gaps in the data one can obtain from current quotations and price-lists. The actual horse-power developed in an engine is proportional to the product of these three things, size, pressure and speed; and if there were none of the gaps just mentioned, it would be possible to formulate a scientific relation between price and actual horse-power. Such formulations in terms of the factors, size and pressure, and also in terms of horse-power, are given in this chapter in numerous diagrams.

A first approximation to a law of costs is obtained by a straight line upon the diagram. A great deal concerning the progress of industrial operations and the profit derivable therefrom can be learnt from investigation on this simple elementary basis of the straight-line law. Without going beyond this first approximation the author of this treatise has attempted to expound various interesting and important results in his two papers, "The Conditions of Financial Profit in Industry," in *The Engineer* of 1899, and "The Laws of Industrial Profit," in *Feilden's Magazine* of 1902.[1]

In terms of horse-power T such a straight-line law would give the price $P = C_1 + C_2 T$; or in terms of cylinder size S, it would make $P = K_1 + K_2 S$, where the C's and K's are constants.

Reference to the numerous price diagrams given below, in Figs. 23 and 24, however, shows that a curve with top-side convexity is needed for useful approximation to the actual figures for steam and gas engines. To express such a curve two kinds of formula

[1] *The Engineer*, 8th and 15th September, 1899. *Feilden's Magazine*, February, 1902.

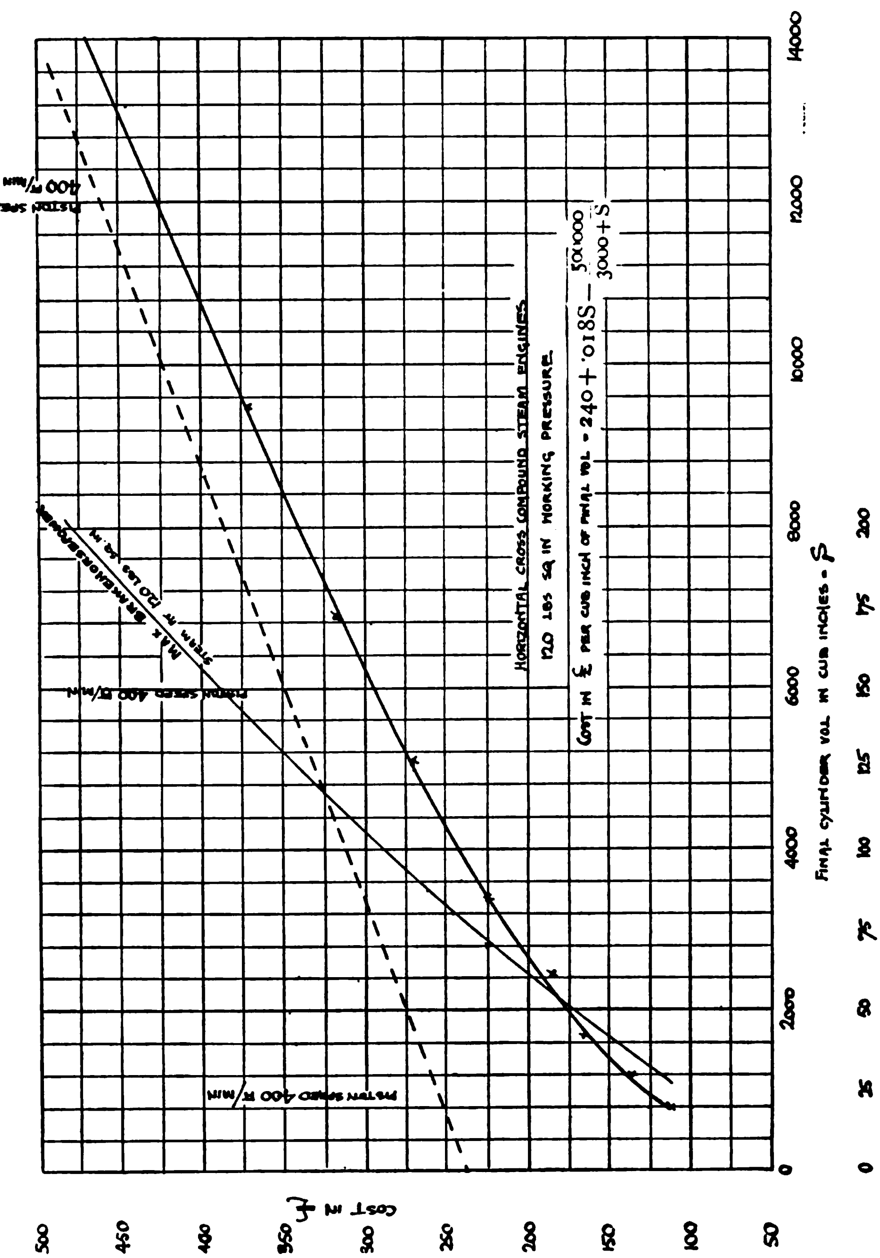


FIG. 23a.—Engine Costs. Horizontal, Cross Compound, Double-acting Steam Engines.

are possible. $K_1 + K_2 S^n$, where n is a power less than 1, would serve; and it would be found that n lies between $\frac{1}{2}$ and $\frac{2}{3}$ for most price-lists. This formula, however, always fails for the upper end of a large range of sizes because of the upward slope diminishing too much. The price increases somewhat rapidly with size as measured by cylinder volume in the case of engines, so long as the sizes are small; but beyond a certain limit the further increase settles down to a nearly uniform rate, that is, to an approximately straight-line law. It is therefore desirable to adopt a formula consisting of the straight-line law with an addition changing rapidly at small sizes, but very slowly, or hardly at all, for large sizes. Such a formula, which the author has found very generally applicable in various industries, is—

$$P = K_1 + K_2 S - \frac{K_4}{K_5 + S}$$

the first two terms giving the straight-line, and the third, involving two extra constants K_4 and K_5, being a quantity to be plotted downwards from the straight-line to the curve giving the prices. In one of the subjoined diagrams the third term is zero, the prices following a straight line; but this does not represent the best commercial practice. In steam engine practice this third term, which is hyperbolic, is invariably a downward plotting as above stated; but in some manufactures it is found to be an upward plotting, the convexity of the curve being on its under side. The straight line $K_1 + K_2 S$ is the asymptotic tangent to the hyperbolic curve, and its inclination gives the steady rate of increase of price per unit increase of size towards which the law settles down at large sizes. The same law applies, only rather less accurately, when horse-power is substituted for size of cylinder; and, in this case, the inclination of the asymptotic tangent gives the extra price charged per extra horse-power.

In our diagrams the size of engine is measured by the cubic inches of volume swept through by the piston in one stroke. In compound and triple expansion steam engines, the low pressure cylinder volume is the measure, irrespective of the size of high, or intermediate, cylinders. In twin engines, or others in which there is a number of cylinders in all of which the steam or gas acts in the same manner, the measure is the combined volume of all these cylinders: thus in twin-cylinder engines the size is twice the volume swept through in one stroke by each piston. Essentially, the measure of size is the volume *finally* (i.e. just before exhaust) occu-

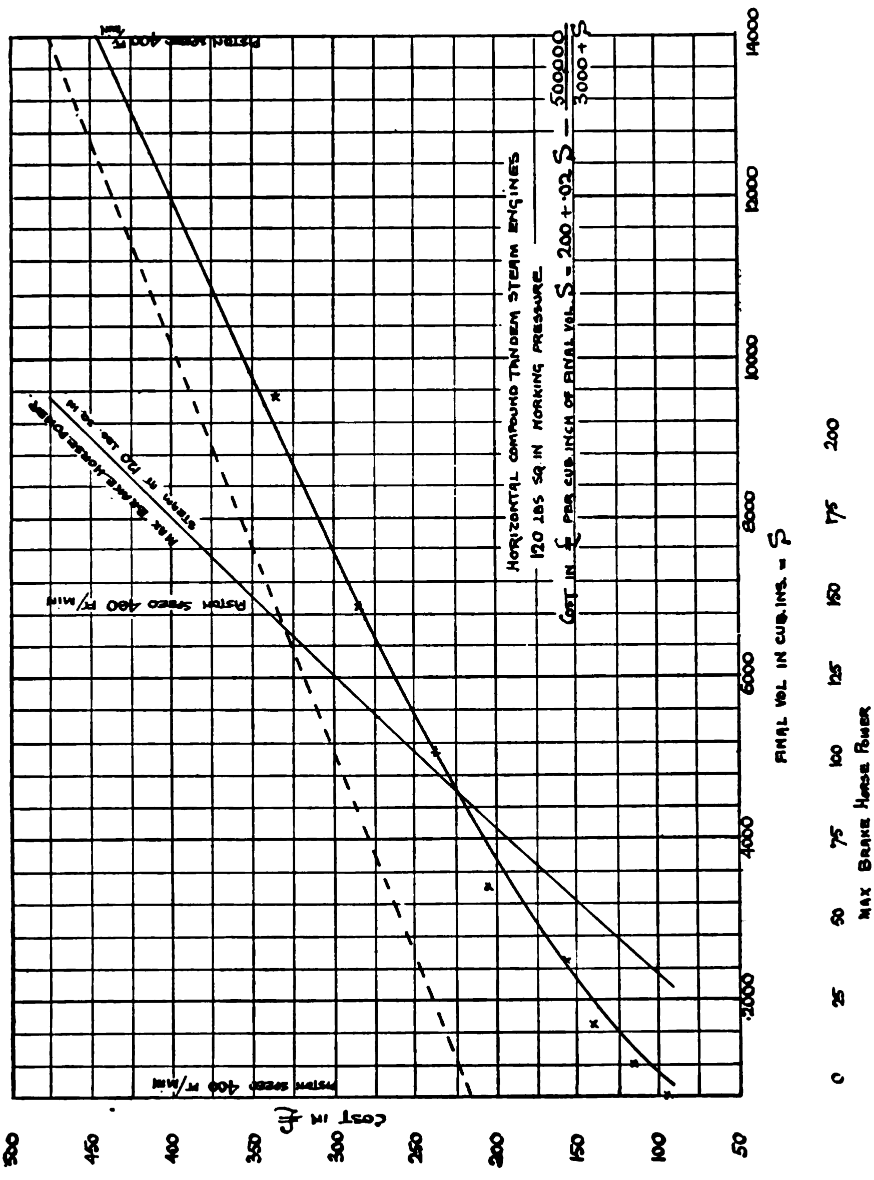


FIG. 23H.—Engine Costs. Horizontal, Tandem, Compound, Double-acting Steam Engines.

pied by the steam or gas that enters with each opening of the admission valves. But a double-acting engine, that is, one in which both ends of the cylinder are used as working chambers, is counted of the same size as a single-acting engine of equal diameter and stroke.

The fifteen diagrams given on Fig. 23 are distinguished by letters A, B, C, etc., etc. The letters indicate distinct classes of engines and various English firms of engineers, who are among the best makers of engines. Their names cannot be attached to the diagrams, the more especially because the discounts generally given are allowed for as deductions from the published price-lists of these firms. Readers, however, may assume that each class of engine is here represented by the best English and Scotch makers of that class.

The method by which the curves have been obtained by Mr. H. M. Hodson is the following. The prices of from 6 to 12 different sizes are first plotted on the diagram and marked by crosses. Nextly a fair curve is pencilled in, passing as nearly as possible through these. The cross-marks have been left in, and their deviations from the curve indicate the erratic irregularity of the pricing of the firm in question. The next step is to draw the asymptotic tangent (shown by the dotted straight line) under the three conditions: (1) that its inclination is approximately the same as that of upper end of the pencilled curve, (2) that this slope is arithmetically well rounded-off to an easy factor for calculation, and (3) that it runs into the vertical at zero S at a well-rounded-off initial height K_1. This height and the slope of this dotted line give the first two constants K_1 and K_2 of the formula. Next two depths, D_1 and D_2, are measured downwards from this line to the pencilled curve, one for a small size, S_1 (*not* close to zero), and the other for a large but not the largest size, S_2. The two other constants, K_4 and K_5, are now obtained by reduction from the two equations—

$$D_1 = \frac{K_4}{S_1 + K_5}$$

$$\text{and } D_2 = \frac{K_4}{S_2 + K_5},$$

this reduction giving—

$$K_4 = (S_2 - S_1)\frac{D_1 D_2}{D_1 - D_2}$$

$$\text{and } K_5 = \frac{S_2 D_2 - S_1 D_1}{D_1 - D_2},$$

N.B.—The piston speed marked on diagram is based on double stroke, not single stroke movement.

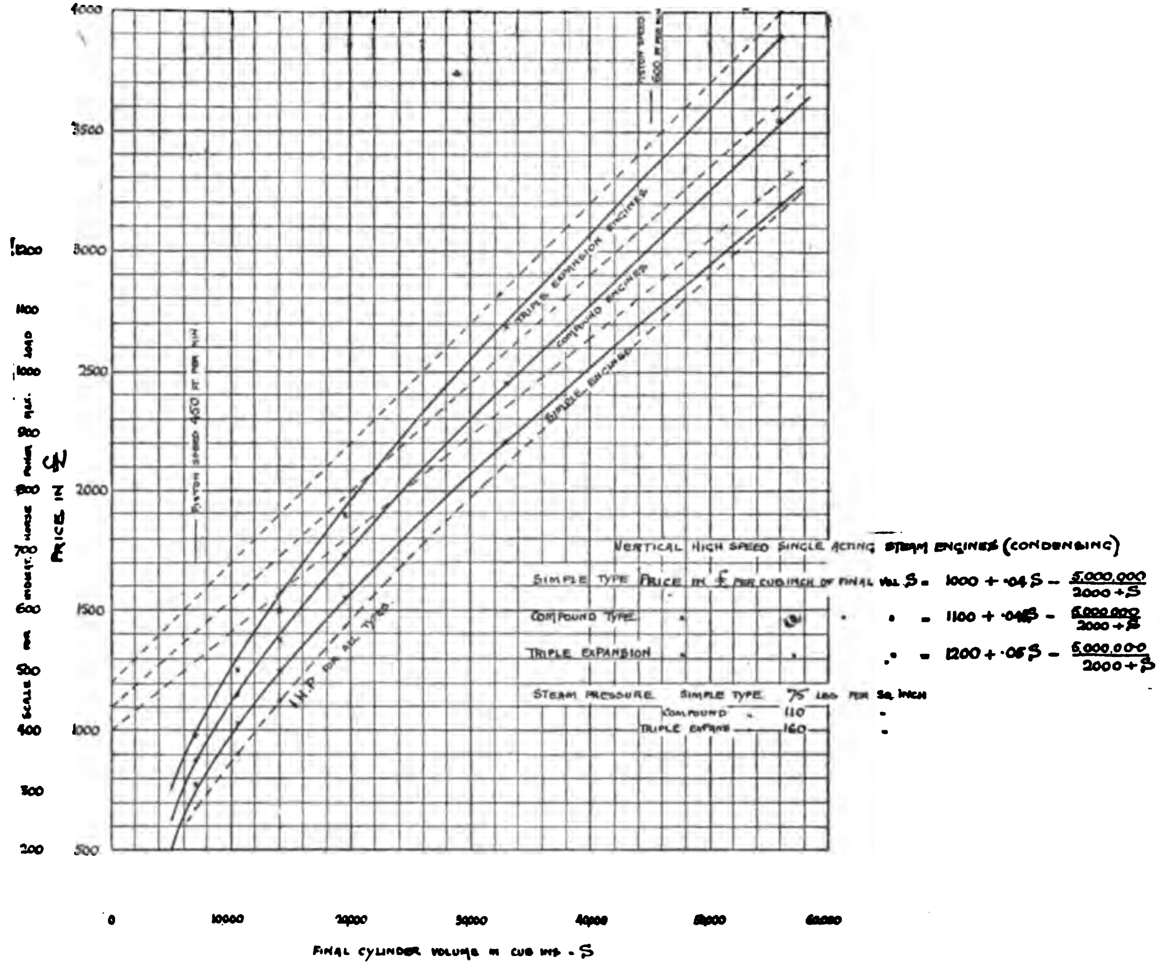


FIG. 231.—Engine Costs. Vertical, High-speed, 3-crank, Single-acting, Condensing Engines. Simple, Compound, and Triple Expansion. The I.H.P. curve co-ordinates power with cylinder volume, and the Indicated Horse-power is the same for all three types for each low-pressure cylinder size.

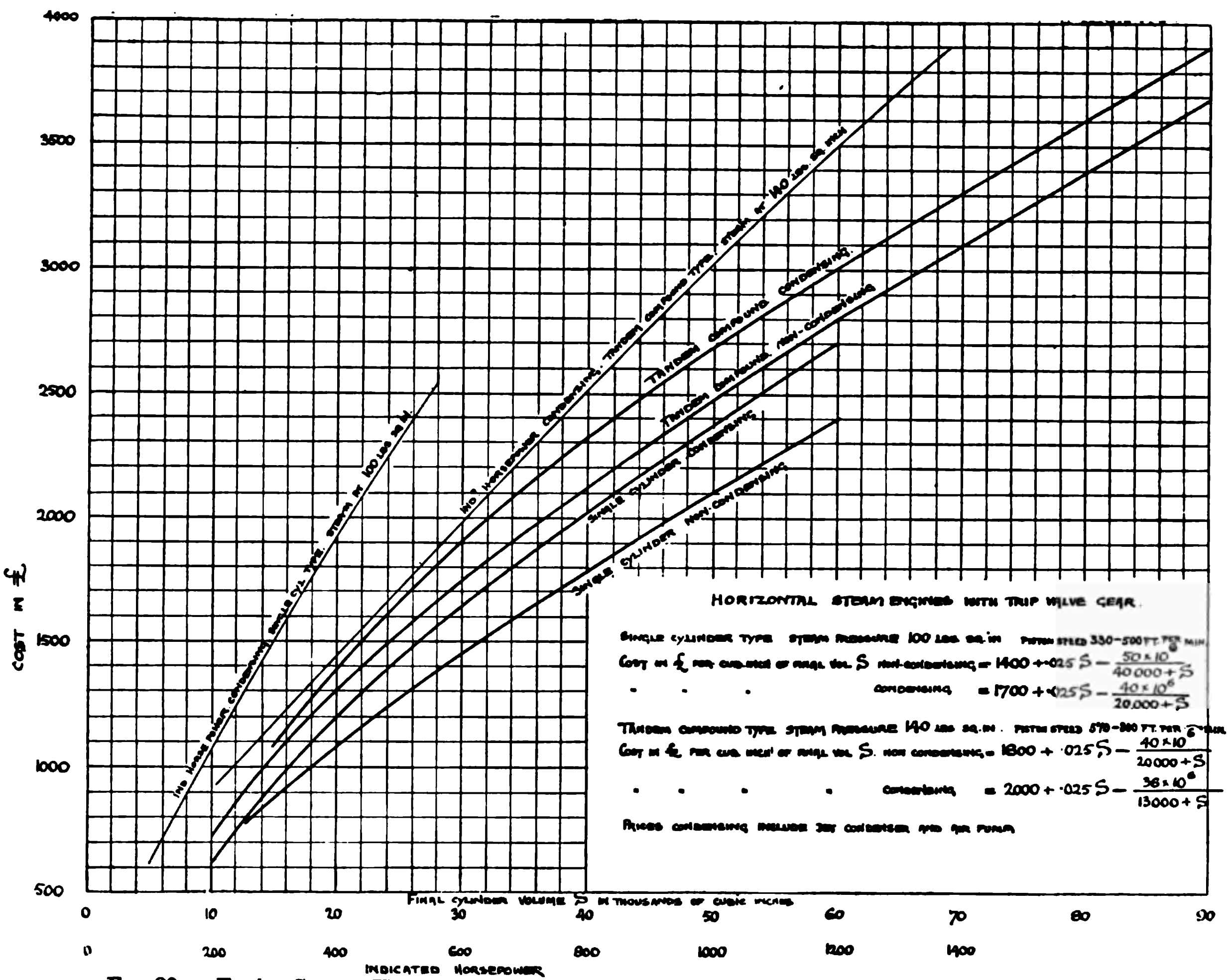
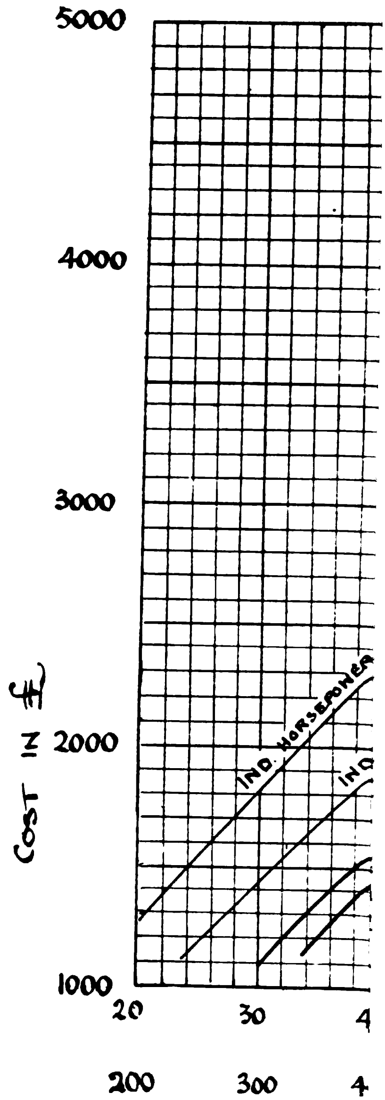


FIG. 23J.—Engine Costs. Horizontal, Double-acting, Steam Engines with trip-valve gear. Simple and Compound Tandem Condensing and non-Condensing.



5000
4000
3000
2000
1000
COST IN £
IND. HORSEPOWER
IND
20
30
4
200
300
4

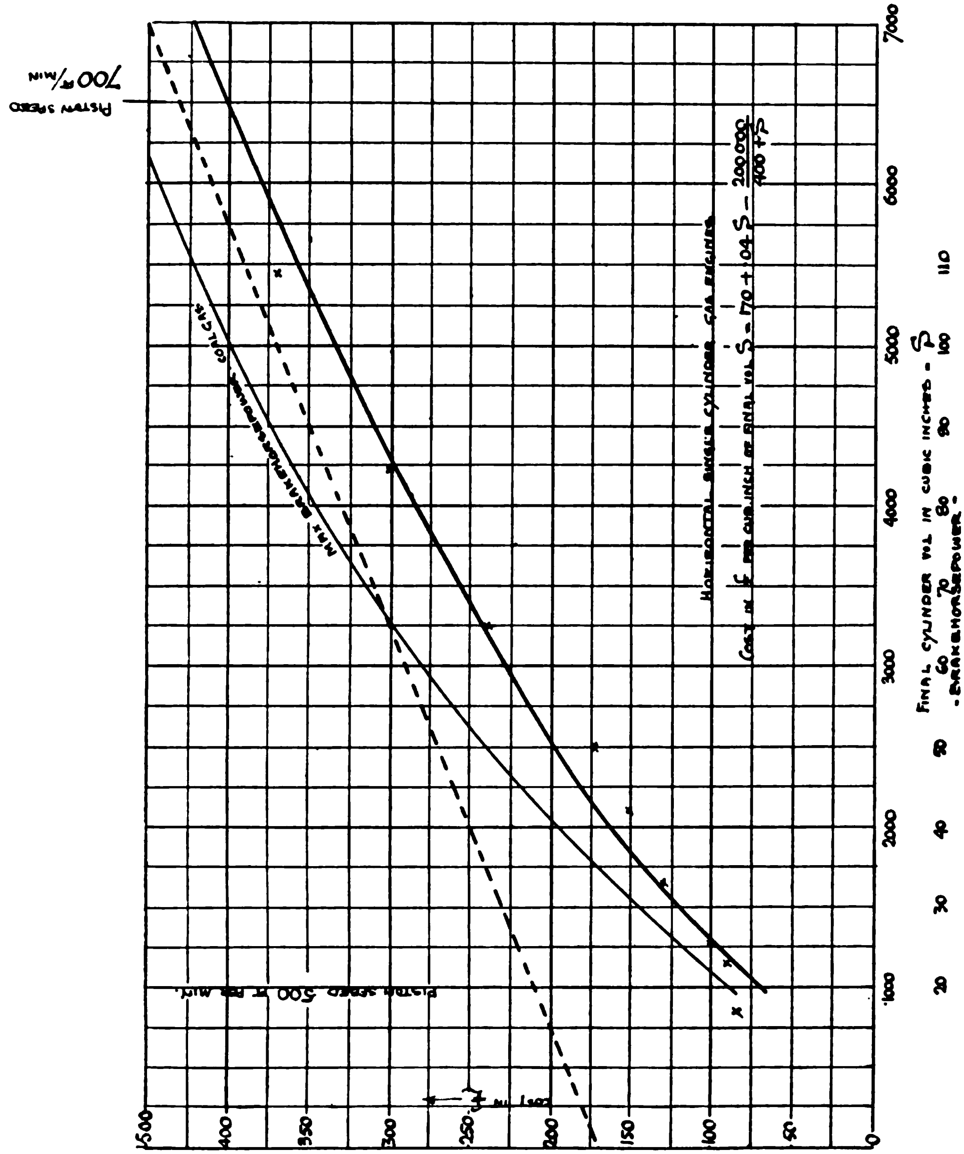


FIG. 23L.—Engine Costs. Horizontal, Single Cylinder Gas Engines. Ordinary type.

or, instead of using the first given value of K_4, K_5 was rounded off from the value as calculated above and then *from the rounded-off value of K_5*, the value—

$$K_4 = \tfrac{1}{2} K_5 (D_1 + D_2) + \tfrac{1}{2} (S_1 D_1 + S_2 D_2)$$

was found, and was itself rounded off from this. It was then examined whether these constants gave $\frac{K_4}{S + K_5}$ nearly correct for the middle of the range of sizes. If they were found to give too flat a curve the value of K_5 was somewhat lessened to another well-rounded-off value, and the value of K_4 recalculated. K_5 must be positive. If K_1 be at first chosen too small, it will be found that $(S_2 D_2 - S_1 D_1)$ is negative; but this is remedied by increasing K_1 by £50 or £100 or some even sum. In all cases it is found possible to describe the actual variation with fair accuracy with four rounded-off factors easy for calculation, so far as the erratic character of some of the price-lists permit. The factors being thus adjusted, a sufficient number of points was calculated from the formula; these plotted in, and a fair curve drawn through them. With a close-drawn asymptotic tangent, the curve becomes more sharply curved when adjusted to pass through the extreme points. The two points from which to make the above calculation should be chosen about one-fifth the length of the curve from its lower and upper ends.

In several of the diagrams, the similar curve co-ordinating the price with horse-power is drawn in fine lines. The horse-power given by these curves is generally the brake-horse-power guaranteed by the makers to be obtained with the maximum pressure permissible and at the speed recommended in the catalogue. In these the figures supplied are more difficult to adjust to fair curves; but the curves approximating most nearly have much the same general character as those co-ordinating price with size.

When the influence of working pressure upon price can be discovered, the price-formula may be modified to take account of it as follows—

$$P = K_1 + K_2 S + K_3 p - K_4 \frac{K_6 + p}{K_5 + S}$$

According to this, the different diagrams co-ordinating P and S

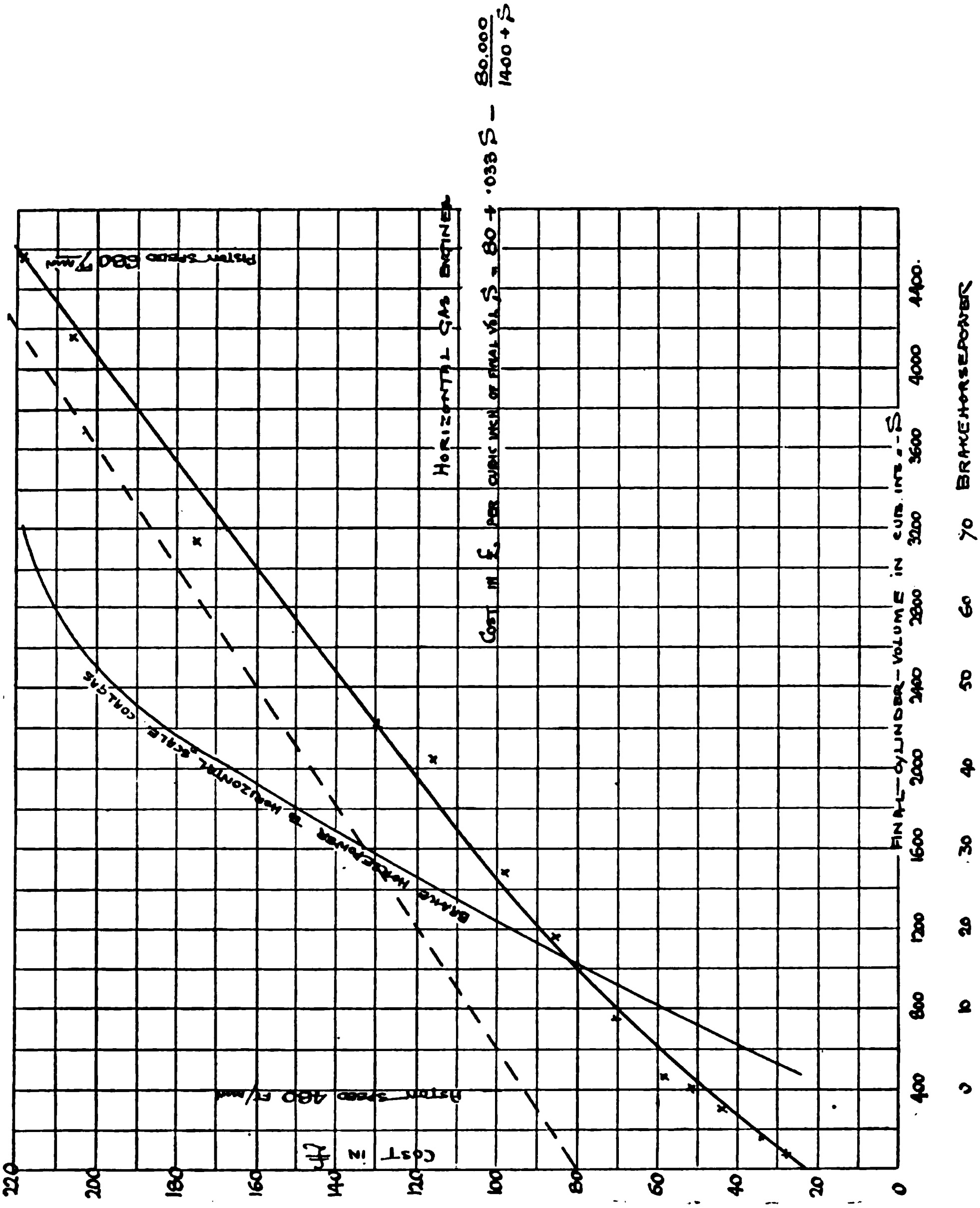


FIG. 23M.—Engine Costs. Horizontal Gas Engines.

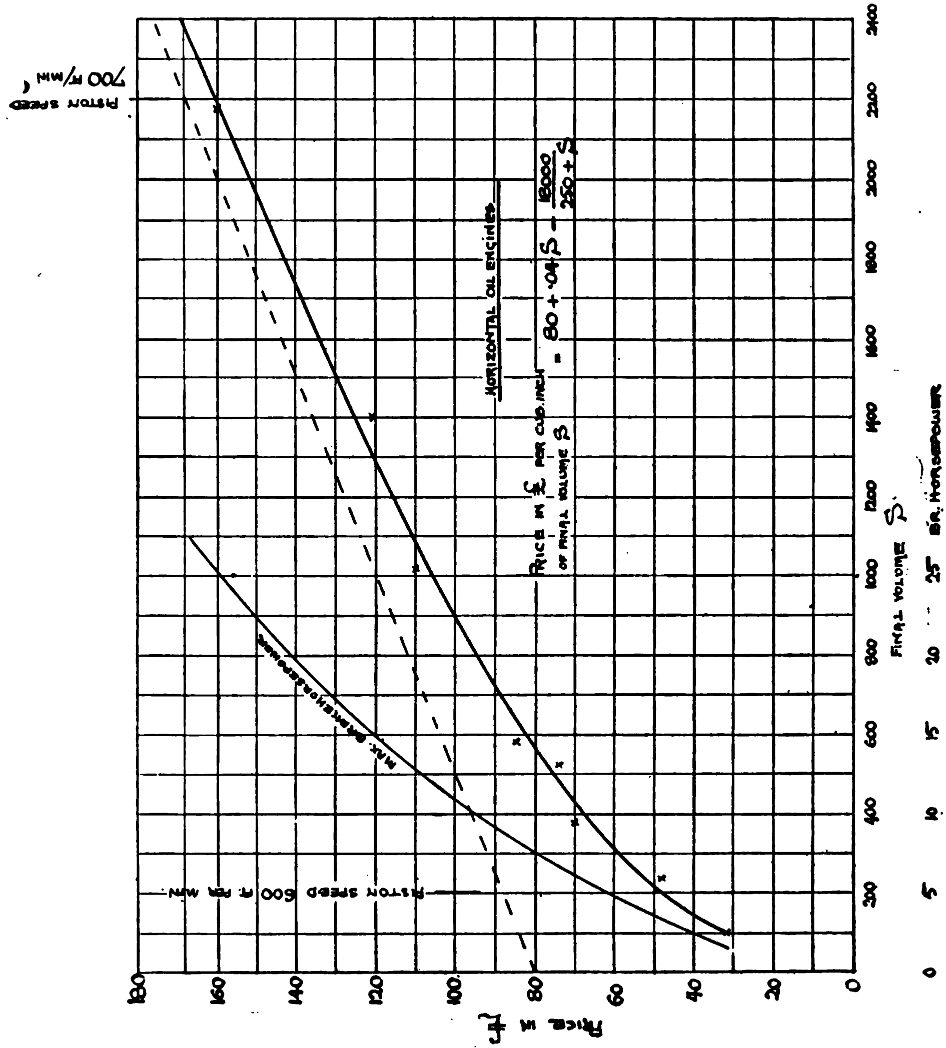


FIG. 23N.—Horizontal Oil Engines.

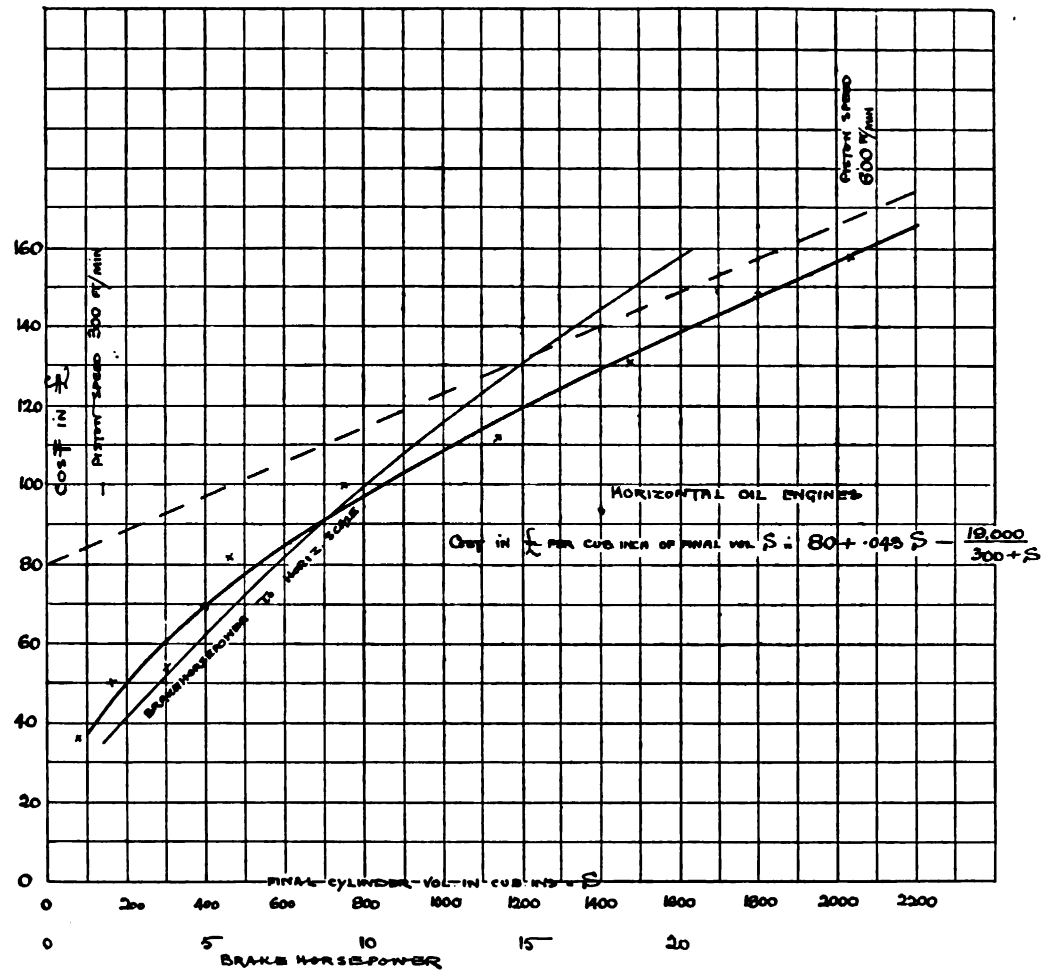


FIG. 230.—Engine Costs. Horizontal Oil Engines.

for different working pressures, have the same slope K_2 for the asymptotic tangent; but this tangent starts at an initial height equal to $(K_1 + K_3 p)$, that is, at a height regularly increasing with the pressure; and the multiplying factor of the hyperbolic term, namely $K_4 (K_5 + p)$, also increases regularly with the pressure.

If one can obtain data to draw two such (P, S) diagrams for the same style of engine but for two different pressures, (taking care to draw the asymptotic tangent at the same slope in both diagrams), then the difference of the initial heights of the two asymptotic tangents divided by the difference of the two pressures gives the pressure co-efficient K_3; and, if one finds the multiplying factors in the hyperbolic terms in the two curves to be C_1 and C_2—the terms being found as $\frac{C_1}{K_5+S}$ for pressure p_1 and $\frac{C_2}{K_5+S}$ for p_2—then the factors K_4 and K_6 are given by—

$$K_4 = \frac{C_2 - C_1}{p_2 - p_1}$$

$$\text{and } K_6 = \frac{C_1 p_2 - C_2 p_1}{C_2 - C_1}$$

$$= \tfrac{1}{2}\left\{\frac{C_1 + C_2}{K_4} - (p_1 + p_2)\right\}$$

the latter method of fixing K_6 being preferable if K_4 be rounded off from the exact result of the first equation.

To illustrate these calculations for difference of pressure, take the diagram of Fig. 23A, in which two diagrams for 80 and 120 lbs./in.^{2} working pressures are given. The pressure difference is 40, and the difference of initial heights of the two asymptotic tangents is $120 - 86 = 34$. Therefore $K_3 = \frac{34}{40} = \cdot 85$, and $K_1 = 120 - \cdot 85 \times 120 = 18 = 86 - \cdot 85 \times 80$. Next $C_1 = 21000$ and $C_2 = 28000$; so that $K_4 = \frac{7000}{40} = 175$; and finally—

$$K_6 = \frac{21000 \times 120 - 28000 \times 80}{7000} = \frac{2520 - 2240}{7} = 40.$$

The complete formula for S in cubic inches and p in lbs. per square inch above atmosphere, is, therefore—

$$\text{Price in £} = 18 + \cdot 1\,S + \cdot 85\,p - 175\,\frac{40 + p}{400 + S}.$$

Different values of the " constants " K are naturally suitable for the various classes of engine, gas, oil, and steam ; condensing and non-condensing ; simple, compound and triple-expansion ; vertical and horizontal ; one, two, or three-crank ; tandem, side-by-side and " cross " compound ; single-acting and double-acting ; slide-valve, Corliss, piston-valve, and " drop-valve " ; " high-speed " and " low-speed " ; and in condensing engines a material difference is found between surface and injection condensation. The factors applying to different makers differ considerably ; but there is a marked tendency to equality in the slope of the asymptote for the same style of engine made by different firms. The formulas for the engine diagrams, Figs. 23A, 23B, etc., are collected together and classified in Table XXIII.

Turning now to Figs. 24A, 24B, etc., which give similar diagrams for boilers, we perceive that the hyperbolic term is seldom required to make the formula follow the price-quotations. This makes evident the makers' custom to frame their price-lists upon the straight-line law, the ordinate being the evaporative power per hour or else the area of the heating surface. This, and the other fact that the constants of different makers approach each other closely, are partly due to the close competition between makers. Two of the diagrams for water-tube boilers give £50 plus $\frac{1}{10}$th the hourly evaporative power in pounds of steam from and at 212°F. This evaporative power is, in water-tube boilers, about 3 lb. per hour per square foot of heating surface ; so that the scale of evaporative power is equivalent to one of heating surface area. In other diagrams a slightly lower co-efficient than $\frac{1}{10}$th is found combined with a higher initial constant ; and the variation from $2\frac{3}{4}$ to $3\frac{1}{4}$ in the evaporation claimed per square foot heating surface accounts for the differences among the co-efficients. In this class of boiler, the evaporative power of any one make and for any one working pressure is closely proportionate to the heating surface, and this latter is almost exactly proportionate to the whole weight of material used in the construction. In all boiler work the cost in wages bears a ratio to that in materials much lower than in engine work ; and thus the evaporative power of this class increases at a very regular rate along with the cost of construction.

In some other classes of boiler, however, this simple straight-line law of prices does not hold. The extra cost per extra square foot of heating surface is very much greater in small sizes than in large sizes ; so that the curve of prices, whether co-ordinated with area of heating surface or with steaming capacity, follows a law

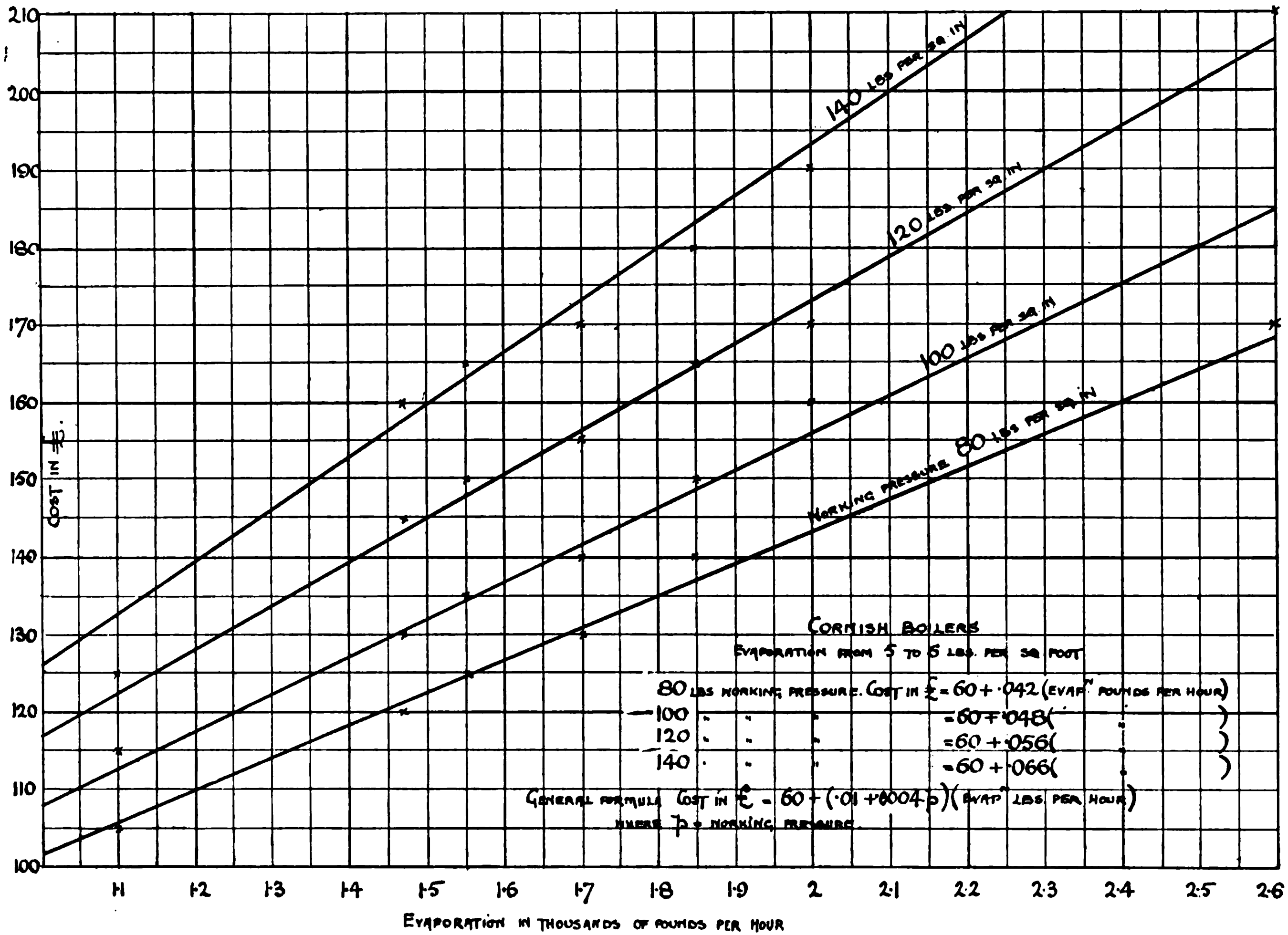


FIG. 24A.—Boiler Costs. Cornish Type for various Working Pressures.

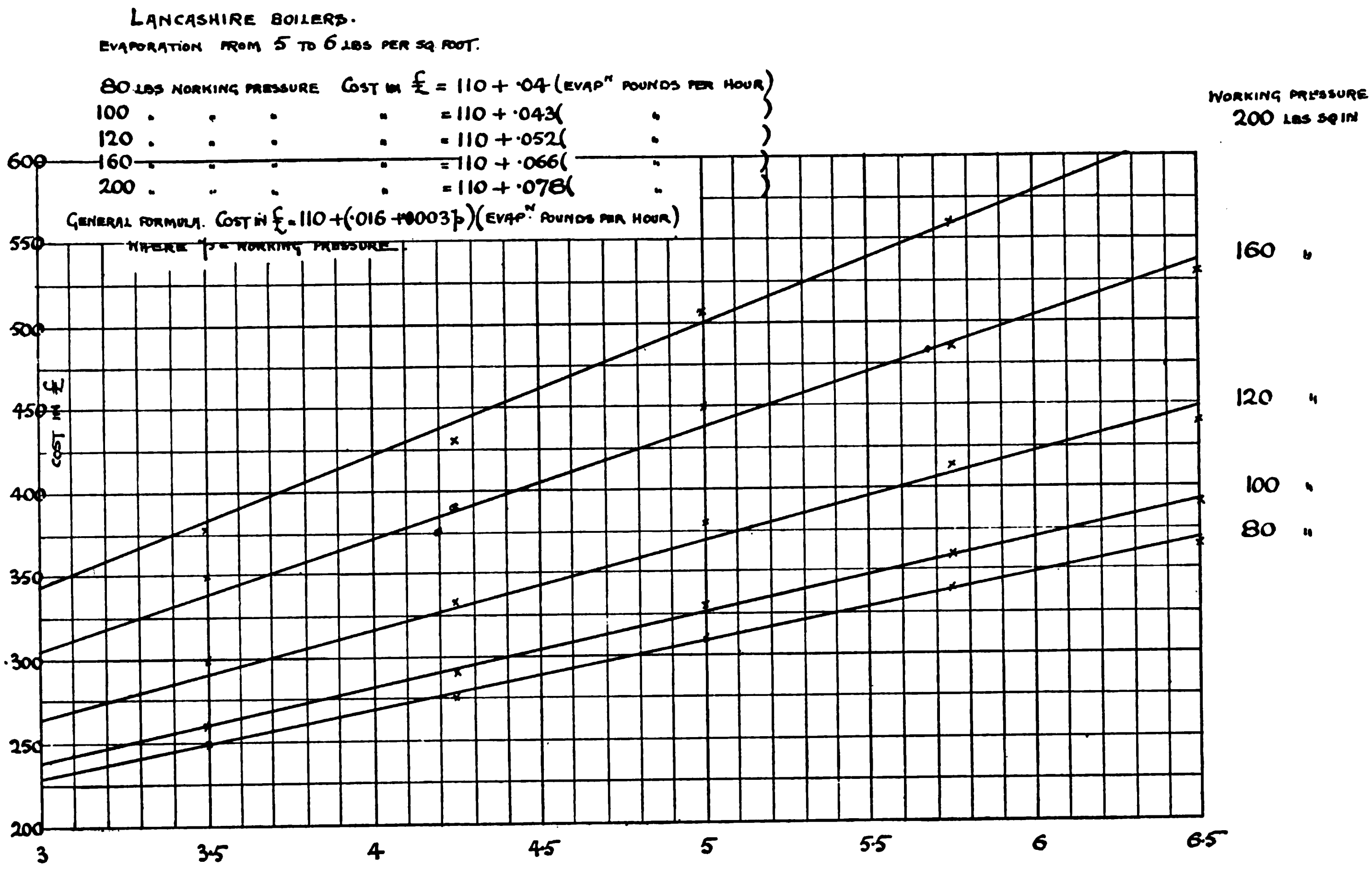


FIG. 24B.—Boiler Costs. Lancashire Boilers for different Working Pressures.

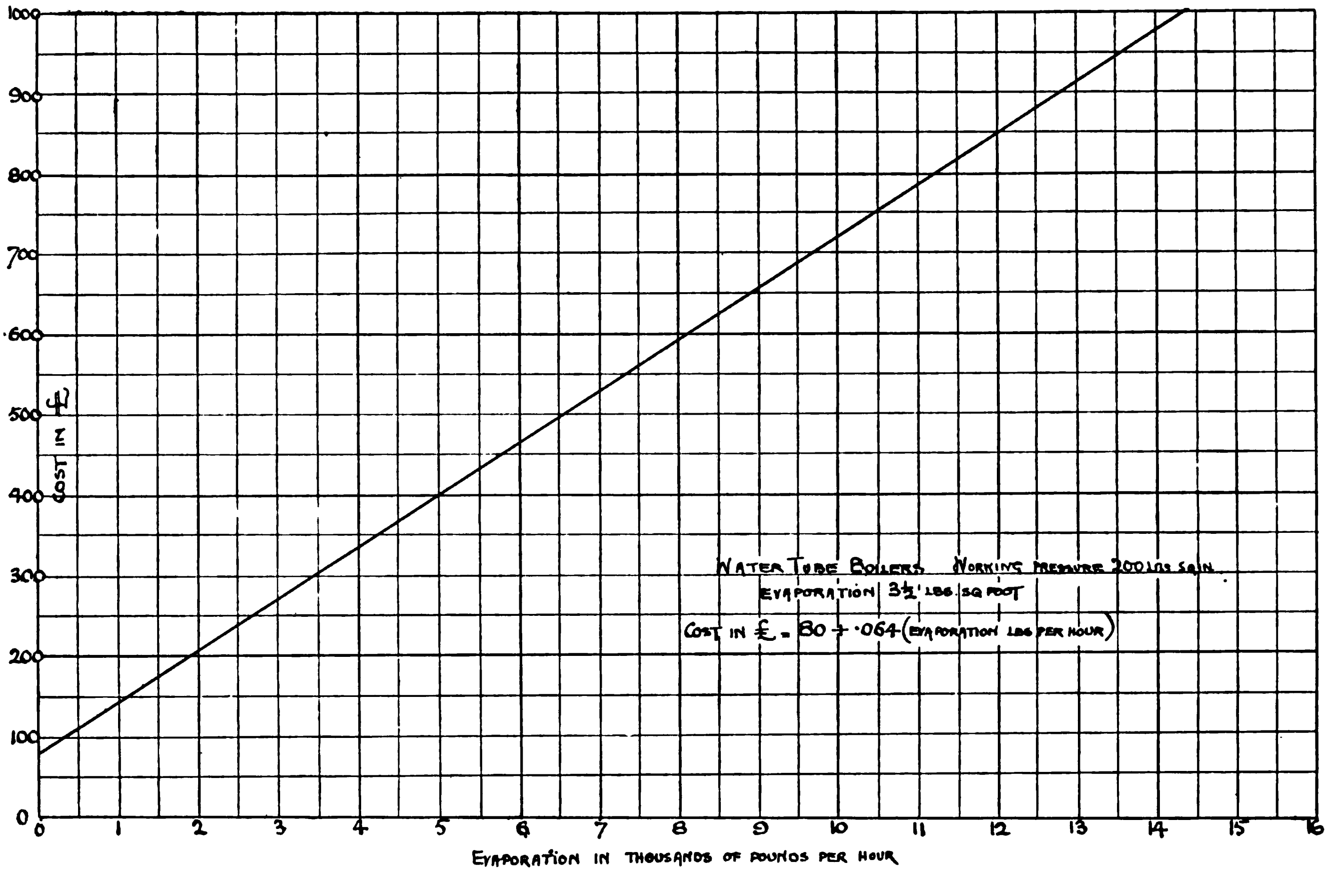


FIG. 24C.—Boiler Costs. Water-tube. Land type.

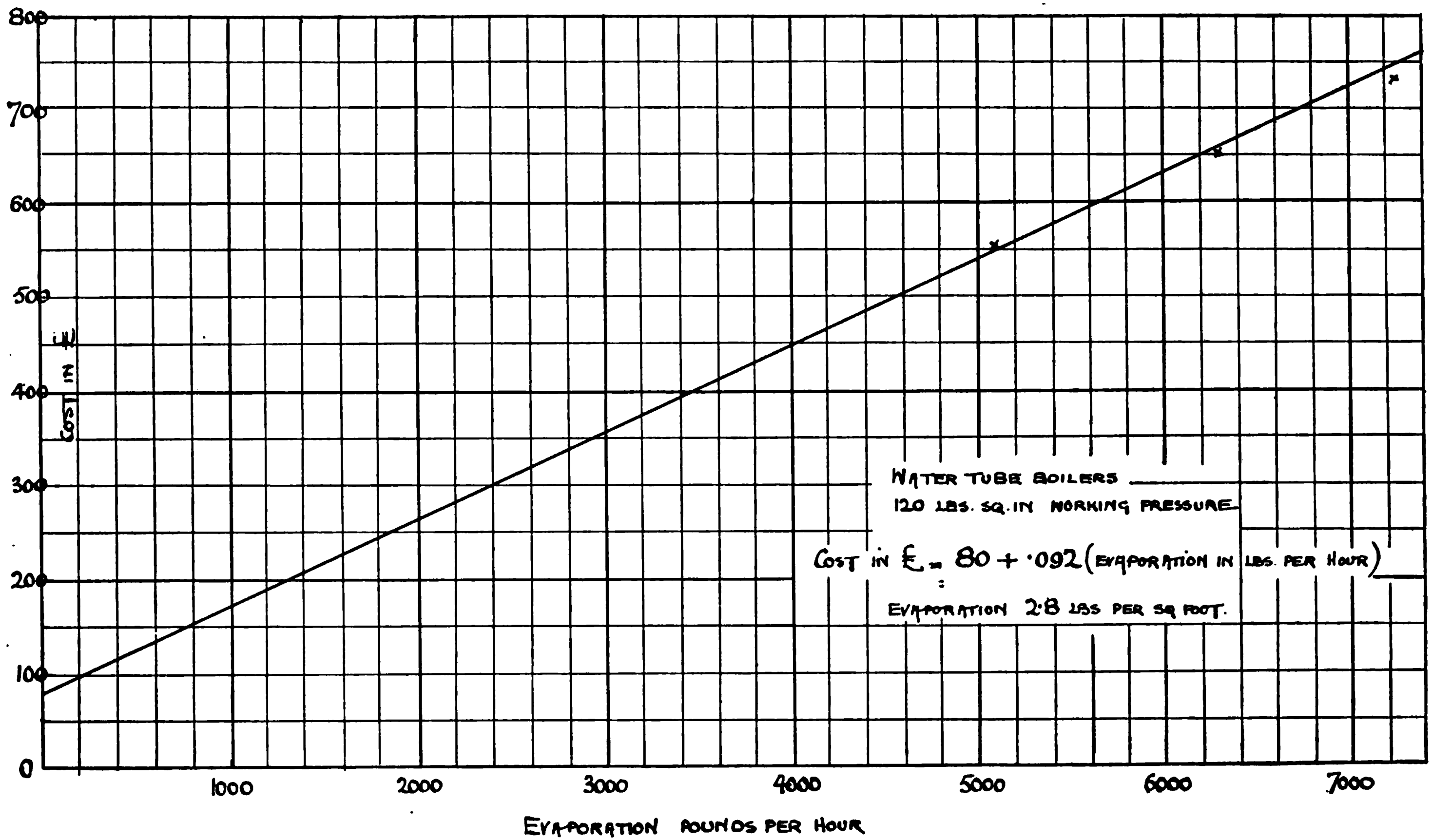


Fig. 24d.—Boiler Costs. Water-tube Boilers. Land type.

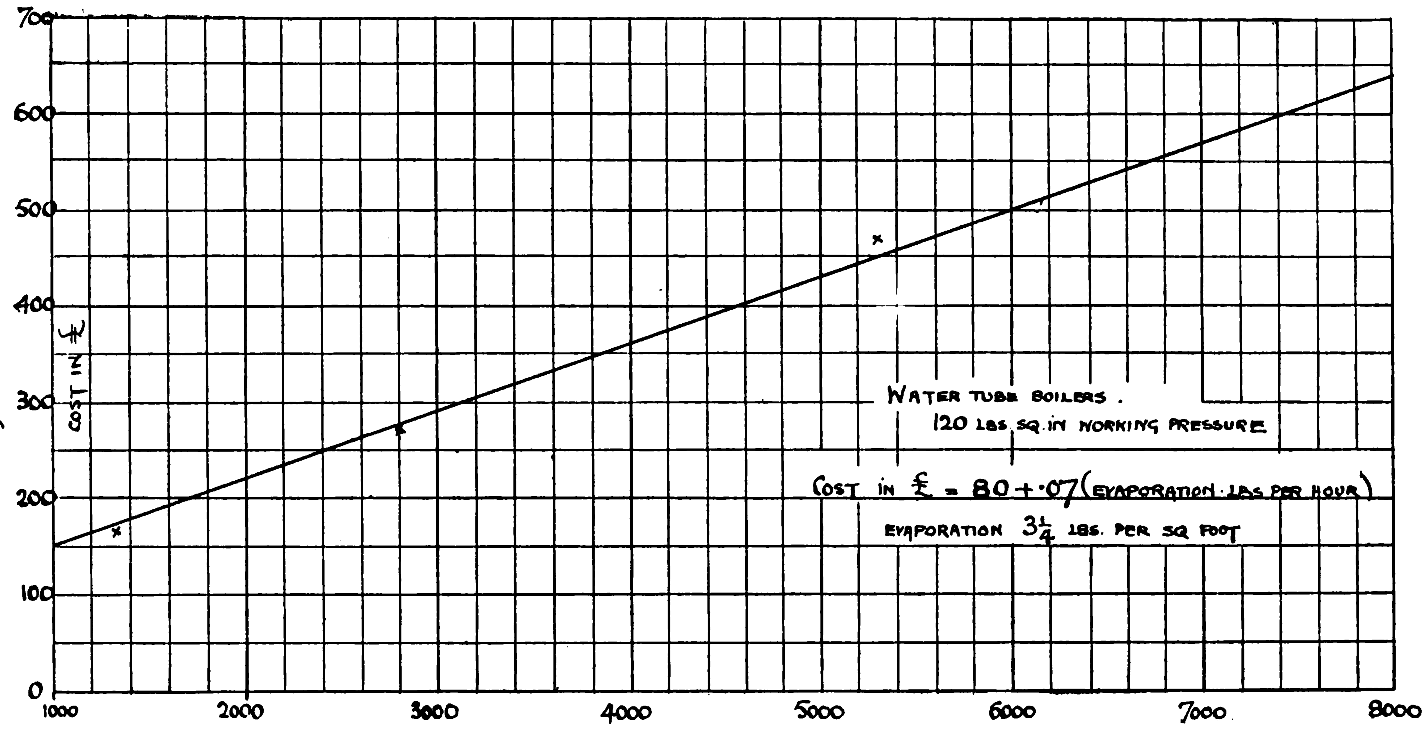


FIG. 24E.—Boiler Costs. Water-tube type.

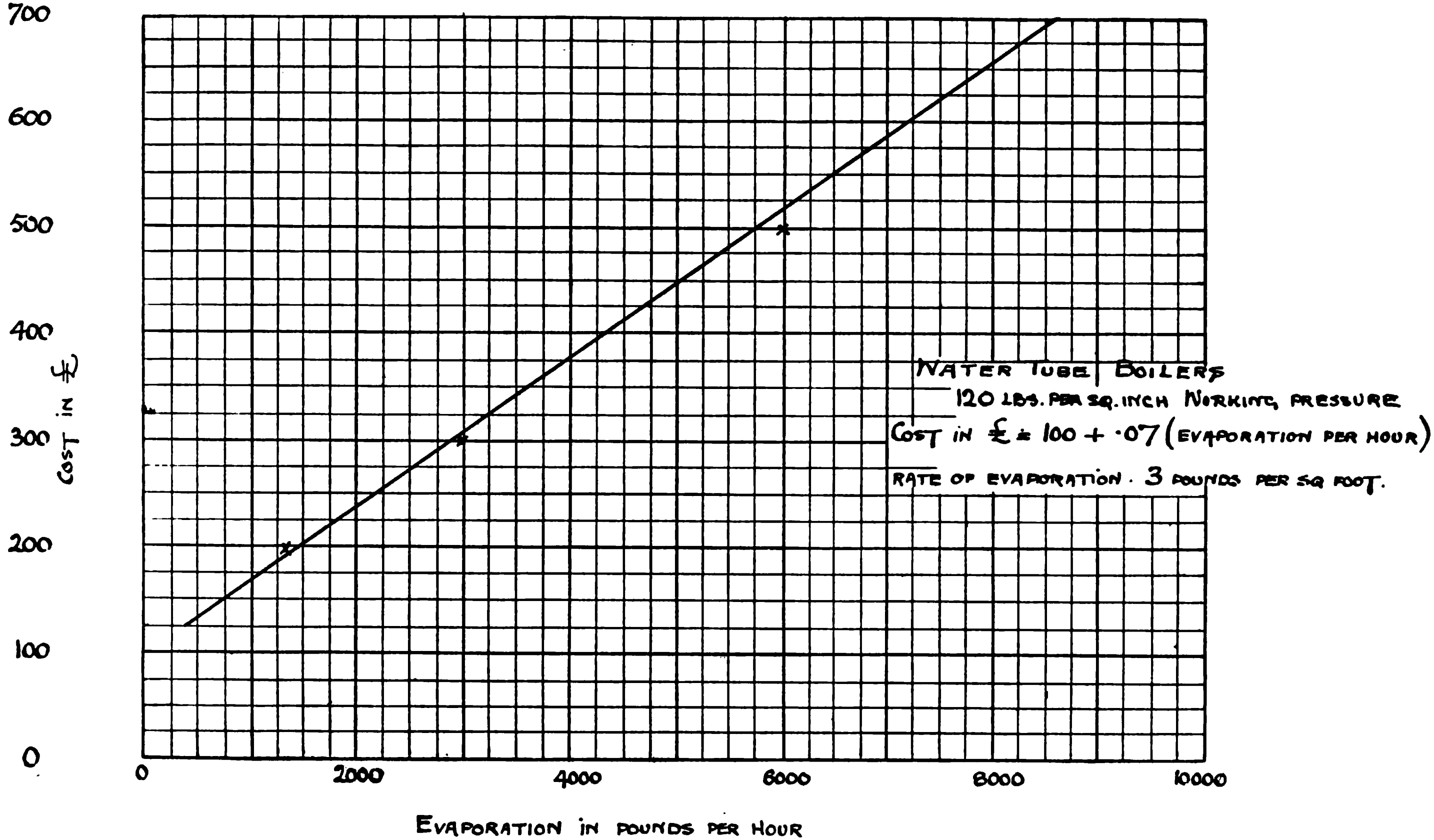


FIG. 24F.—Boiler Costs. Water-tube. Niclausse Marine type.

similar to that explained for engines. The steaming power or evaporation from and at 212°F. per hour, is an accurate measure of the heat transpower, the special unit employed being 966·1 Br. Heat units per hour = 750,000 ft. lbs. per hour = 12,500 ft. lbs. per minute instead of the regulation 33,000 per "horse-power"; so that the "evaporative power" is $2\frac{5}{8}$ times the heat horse-power. Diagrams have already been given in Chapter II, Fig. 18, showing how the heat horse-power varies with the heating surface according to the efficiency demanded, the air-admission to the furnace, and the steam pressure. The prices demanded per unit of evaporative power may be taken as all conforming to the same average efficiency; but one style of furnace permits of more careful and economical regulation of the air-admission than another, while high-pressure boilers undoubtedly require relatively more heating surface than low pressure ones. The cost to the maker of each style is nearly in direct proportion to the extent of heating surface; but competition between different styles, or classes, of boilers prevents the prices following the makers' costs quite closely.

The diagrams given in Figs. 24A, 24B, etc., and the formulas in Table XXIV illustrate how this law runs for several patterns of boiler. It must be understood that the prices here given do not include the cost of boiler "setting," nor of buildings for the housing of the boilers and coal or other fuel.

Here there is little difficulty in ascertaining the influence of working pressure upon the cost, and this influence is shown on the diagrams by the series of straight lines for different pressures. The slopes of these lines vary nearly uniformly with the pressure, and in the individual price-lists the deviation from this uniformity is found to be quite irregular. Very commonly the "initial constant" or starting height of the straight line is not changed by change of pressure, which means that for very small sizes the design and cost of the boiler depends little, if at all, upon considerations of strength. The "general" formulae given on the diagrams and in Table XXIV show clearly this constancy of the starting point and uniform gradation of the slope with pressure.

The next eight diagrams, Figs. 25 to 32, give the results of careful estimates for complete plants for all horse-powers up to 500. They are in pairs, one of each pair being devoted to capital outlay and the other to annual costs. The first pair refers to vertical compound steam engines with water-tube boilers working at 120 lbs. per square inch gauge pressure without condensation. The second pair refers to similar steam plants working with surface condensa-

TABLE XXIV

FORMULAE FOR PRICES OF BOILERS IN £ IN TERMS OF E=EVAPORATIVE POWER IN LBS. OF WATER PER HOUR.

Diagram Letter in Fig. 24.	Type of Boiler.	Working Pressure. lbs. per square inch.	Formula for Price. £ in terms of E.
A	Cornish.	80	60 + ·042 E
,,	,,	100	60 + ·048 E
,,	,,	120	60 + ·056 E
,,	,,	140	60 + ·066 E
			General 60 + (·01 + ·0004 p) E
B	Lancashire	80	110 + ·040 E
,,	,,	100	110 + ·043 E
,,	,,	120	110 + ·052 E
,,	,,	160	110 + ·066 E
,,	,,	200	110 + ·078 E
			General 110 + (·016 + ·0003 p) E
	WATER TUBE		
C	Babcock Land Type	200	80 + ·064 E
D	,, ,, ,,	120	80 + ·092 E
E	Sinclair ,, ,,	120	80 + ·07 E
F	Niclausse Marine Type	120	100 + ·07 E

NOTE.—For Cornish and Lancashire boilers the rate of Evaporation per hour per sq. foot of heating surface is 5 to 6 lbs.; for Water-Tube boilers it is 3 to 3½ lbs.

tion; the third pair to gas engines with Dowson gas-producers; and the fourth pair to Diesel oil engines. The capital outlays include all necessary buildings. The annual costs include 4½ per cent. interest on capital outlay; ground rent at 5 per cent. of capital value of ground; 4½ per cent. for maintenance, repairs and depreciation of buildings, except only 3 per cent. for such buildings as chimneys, hot wells and other tanks; and for maintenance, repairs and depreciation of engines 9 per cent., of condensers 7 per cent., and of boilers 13 per cent. on small sizes to 11 per cent. on large sizes. The other annual costs included are fuel, lubricant and other materials, and wages for engine driving, stoking, removal of ash. As the cost of fuel is a large proportion of the total, these diagrams would be of no value without the price per ton of the fuel used being stated. The prices stated on the diagrams were selected as suited to the neighbourhood of London. A statement will be found below of the modifications of the results for other fuel prices likely to be charged in other parts of Britain.

In these diagrams the costs are co-ordinated with the brake horse-power, the mechanical efficiency, or ratio of brake to cylinder indicated power, being assumed as from 80 to 83 per cent. In the non-condensing steam engines from 4 lbs. for small to 3 for large sizes is taken as the coal consumption per ind. h.p. hour; and from 3 to 2 lbs. per ind. h.p. hour in the condensing engines.

Referring now to Fig. 25, the capital outlay on non-condensing steam plant, it is seen that the cost of the engine and its accessories alone runs nearly on a straight-line law, being approximately £120 plus £2 18*s*. per brake horse-power, or, taking sizes not less than 150 brake horse-power only, more nearly £75 plus £3 per horse-power. Allowing for the slight curvature of the diagram, it is more accurately represented by—

$$50 + 3\,T + \frac{7000}{50 + T}$$

in £ where T = brake horse-power.

The prices of the boilers follow a curve very similar to that for the engines and hardly distinguishable from it. For small sizes the boiler costs a little more than the engine, and for large sizes a little less than the engine, the two curves crossing near 200 horse-power. The approximate straight line for the boiler prices is—

$$£255 + 2{\cdot}3\ T.$$

The cost of the chimney is from one-third to one-fourth of that of the boiler. The rest of the capital outlay is on engine, boiler and

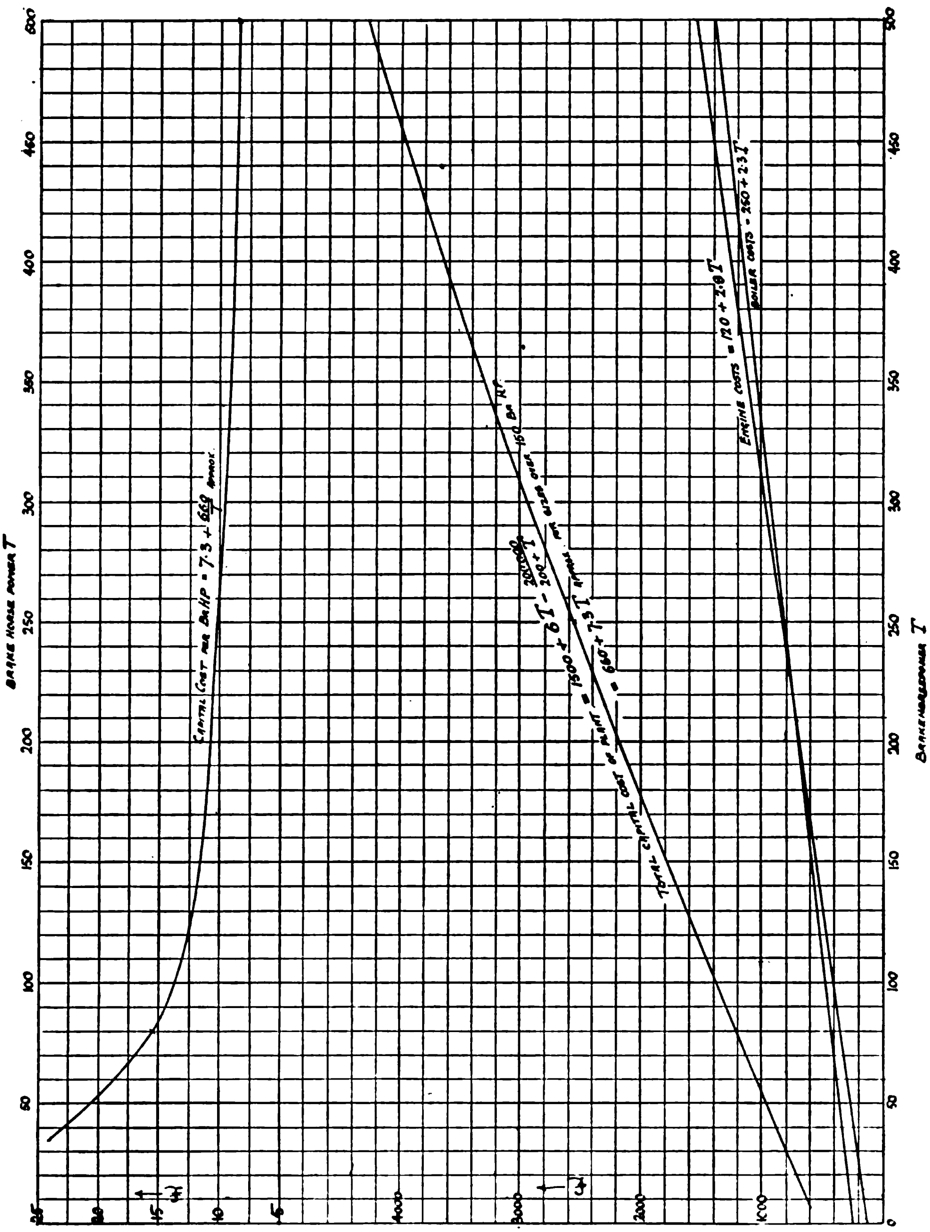


FIG. 25.—Capital Costs of Power Plants. Non-Condensing, Vertical Steam Engines and Water-tube Boilers.

coalhouses, and on foundations. The total is set out as a curve which is accurately represented by the formula—

$$1500 + 6\ T - \frac{200{,}000}{200 + T}$$

and approximately for sizes over 150 br. h.p. by the straight line—

$$660 + 7{\cdot}3\ T.$$

The three straight-lines making up this total are—

Engine, etc.	75 + 3·0 T
Boiler, etc.	255 + 2·3 T
Buildings, etc.	330 + 2·0 T
Total	£660 + 7·3 T.

The upper curve on the diagram reduces this to per brake horse-power, and is approximately the above divided by T, or—

$$£7{\cdot}3 + \frac{660.}{T}$$

The next diagram, Fig. 26, gives the annual costs for the same plants. The lowest line gives the total charge for interest, rent, depreciation, maintenance and repair. The average percentage on the total cost is not quite the same for small and large sizes, but in these particular plants it varies little from 12½ per cent. This curve is thus nearly a straight line, and it is best represented by the formula—

$$£90 + {\cdot}9\ T.$$

The fuel curve is calculated for coal at 16*s.* per ton. The working year is taken as 50 × 54 = 2,700 hours; but allowance is made for burning fuel outside working hours in getting up steam and maintaining it during meal hours. This allowance is no less than 750 hours per year in addition to the above 2,700, or 28 per cent. extra. The amount of steam, and also that of coal, used per horse-power-hour is greater for small than for large sizes. The coal is here taken at 4 lbs. per br. h.p. hour for the smallest sizes, diminishing to 3 lbs. for the largest sizes. The accurate formula for the curve is—

$$220 + 4\ T - \frac{6500}{20 + T};$$

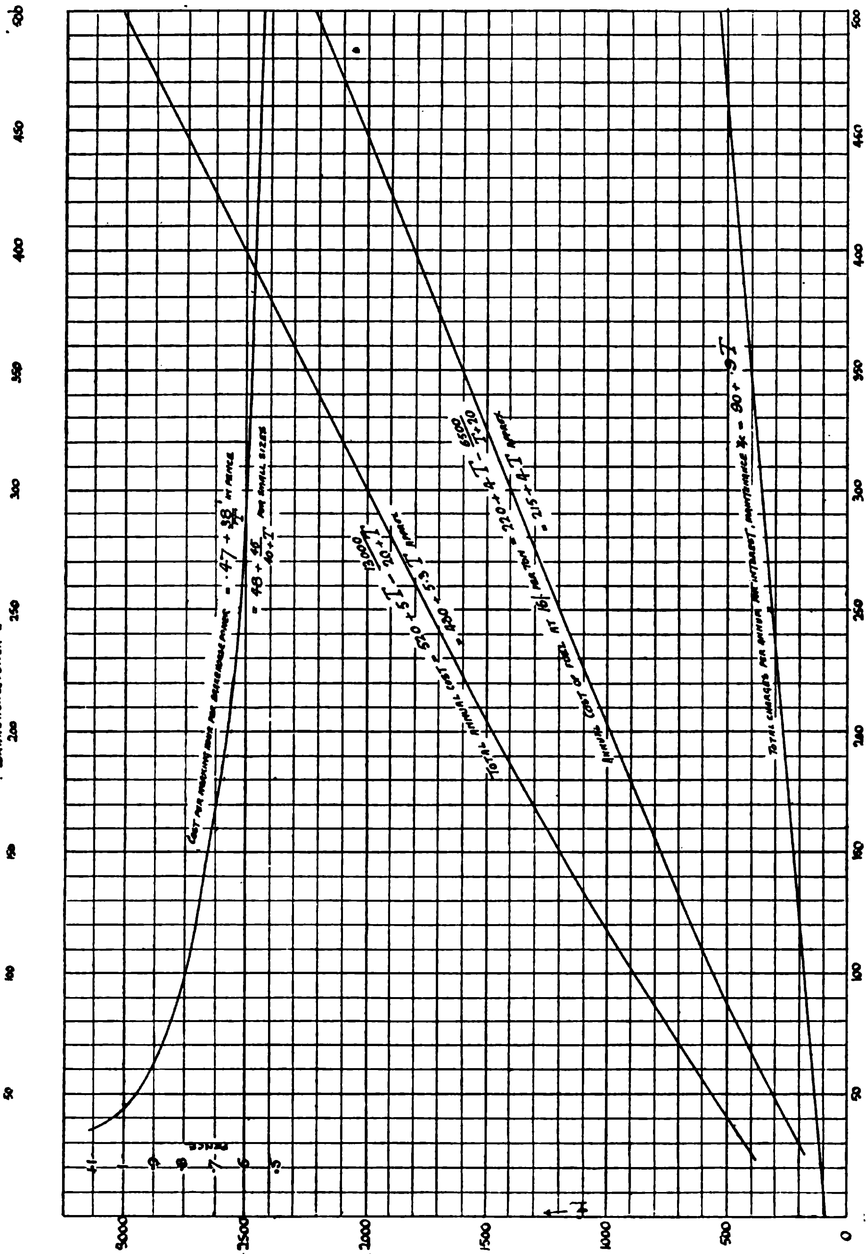


FIG. 28.—Annual Costs of Power Plant. Non-Condensing Vertical Steam Engines and Water-tube Boilers.

but, taking it as a straight line, it runs nearly as—

$$215 + 4\ T$$

for the larger sizes.

These are the two most important items in the total annual cost. The remainder consists of lubricants, etc., and wages. The curve of this total is given accurately by—

$$520 + 5\ T - \frac{13000}{20 + T};$$

and, as an approximate line, by—

$$430 + 5{\cdot}3\ T;$$

but this straight line errs in excess by as much as £150 for small sizes between 30 and 50 horse-power. At and above 150 horse-power, it is fairly correct.

For these larger sizes, taking it as a straight line, the whole annual cost may be considered as made up of—

Interest, Depreciation, Maintenance, etc. .	90 + 0·9 T
Fuel at 16*s.* per ton	215 + 4·0 T
Wages, Lubricant, etc., etc. . . .	125 + 0·4 T
Total	£430 + 5·3 T

But in this account the last line is misleading, the wages, lubricant, etc., by itself running much more nearly according to the straight line 90 + ·5 T. If this be added to the two other straight lines, there is obtained 395 + 5·4 T.

The upper curve of the diagram gives this total annual cost reduced to per brake horse-power-hour and in pence. The working year is taken as 2,700 hours; and, if one throws out of the reckoning the smallest sizes, and calculates from (430 + 5·3 T) £ per year, the cost per horse-power-hour in pence would be—

$$\left(\frac{430}{T} + 5{\cdot}3\right)\frac{240}{2700} = {\cdot}47 + \frac{38}{T}.$$

But there is written on the curve on the diagram a formula which allows more accurately for the deviation of the small sizes from the straight-line law, namely—

$${\cdot}48 + \frac{45}{40 + T} \text{ pence per brake horse-power-hour.}$$

Leaving sizes under 150 b. h.-p. out of sight, the ratio of the total *annual* cost to the total capital outlay is

$$\frac{430 + 5{\cdot}3\ \mathrm{T}}{660 + 7{\cdot}3\ \mathrm{T}} = \text{nearly } {\cdot}7,$$

varying only from ·68 at T = 150 to ·72 for 500 horse-power. For lower powers the ratio taken from the curves is about ·63 at 100 and ·55 for 50 horse-power.

The ratio of the total annual cost to the capital outlay on engine alone is—

$$\frac{430 + 5{\cdot}3\ \mathrm{T}}{75 + 3{\cdot}0\ \mathrm{T}},$$

which ratio runs from 2·3 for 150, to 2·0 for 500 horse-power.

In considering these ratios, however, it must not be forgotten that the coal is taken at 16*s.* per ton and that it accounts for from $\frac{2}{3}$ to $\frac{3}{4}$ of the total annual cost. If the fuel cost only 8*s.* per ton, these ratios would be reduced to 0·5 and 1·5 respectively.

The next pair of diagrams, Figs. 27 and 28, refers to surface-condensing engines of similar style with similar boilers.

Here the engine, condenser and accessories cost, of course, more than in the non-condensing plant; and the curvature of this part of the diagram is more pronounced in that part referring to small sizes than in the non-condensing plant. The diagram and the table give the formula which enables the cost for all sizes to be calculated with considerable accuracy. But the curvature above 150 brake horse-power is slight, and for this and larger sizes the straight line—

$$£130 + 4\ \mathrm{T}$$

is very nearly correct.

The boilers necessary are smaller and cost less than for non-condensing plant because of the smaller quantity of steam used. The saving in boiler cost is more apparent in the larger than in the smaller sizes, and from 150 b. h.p. upwards the boiler cost may be fairly well represented by the straight line—

$$215 + 2\ \mathrm{T}.$$

The error involved in neglecting the curvature is naturally greater if one applies the simpler rule to the total cost; but in this case the subjoined account does not give anywhere an error much exceeding £50 in the total for sizes over 150 b. h.p.

Engine, Condenser, etc.	£130 + 4 T
Boiler, etc.	215 + 2 T
Buildings, etc.	405 + 2 T
Total	£750 + 8 T

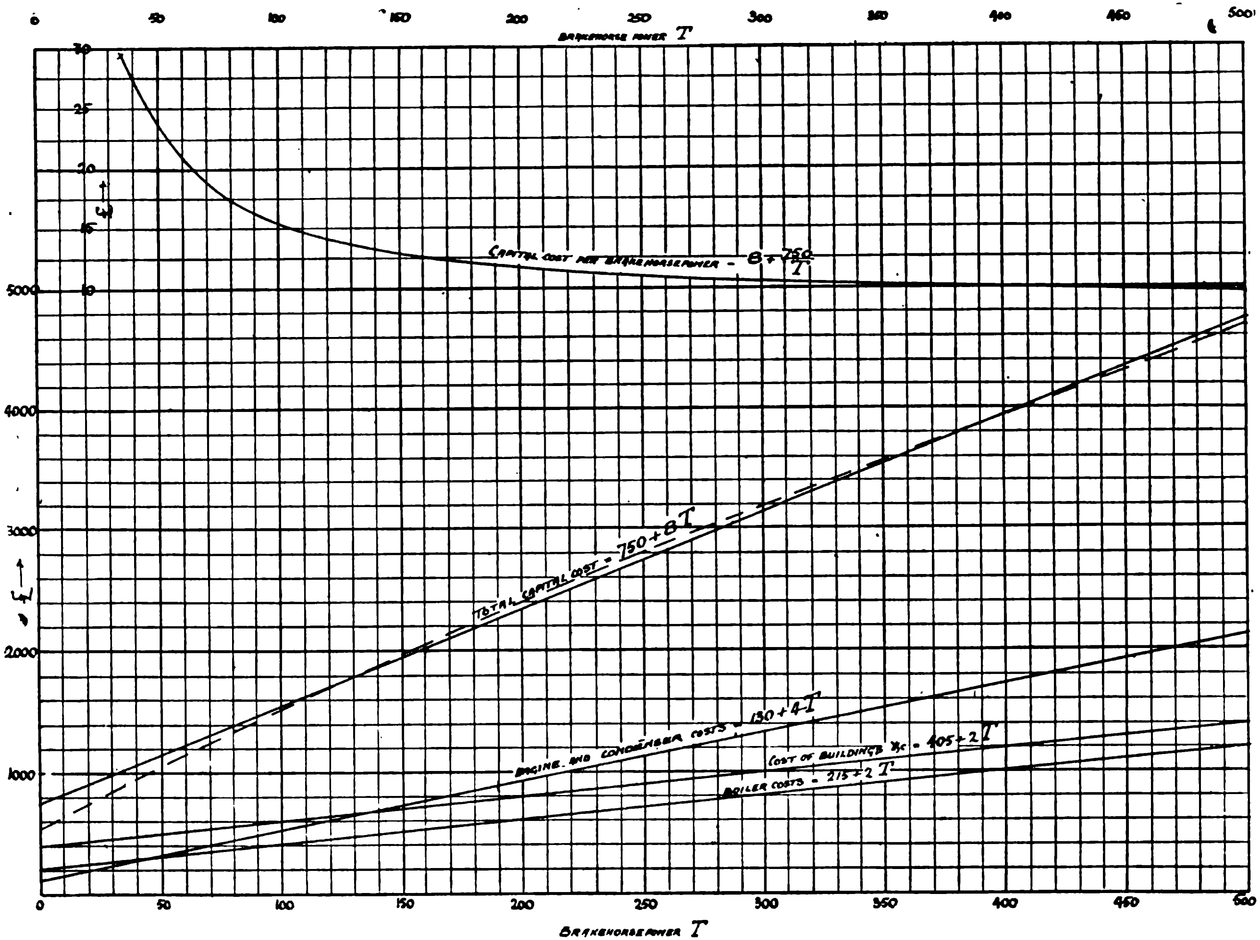


Fig. 27.—Capital Costs of Power Plants. Surface Condensing Vertical Steam Engines and Water-tube Boilers.

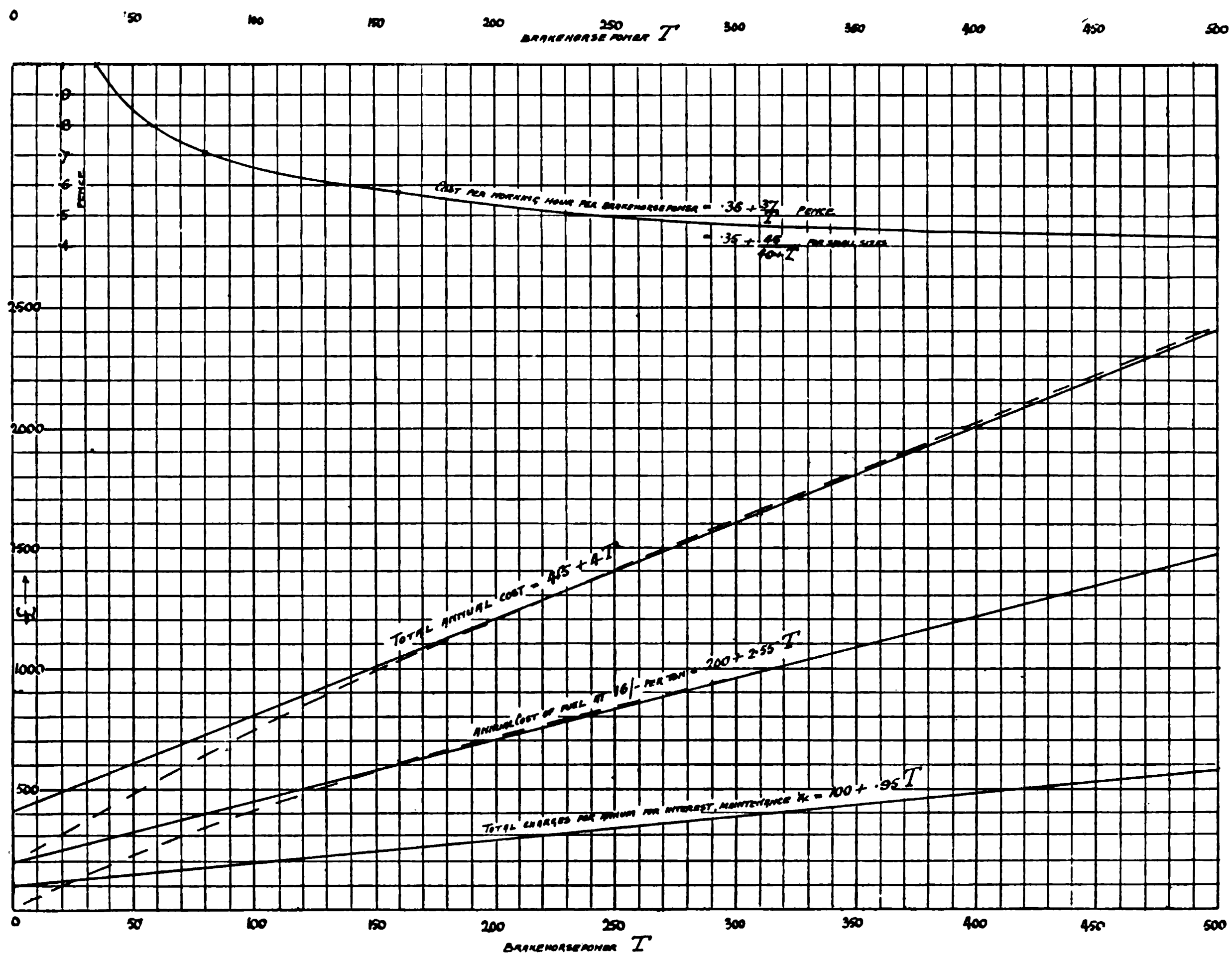


FIG. 28.—Annual Costs of Power Plants. Surface Condensing Vertical Steam Engines and Water-tube Boilers.

This reduced to per horse-power is given by the topmost curve, and is, putting aside the small sizes—

$$£8 + \frac{750}{T}.$$

Examining in the same way the curves on Fig. 28 for annual costs, it may be found that for sizes between 150 and 500 brake horse-power, the approximate straight-line variations are as follows—

Interest, Depreciation, Maintenance, etc. .	£100	+	0·95 T
Fuel at 16*s*. per ton	200	+	2·55 T
Wages, Lubricant, etc. . . .	115	+	0·5 T
Total	£415	+	4 T.

Here the fuel is computed at 3 lbs. per br. h.p. hour for the smallest engine, decreasing to 2 lbs. for the largest.

Comparing these with the previous summary for non-condensing engines, it is seen that the first item is only very slightly greater in spite of the extra cost of condensing plant. In fact, by a curious coincidence, the saving due to the smaller boiler required almost exactly compensates for the extra annual capital charge for condenser, circulating and air-pumps and cooling tanks. This compensation does not appear clearly in the capital outlay account, because it is largely due to the different rates for depreciation and repairs assumed for boilers and for condensing plant. In the fuel account the saving is not very marked for small sizes; but it becomes extremely important for large sizes, the two coefficients of T being 2·55 and 4. In the account for wages, lubricant and sundries, there is a very small difference similar to that in the capital charges. There is here also a curious compensation at work: the extra cost of attendance and lubrication on condenser and pumps being almost neutralized by the saving in the cost of the removal of a largely decreased quantity of ash proportionate to the decrease in fuel consumption.

The top curve of the condensing engine diagram reduces the total annual cost to pence per brake horse-power-hour, again assuming 2,700 working hours per year. From the straight-line approximation, this amount would be found as—

$$\left(\frac{415}{T} + 4\right)\frac{240}{2700} = \cdot 36 + \frac{37}{T}.$$

This will not apply to sizes much under 150 horse-power, and a formula adhering more closely throughout the whole range of size is

$$\cdot 35 + \frac{45}{40 + T}$$

which is, for all sizes, just ·13 penny less than for non-condensing plant.

In these condensing engines the ratio of total annual cost to total capital outlay is about ·51 at 500 h.p., increases to ·52 at 150, and then diminishes again to ·48 at 100 and to ·43 at 50 horse-power. This ratio is ·2 lower than in non-condensing engines. The ratio between the total annual costs in the two classes of engines for like horse-power is $\frac{415 + 4\,T}{430 + 5{\cdot}3\,T}$, which is ·79 at 500 and ·83 at 150 horse-power.

To illustrate how these results are affected by the price of coal the following figures will suffice.

TOTAL COST PER BRAKE HORSE-POWER-HOUR IN PENCE.

		Price of Coal. Shillings per Ton.		
		8	16	22
Non-Condensing Engines	50 br. h.p.	·73	·99	1·19
	500 ,, ,,	·35	·55	·70
Condensing Engines	50 br. h.p.	·68	·87	1·01
	500 ,, ,,	·30	·43	·53
Ratio $\frac{\text{Condensing}}{\text{Non-Condensing}}$	50 br. h.p.	·93	·88	·85
	500 br. h.d.	·86	·78	·76

The next pair of diagrams, Figs. 29 and 30, give similar results for power plants consisting of Crossley gas engines working with Dowson producer gas. In the higher ranges above 200 horse-power the costs do not conform well to the general laws seen to be applicable to steam engines, namely gradual and regular decrease of cost per horse-power as the size increases. It is believed that this irregularity is temporary and will disappear when the art of manufacture of large gas engines has settled down on lines ruled by longer experience. It is only a very few years since the first attempts were made to introduce on the market gas engines for large powers, and

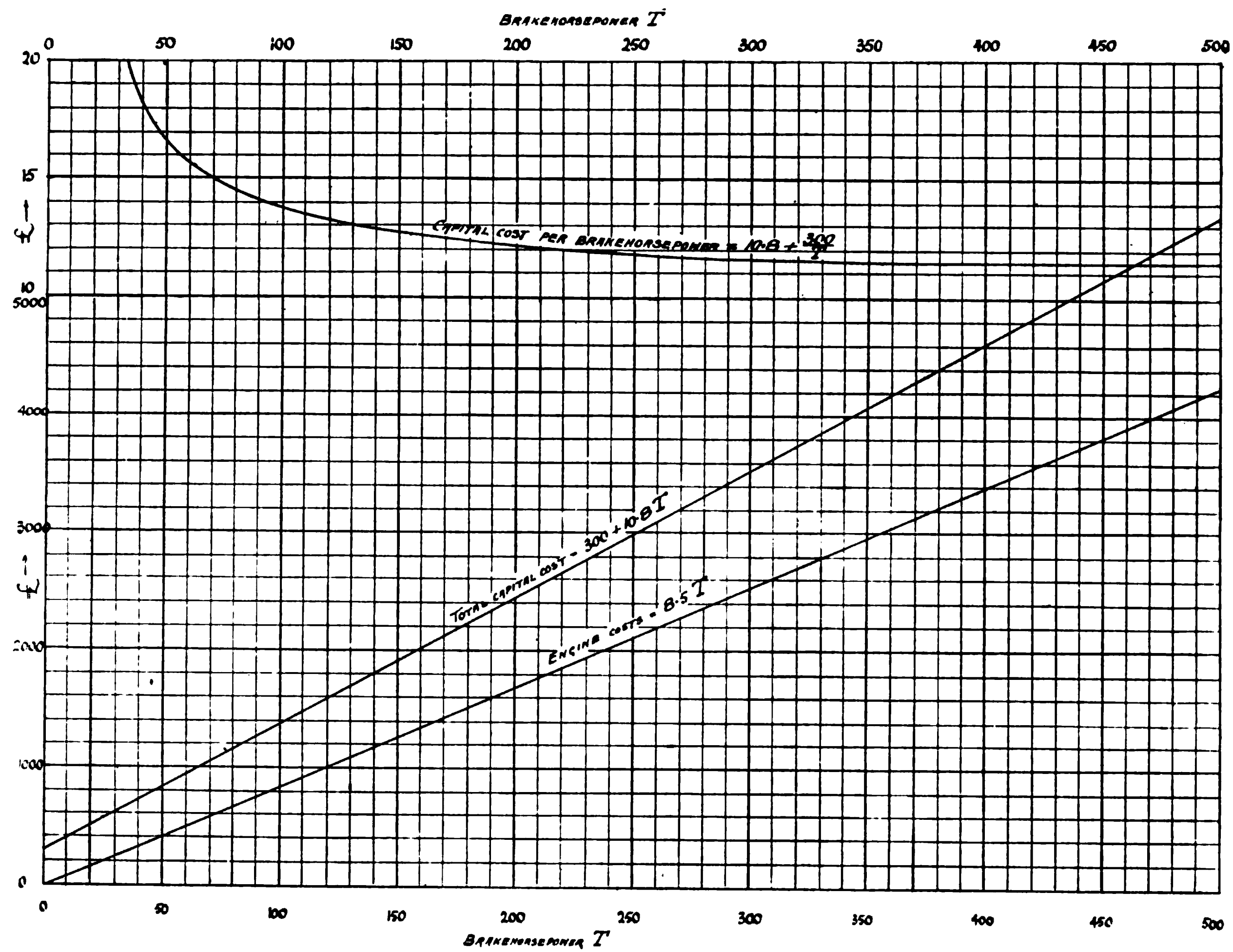


FIG. 29.—Capital Costs of Power Plants. Gas Engines and Dowson type Producer Plants.

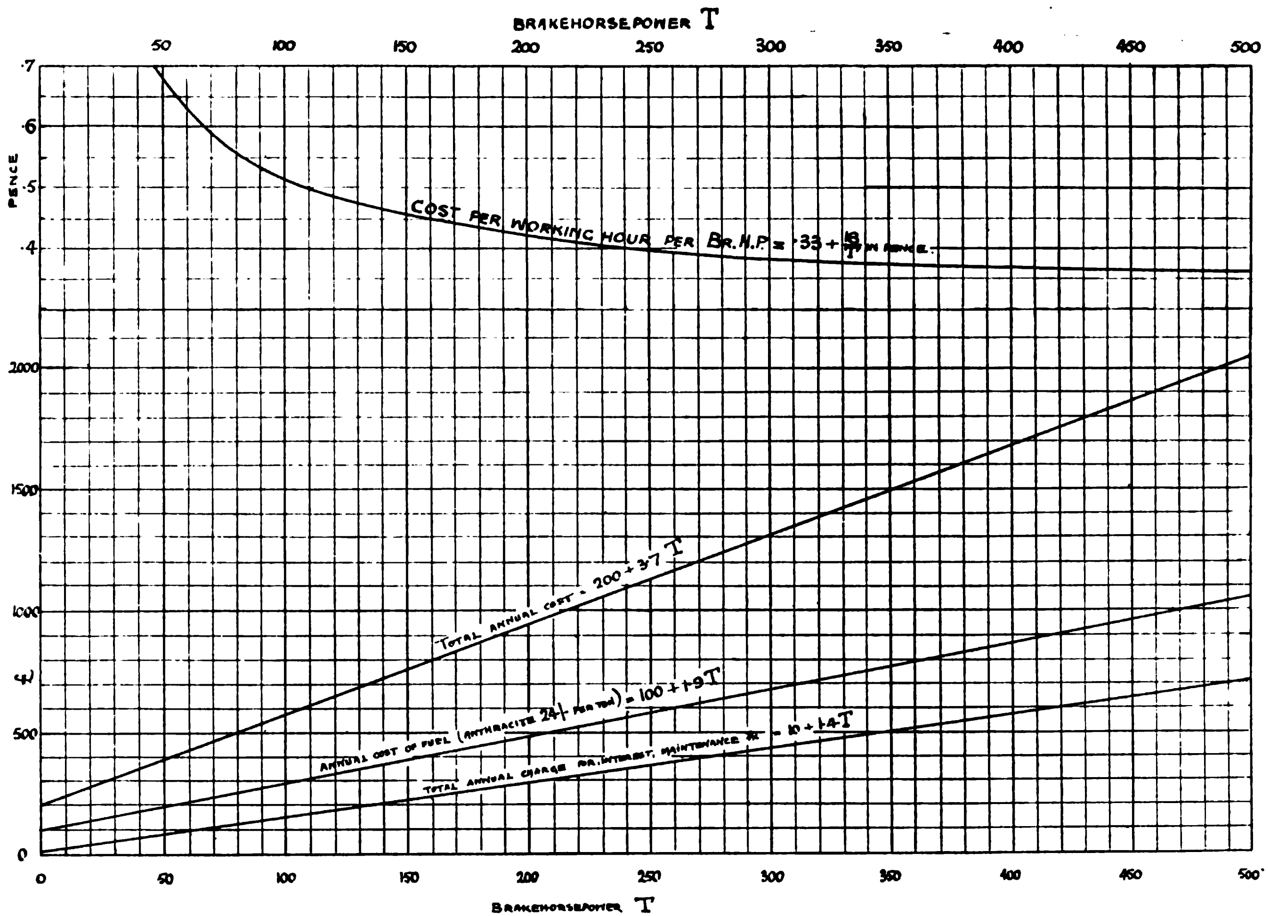


FIG. 30.—Annual Costs of Power Plants. Gas Engines and Dowson type Producer Plants.

this because there are real difficulties to be overcome in the making of large sizes. The expense of overcoming these—and they are common to gas and oil, or all internal combustion engines—results, at present, in the cost of construction above certain limits increasing rather faster than does the size. In fact, it is found that the curvature of many of the diagrams is upwards in the higher ranges of size, while it remains downwards, as in steam plants, through the lower ranges.

Nevertheless, if one be content with straight-line approximations which involve from £10 to £50 or £60 errors at certain points, these have a general resemblance to those for steam plants. In the example here taken, namely of Dowson Producers feeding Crossley gas engines up to 500 horse-power, the straight lines are as follows:—

Capital Outlay.

Engines and Accessories . . .	8·5 T
Producers and Buildings, etc. . .	300 + 2·3 T
Total	300 + 10·8 T

Total Annual Cost.

Capital Charges, Depreciations and Repairs	10 + 1·4 T
Fuel, Anthracite at 24*s*. per ton . .	100 + 1·9 T
Wages, Lubricant, etc. etc. . . .	90 + 0·4 T
Total	200 + 3·7 T

Reducing this last to per brake horse-power-hour, at 2,700 hours per year, the cost in pence is—

$$\cdot 33 + \frac{18}{T}.$$

The constant here, ·33, is only slightly less than for condensing steam engines, but the second term is only one-half of that for such steam plant. The advantage of gas over steam power is large for small powers, but decreases as the power increases.

In these gas engines the ratio of total annual cost to capital outlay varies from ·36 at 500 to ·45 at 150 horse-power. The fuel is almost exactly one-half of the total annual cost. Anthracite sometimes costs more and sometimes less than 24*s*. per ton, the price here assumed; but its variation in time and place is not so great as that of steam coal. No allowance is here made for the value of any "residuals" from the gas producers. In large gas power-plants if

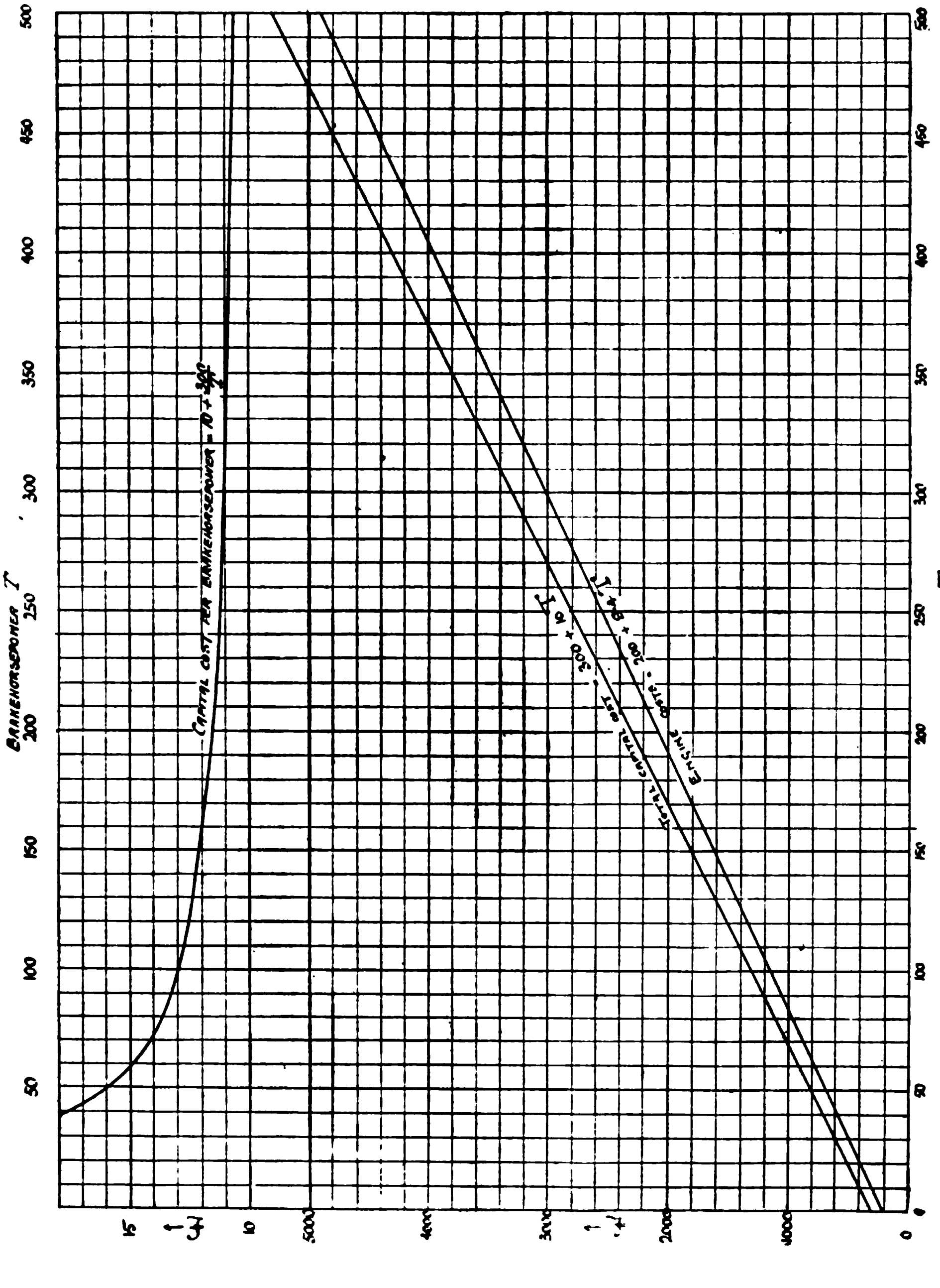


FIG. 31.—Capital Costs of Power Plants. Diesel Oil Engines.

Mond gas producers be used with recovery of ammonia, these residuals have a value which must not be neglected in the calculation; but the additions to the producing plants needed for this recovery are hardly repaid by this extra value unless the power be as much as 900 or 1,000 horse-power.

If on the other hand, rich lighting gas be used from town mains, then in the price paid for the gas the value of the residuals is already accounted for. The price in England varies from 2*s*. to 4*s*. per 1,000 cubic feet. Taking 2*s*. 6*d*. as an average, and 18 cubic feet per brake horse-power-hour as the consumption in the best modern engines, we have the cost of fuel $\frac{30 \times 18}{1000} = \cdot 54$ penny per brake horse-power-hour. At 24 horse-power the same figure would be given by our formula 100 + 1·9T for Dowson gas using 24*s*. anthracite. For larger powers the Dowson gas would be less, but this comparison is slightly unfair because it does not take account of the capital charge and the wages on the producers. As the capital charges due to the producer are (10 + ·07 T), which reduced to a fraction of a penny per horse-power-hour is $\left(\cdot 006 + \frac{\cdot 9}{T}\right)$. Allowing for this and a part of the wages, 35 to 40 h.p. is the limit above which the Dowson producer gives less total fuel charges.

The next pair of diagrams, Figs. 31 and 32, give similar estimates for Diesel engines worked with crude petroleum at 45*s*. per ton.

The Capital Outlay is—

Engines, Tanks, Piping, etc.	200 + 9·4 T
Buildings	100 + 0·6 T
Total	£300 + 10 T

This is slightly less than for the Crossley-Dowson gas plant. It increases much more rapidly than in either of the steam plants considered, but starts from a lower level. The total annual cost is—

Capital Charges	30 + 1·4 T
Fuel	15 + 1·05 T
Wages, Lubricants, etc.	65 + 0·45 T
Total	£110 + 2·9 T

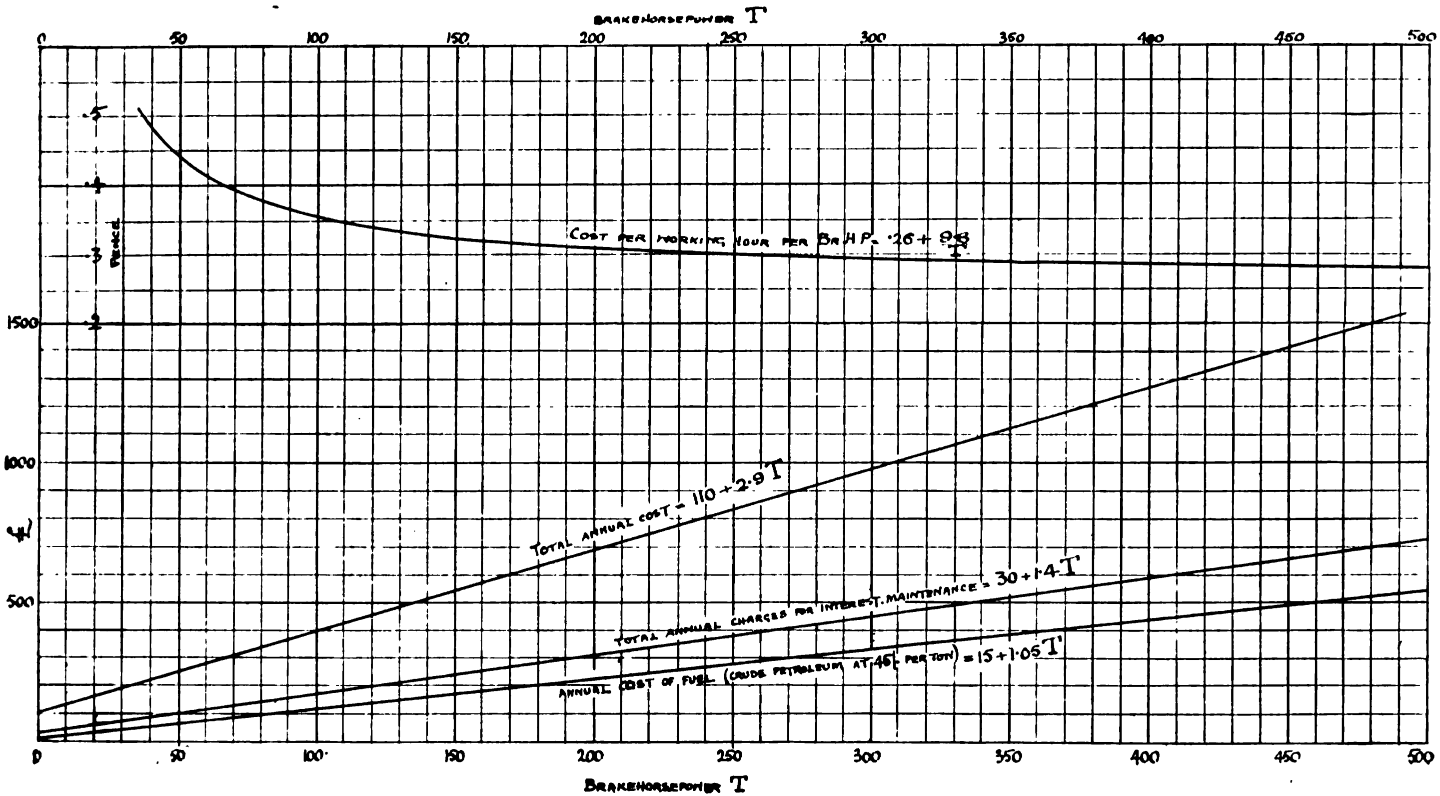


Fig. 32.—Annual Costs of Power Plants. Diesel Oil Engines.

This reduced to per working hour of a 2,700 hours' year and to per brake horse-power is, as a fraction of a penny,

$$\cdot 2\cdot 6 + \frac{9\cdot 8}{T}.$$

The chief remarkable point about these costs, is the very low figure for fuel. Unfortunately the price of petroleum varies very much from place to place and from time to time, and under adverse circumstances may be twice as much as it is here reckoned.

The straight-line formulae applicable from about 150 to 500 brake horse-power to the four kinds of power plant and shown in detail in the eight diagrams, Figs. 25 to 32, are here collected into a set of Tables for convenience of reference and comparison.

In considering all these formulations of cost per horse-power hour, it should not be forgotten that one horse-power hour is simply 1,980,000 ft.-lbs. of work, or 2 millions less 1 per cent. ft.-lbs. The costs so expressed, therefore, do not include any time-rate measurement, although it is the invariable rule, as seen in all the results quoted, that this cost per 2×10^6 ft.-lbs. is much greater for small time-rates of doing work than for large time-rates.

In Table XXXIII and Fig. 33 are reproduced some interesting calculations of cost of steam power in the United States of America from a letter in the American journal, *The Engineer*, of March 16, 1903, by Mr. W. O. Webber. His figures for "cost of plant" include that of land, buildings, chimneys, tanks, boiler setting and covering, steam pipe covering, etc., etc. For the land and buildings, inclusive of chimney, in the case of a 60 horse-power plant, $6,200 were provided, out of a total prime cost of $12,000. The boiler and accessories cost nearly $2,700, while the engine and accessories cost $2,300 nearly. The remainder was for shafting and belting. The total was $200 per horse-power; but in his table Mr. Webber gives $180 per horse-power for this size. He estimates the total of the "fixed charges" as 14 per cent. of the prime total cost. His table and diagram are divided into three sections, for Simple Engines up to 80 H.P.; for Compound Condensing Engines up to 1,000 H.P.; and for Triple Expansion Condensing Engines up to 2,000 H.P. For the smaller sizes increase of price of coal from $2 to $5 per ton, raises the total annual cost by 50 per cent., which ratio again applies to the largest sizes; while at intermediate power this percentage is a little higher. At $5 per ton for coal, the total annual cost per horse-power bears to the total cost of plant a ratio which for simple engines varies, in the direction of increasing size, from

TABLES XXV–XXXII

STRAIGHT-LINE FORMULAE FOR CAPITAL COSTS AND FOR COSTS PER YEAR OF 2,700 WORKING HOURS IN £ OF COMPLETE POWER PLANTS IN TERMS OF BRAKE HORSE-POWER T.

No. of Table and Diagram.	Description of Plant.	Capital Costs.		Annual Costs.		Pence per Brake Horse-Power-Hour.	Ratio of Annual Cost to Capital Cost, at 150 H.P.	at 500 H.P.	Ratio of Annual Cost to Capital Cost of Engine. at 150 H.P.	at 500 H.P.
25 and 26	Steam. Vertical Compd. Engines. Non-Condensing. Water-Tube Boilers. 120 lbs. p. sq. inch.	Engine, etc.	75 + 3·0 T	Interest, Depreciation and Maintenance	90 + 0·9 T					
		Boiler, etc.	255 + 2·3 T	Fuel at 16*s.* per ton	215 + 4·0 T					
		Buildings, etc.	330 + 2·0 T	Wages, Lubricant, etc.	125 + 0·4 T					
		Total	660 + 7·3 T	Total	430 + 5·3 T	$\cdot 48 + \frac{45}{40+T}$	·08	·72	2·3	2·0
27 and 28	Steam. Vertical Compd. Engines. Surface-Condensing. Water-Tube Boilers. 120 lbs. p. sq. inch.	Engine, etc.	130 + 4 T	Interest, Depreciation and Maintenance	100 + 0·95 T					
		Boiler, etc.	215 + 2 T	Fuel at 16*s.* per ton	200 + 2·55 T					
		Buildings, etc.	405 + 2 T	Wages, Lubricant, etc.	115 + 0·5 T					
		Total	750 + 8 T	Total	415 + 4 T	$\cdot 35 + \frac{45}{40+T}$	·52	·51	1·39	1·13
29 and 30	Gas. Crossley Engines with Dowson Producer Gas	Engine, etc	8·5 T	Interest, Depreciation and Maintenance	10 + 1·4 T					
		Producers and Buildings, etc.	300 2·3 T	Fuel at 24*s.* per ton	100 + 1·9 T					
				Wages, Lubricant, etc.	90 + 0·4 T					
		Total	300 + 10·8 T	Total	200 + 3·7 T	$\cdot 33 + \frac{18}{T}$	·39	·36	·59	·48
31 and 32	Diesel Oil Engine.	Engine, Tanks, etc.	200 + 9·4 T	Interest, Depreciation and Maintenance	30 + 1·40 T					
		Buildings, etc.	100 + 0·6 T	Fuel at 45*s.* per ton	15 + 1·05 T					
				Wages, Lubricant, etc.	65 + 0·45 T					
		Total	300 + 10 T	Total	110 + 2·9 T	$\cdot 26 + \frac{9\cdot 8}{T}$	·30	·30	·34	·32

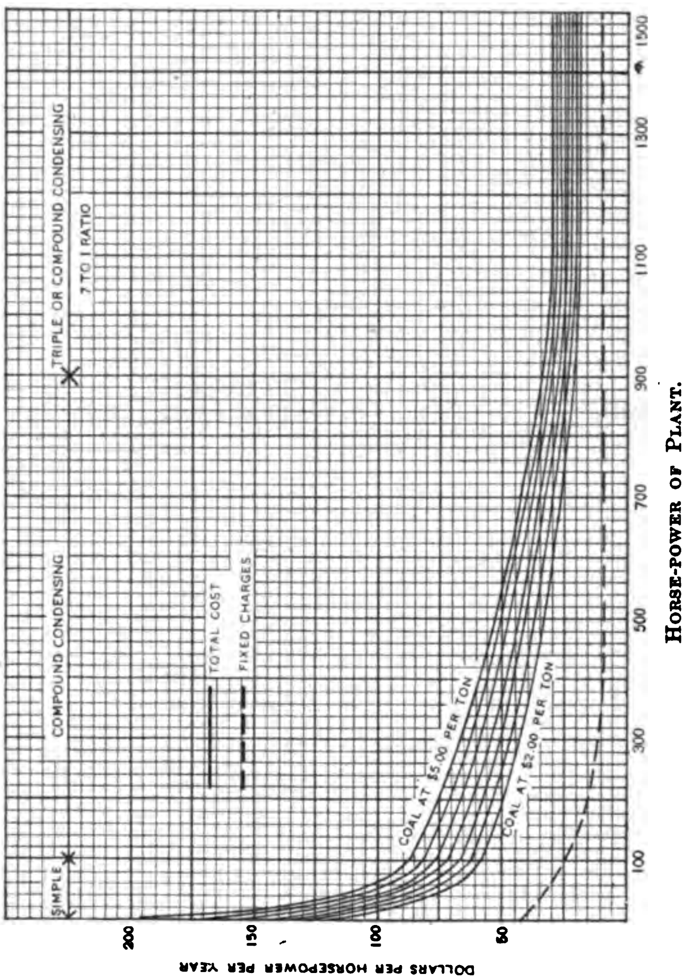


FIG. 33.—American Costs of Steam Power.

	SIMPLE.				COMPOUND CONDENSING.										TRIPLE.	
Size of plant, h. p.	20	40	60	80	100	200	300	400	500	600	700	800	900	1000	1500	2000
Cost of plant per h. p.	$200 00	$190 00	$180 00	$175 00	$170 00	$146 00	$126 00	$110 00	$96 00	$85 00	$76 00	$69 00	$64 00	$60 00	$58 00	$56 00
Fixed charges, at 14 per cent.	28 00	26 60	25 20	24 50	23 80	20 40	17 65	15 40	13 45	11 90	10 65	9 65	8 95	8 40	8 12	7 85
Coal per h. p. h. in lbs.	12 00	10 00	9 00	8 00	7 00	6 50	6 00	5 50	5 00	4 50	4 00	3 50	3 00	2 50	2 00	1 50
Cost, at $4.00 per ton	66 00	55 00	49 50	44 00	38 50	35 70	33 00	32 00	27 50	24 70	22 00	19 20	16 50	13 75	11 00	8 25
Attendance, 10h. basis	30 00	20 00	15 00	12 00	12 00	10 00	8 60	7 25	6 20	5 40	4 70	4 15	3 75	3 50	3 25	3 00
Oil, waste, and supplies	6 00	4 00	3 00	2 60	2 40	2 00	1 72	1 45	1 24	1 08	94	88	75	70	65	60
With coal at $5.00 per ton	146 50	119 35	105 07	95 10	86 40	77 10	69 22	61 90	55 29	49 28	43 79	39 73	34 05	29 80	25 77	21 75
With coal at $4.50 per ton	138 25	112 47	98 80	89 60	81 50	72 60	65 07	58 10	51 79	46 18	41 04	36 28	32 00	28 05	24 39	20 72
With coal at $4.00 per ton	130 00	105 60	92 70	84 10	76 70	68 10	60 97	56 10	48 39	43 08	38 09	33 83	29 95	26 35	23 02	19 70
With coal at $3.50 per ton	121 75	98 72	86 51	78 60	71 90	63 70	56 82	50 50	45 04	39 98	35 54	31 48	27 87	24 60	21 64	18 67
With coal at $3.00 per ton	113 5	91 85	80 32	73 10	67 00	59 20	51 67	46 70	41 49	36 88	32 79	29 03	25 80	22 90	20 27	17 65
With coal at $2.50 per ton	105 25	84 97	74 13	67 60	62 30	54 75	48 59	43 00	38 09	33 83	30 04	27 18	23 75	21 20	18 89	16 60
With coal at $2.00 per ton	97 00	78 10	67 95	62 10	57 45	50 25	44 47	40 10	34 64	30 73	27 29	24 23	21 70	19 47	17 52	15 57

TABLE 33.—American Costs of Steam Power in Dollars per Brake Horse-power Year.

·73 to ·55; for compound engines from ·50 up to ·55 and down again to ·50; and for triple-expansion engines from ·45 to ·39.

One Board of Trade Unit of electrical power is one kilowatt hour and equals $\frac{1 \cdot 98 \times 10^6}{\cdot 746} = 2 \cdot 654 \times 10^6$ ft.-lbs. This measure is mainly used for the electrical output between the terminals of the generating dynamo. An efficiency of 88½ per cent. as between this output and the similar output of the driving engine (which is the brake horse-power of this engine), would require exactly 3 million ft.-lbs. work to be done by the engine per B.T.U. If the efficiency were 89½ per cent., then exactly 1½ engine brake horse-power would yield one kilowatt output from the dynamo. Although the combined mechanical and electrical efficiency of the dynamo is generally lower than this, this relation is worth fixing in mind.

Regarding the costs of power for electrical purposes much useful information is to be had from the yearly returns made by the electrical companies to the Board of Trade. In 1898 the statistics then available were collated by Mr. Robert Hammond in a most valuable paper read before the Institute of Electrical Engineers in March, 1898 (vol. xxvii); and in November, 1899 (vol. xxix) Mr. John Holliday and Mr. R. E. Crompton read interesting papers on the "Cost of Steam Raising" and on the "Influence of Cheap Fuel on Cost of Electrical Energy." In these statistics the capital outlay on the electrical machinery is not separated from that upon the steam-power plant, and the inefficiency of the dynamo cannot be distinguished from that of the whole plant including the inefficiency of the driving engines. These statistics therefore do not give figures of direct use in the investigation attempted in this treatise. Nevertheless the figures in these papers will serve to show what is accomplished in some of the best managed steam power stations of large size.

In comparing with the figures given above it is needful to bear in mind (1) that 1 B.T. Unit corresponds to rather more than 1½ engine brake horse-power-hour, (2) that the "units sold" may be from 10 to 25 per cent. less than those actually generated, the excess being used throughout the works and in distribution, (3) that capital charges, depreciation and maintenance are estimated on very variable bases, and (4) that the "load-factor" of electric lighting central stations is very low, sometimes being as little as 10 per cent. and seldom more than 25 per cent. The load on tramway stations, although very unsteady from minute to minute, is more regular in respect of ratio of average to maximum; and when traction and

lighting are both supplied from one station, the load-factor is often capable of being raised much higher to 30 or 40 per cent.

Mr. Holliday's report refers to boilers alone. They were Lancashire boilers, and with natural draught six of these evaporated 136 million pounds of water in 35,000 hours at the rate of 9·6 lbs. evaporation per lb. of coal. The coal cost on the average 14*s*. 5*d*. per ton. What are termed the "working expenses" are given as—

TABLE XXXIVA

	£	Per cent.
Fuel	4,554	87·2
Wages	327	6·2
Ash Removal	75	1·4
Water	226	4·3
Light and Stores . . .	44	0·9
Total	5,226	100

Some of Mr. Holliday's items are combined here, and for the reduction to percentages Mr. Holliday is not responsible. 30 lbs. evaporation per hour being commonly allowed per nominal horse-power, this total gives 0·276 of a penny per 30 lbs. evaporation. Of this 0·276, the coal accounts for 0·24.

In three forced-draught Lancashire boilers 72 millions lbs. of water were evaporated in 20,600 hours by 4,900 tons of breeze and coke, costing 4*s*. 3½*d*. per ton, being burnt on Perret's special breeze furnace; this evaporation being about 6½ lbs. water per lb. of fuel. The costs, after rounding off odd figures, are—

TABLE XXXIVB

	£	Per cent.
Fuel.	1,050	64·7
Wages	300	18·5
Ash Removal	120	7·4
Water	120	7·4
Light and Stores . .	33	2·0
Total	1,623	100

Here naturally the fuel is a much smaller percentage of the total. This total is at the rate of 0·162 penny per 30 lbs. of evaporation, and of this the coal costs 0·105 penny.

On the same subject of the influence of cheap fuel, Mr. Crompton gives the following table of costs in the New Chelmsford works, where the tools are driven by electric motors distributed along the line-shafts and driving direct on these without belt or other gearing. The "units" referred to are kilowatt-hour units delivered by these motors to the tools. The output upon which the measurements were taken was 368,000 units in 49 weeks.

	Cost Pence per B.T. Unit.	
	With Coal at 17*s.* 5*d.* per Ton.	With Coal at 5*s.* 4*d.* per Ton.
Coal	·488	·150
Stores	·029	·029
Wages	·236	·236
Repairs	·016	·016
Total	·769	·431
Ratio of Coal to Total	·64	·35

In the figures in the second column, which is only an estimate, the cost of coal is reduced in exact proportion to the price per ton and the other items are left unchanged. This method of estimating is hardly reasonable when the two prices are so completely different as to mean totally different qualities of fuel. In the two tables quoted above from Mr. Holliday, the coal prices per ton were in the ratio 4*s.* 3½*d.* to 14*s.* 5*d.*, or ·3, while the coal costs per 30 lbs. of water evaporated were in the ratio $\frac{\cdot 105}{\cdot 240} = \cdot 44$. There is 50 per cent. difference between these two ratios.

The following Table XXXV gives a summary of costs in the electric lighting stations of a few of the large provincial cities and in London districts. The figures are calculated from the Board of Trade returns, the costs being condensed into four main accounts and the figures being rounded off. The first column gives the annual electric output from the station, it being necessary to consider the costs per unit in connection with the size of the station. As all our curves show, the cost per unit goes down at first rapidly and then more slowly with increasing size of plant. Mr. Hammond's paper above referred to was written in order to demonstrate this decrease. Since the date of it the outputs of all these stations have increased in a very large ratio, and the cost per unit has gone

TABLE XXXV

Cost in Large Electric Lighting Central Stations, Year 1901–2

Place.	Annual Output Millions B.T.U. Sold.	Capital Expenditure £,1000 Unit.	Capital Expenditure per Million B.T.U. of Annual Output Unit £1,000.	Price of Coal per ton. s.	d.	Cost in Pence per B.T. Unit Sold. Coal.	Wages, Oil, Stores.	Maintenance and Repairs.	Rent, Rates, Taxes and Management.	Total.
Bristol	2·75	385	140	—		·92	·34	·21	·52	1·9
Edinburgh	7·76	766	99	7	2	·34	·17	·10	·30	0·91
Glasgow	9·28	963	104	7	2½	·35	·23	·14	·31	1·03
Leeds	3·06	505	165	—		·27	·22	·07	·22	0·78
Liverpool	20·02	1188	58	—		·50	·22	·04	·20	0·96
Manchester	10·50	1164	111	—		·47	·22	·19	·47	1·35
London—										
Charing Cross and Strand (West End)	6·55	665	102	17	6	·85	·33	·16	·39	1·73
Kensington & Knightsbridge	3·06	325	106	18	6	·73	·32	·28	·92	2·25
Metropolitan Electric	11·12	1620	146	—		1·58	·53	·33	·57	3·01
St. James and Pall Mall	5·93	423	71	14	0	·84	·39	·23	·56	2·02
Westminster	8·93	856	96	15	4½	1·04	·35	·20	·65	2·24

down considerably, although not in the ratio expected by Mr. Hammond, who anticipated that the cost in Leeds would be ¾*d.* when the output reached 5 million. It is true that the subjoined table shows only ·78*d.* for Leeds; but when one compares the cost of maintenance and repairs entered for Leeds, Liverpool, and in a less degree for Edinburgh, with those entered for other places, it becomes apparent that different methods of account-keeping prevail in these different places. It should be noted that neither depreciation nor sinking fund is included in these costs; while it is well understood that the accounts for maintenance, repairs and depreciation are complementary. If maintenance be well maintained there is correspondingly little real depreciation, while what is written off for depreciation in any one year is apt to depend entirely on the profits of that year. The local differences of methods of dealing with "maintenance and repairs," and "Rent, etc., and Management," deprive these columns of value as guides to engineering practice. The only trustworthy comparison must be made by reference to the columns "coal" and "wages, oil and stores," the former, of course, in connection with the price of coal. The coal prices are not given in the Board of Trade returns, and in the table the prices given are extracted from Mr. Hammond's paper. Mr. Hammond, unfortunately, could not get this price in all cases, and he explained that the average price given him by the resident engineer was not always very accurately obtained. The table, however, is interesting as showing the enormous differences in capital outlay in proportion to work done, and the similarly surprising differences in all the items of cost per B.T.U., except in "wages, oil and stores." The differences in cost of fuel are evidently not wholly accounted for by those in the prices per ton, nor by the size of the output.

In all the figures given, except in this last table from which it is purposely excluded, depreciation has been taken as some uniform per cent. per year of the original capital outlay. The percentage, of course, varies with the kind of plant, being different for boilers, engines, condensers, buildings, etc. This custom of applying a uniform yearly percentage to cover depreciation must perforce be followed, because it is the invariable existing practice of accountants. But that it is not a rational practice was demonstrated in an article by the author of this treatise in *Feilden's Magazine* of March, 1900. It is there shown how to calculate the "Present Value" of any plant as a profit earning apparatus. This depends on the profits to be earned up to the end of the life of the machine;

and this life is a " probable " life, necessarily a matter of estimate and, in a certain degree, of speculation. The strict proper estimate may vary from year to year, not only by reason of reduction of length of life still to run, but also from " antiquation " of design or pattern, and from change of cost of production of new plant capable of being substituted. Even when there is no risk of " scrapping " being necessary in consequence of " antiquation," and when no accidents intervene to change the original estimate

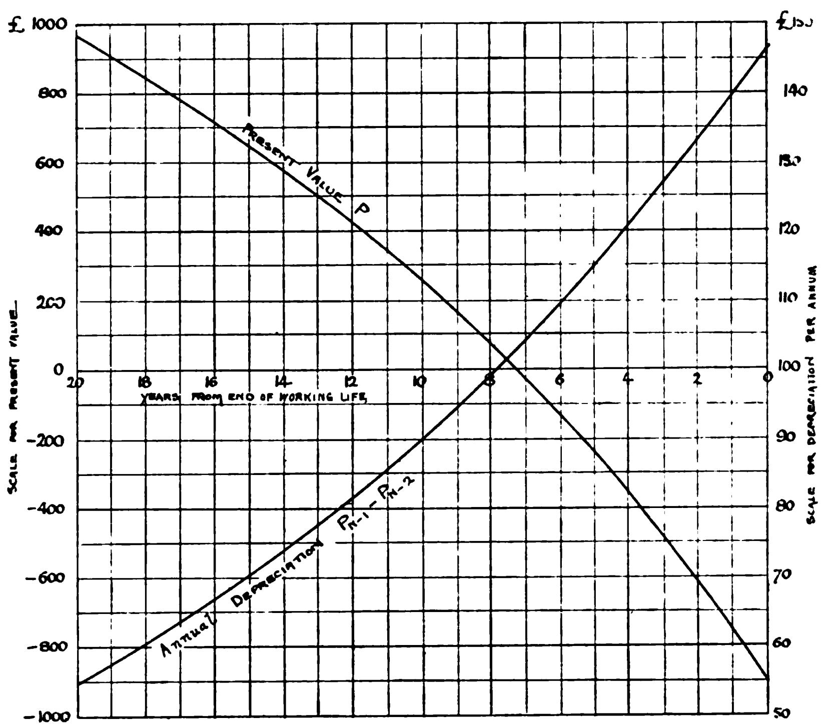


FIG. 36.—Annual Depreciation of Present Intrinsic Value with Steady Earning Capacity.

of total length of working life, the " present intrinsic value " does not decrease at a uniform rate, but only slowly at first and very rapidly as the end of the working life is approached. The true " depreciation " is evidently the decrease in any year of the " present value." The results of the mathematical calculation of these values when no break in the earning power or prospect of life of the plant occurs, is given in the following two Tables, XXXVI and XXXVII, and the two corresponding diagrams, Figs. 36

and 37. In Table XXXVI the earning power of, or the net revenue obtainable from, the plant is supposed to be uniformly maintained at £100. In the next table it is supposed to decrease uniformly from £100 at the start by £2 per year to £60 at the end of 20 years. The first cost is taken at £1,000, and the nett revenue is reckoned as being obtained after paying £50, or 5 per cent., interest on this first cost. In both

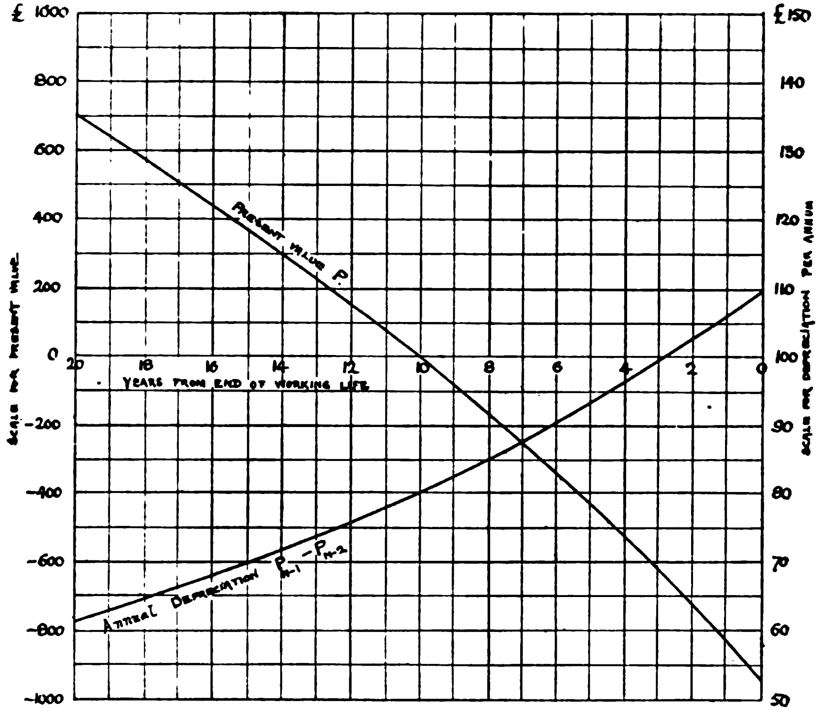


FIG. 37.—Annual Depreciation of Present Intrinsic Value with Earning Capacity decreasing 2% per Year.

cases the "present value" is obtained on the supposition that the £1,000 capital outlay is to be repaid at the end of the life of the plant; but the real value of the depreciation, or change of present value, does not depend upon whether this first cost is, or is not, to be repaid; that is, does not depend upon whether the plant has been laid out with borrowed capital or otherwise.

It may be noticed that in a life of 21 years the "present value" changes from positive to negative 7 years in the one case, and 10 years in the other, before the end of the life. This negative sign

merely means that the "present value" is here calculated as including this liability to pay £1,000 at the end of the term.

In both cases the depreciation per year increases at a very nearly steady rate. In the first table it increases from £57 in the 21st year before the end to £143 in the last year. In the case of gradually diminishing earning power, it increases from £62 in the 21st year before the end to £108 in the last year.

This rationale of depreciation is, however, in view of existing custom, of mainly theoretic interest. It is impracticable to introduce it as a working method into the accounts of industrial companies under present circumstances.

TABLE XXXVI

Annual Depreciation of Present Intrinsic Value

First Cost £1,000. Rate of Interest 5 per cent.

Uniformly maintained Nett Revenue £100, after paying £50 Interest.

Number of Working Years still to Run. $N-1$	*Ultimate Value* of Future Earnings less First Cost. U	*Present Value* of Future Earnings less First Cost to be Repaid at End. P_n	*Annual Depreciation* of Present Value during $(N-1)$th Year. $P_{n-1}-P_{n-2}$
20	2574	969	56·6
19	2307	913	59·4
18	2053	853	62·4
17	1813	791	65·5
16	1584	726	68·7
15	1366	657	72·1
14	1158	585	75·7
13	960	509	79·5
12	771	429	83·5
11	592	345	87·7
10	420	258	92·1
9	258	166	96·7
8	103	70	101·5
7	− 45	− 32	106·6
6	− 186	− 139	111·9
5	− 320	− 251	117·5
4	− 447	− 368	123·4
3	− 569	− 491	129·6
2	− 685	− 621	136·1
1	− 795	− 757	142·9
0	− 900	− 900	

Formulae—

$$U = 2000 \times 1{\cdot}05^{n} - 3000\ ;\ P = 2100 - \frac{3000}{1{\cdot}05^{n-1}}\ ;\ P_{n-1} - P_{n-2} = \frac{150}{1{\cdot}05^{n-1}}$$

TABLE XXXVII

Annual Depreciation of Present Intrinsic Value.

First Cost £1,000. Rate of Interest 5 per cent.

Nett Revenue, after paying £50 Interest on First Cost, £100 during the 21st year from end of life, and decreasing by £2 per year to £60 during the last year.

Number of Working Years still to Run. N—1	*Nett Revenue.* Average of N Years from end. R—W	*Ultimate Value* of Future Earnings less First Cost. U	*Present Value* of Future Earnings less First Cost, to be Repaid at end. P	*Annual Depreciation* of Present Value During (N—1)th Year. $P_{n-1}-P_{n-2}$
20	80	1858	700	62
19	79	1612	638	64
18	78	1382	574	66
17	77	1166	509	68
16	76	964	441	69
15	75	774	372	71
14	74	596	301	73
13	73	431	228	75
12	72	275	153	77
11	71	130	76	79
10	70	– 6	– 3	81
9	69	– 132	– 84	83
8	68	– 250	– 167	86
7	67	– 360	– 253	89
6	66	– 463	– 342	92
5	65	– 558	– 434	95
4	64	– 646	– 529	98
3	63	– 729	– 627	101
2	62	– 805	– 728	104
1	61	– 876	– 832	108
0	60	– 940	– 940	

Chapter IV

Kinetic Energy and Resilience.

Kinetic Transpower and Resilient Transpower.

In the previous chapters there have been dealt with the general theory of industrial economy, and the physical and commercial data of power production from heat as this special industry is carried on at present. We have now to seek to apply these data and that theory to the economic improvement of Mechanical Power Production.

This treatise is an effort to contribute towards presenting the relation of heat to mechanics in a form which is more complete, and therefore more applicable to engineering needs, than that which has been developed by the great scientific men of fifty years ago and has been repeated in so voluminous a fashion in the text-books of the last quarter century. The thermodynamics of the pure scientists has certainly done much towards helping us to understand one aspect of the whole matter—namely, the influence of *temperature* in the phenomena—but not a great deal in other directions ; and as temperature is only one influence that *forces* itself into the design of heat machinery, the natural and inevitable result is that the industrial and commercial development of power plant has been guided only in a secondary degree by the maxims of the thermodynamic treatises. Practice framed on bases in which even only one important influence is overlooked, is necessarily ruinous. Every such influence must make itself fully felt in all practice which results successfully. The great success of steam and other heat engines has arisen under several other dominating forces besides those of the thermodynamic theory found in German and English books, although obedience to the physical truths established by Meyer, Joule, Clausius, and Rankine is, of course, one of the conditions of practical success. What is here said has no reference to the splendid refinements in the mechanism of these machines,

such as those in valves, governors, glands and packings, guides and bearings.

This true, but deficient, theory seems to fail in affording practical guidance in three main directions.

First, in the depths and breadths of the pursuit of many temperature-relations, it loses sight of—at least, fails to keep the necessary attention riveted upon—the fact that, in so far as the production of mechanical power is concerned, heat is of use solely as a means of creating *resilience*; that it is resilience that does all the work; and that *stress*—i.e. pressure, or tension, or shear—along with *unstraining*, are the two only factors which are of direct consequence in the engineering of mechanical power.

Secondly, although physicists are particularly well aware that all physical change occurs in *time*, yet, from first to last, this theory contains no reference whatever to the *time-element*. Now, the life of man and of the generations of men is short; and, ever since we emerged from the luxuries of the Euphrates Valley, and more especially since we created the United States of America, the time-element has been the chief domineering factor in industrial and commercial life. This is overwhelmingly the law for machinery, because its function is material production, and in such production there are only two things to be aimed at—quality and speed. It is reported that in academic cloisters sometimes time passeth unheeded, but engines framed solely in accordance with a theory in which time is of no consequence, are of no use in the industrial world.

The truths of pure geometry are expressed without reference to this time-element, but they become of greater vital interest when translated into terms of kinematics in which the time-element is essential. These again assume really practical importance only when translated into dynamics by the insertion of the mass element. Even in pure geometry the fact really important to the land surveyor and the farmer, or to the architect and owner of a building, is the approximate preservation of the geometrical dimensions of the fields surveyed, or of the structure built, unchanged during the flux of time. Because the change in the area of these fields and in the size of these buildings is approximately zero, the time flux disappears from the mathematical expression of the important fact; but herein the mathematical expression is deceptive, because, although correct, it is incomplete, the really important fact being that time flows on, thereby permitting of the raising of crops and the collection of rent, while no, or not much, geometrical change takes place.

Thirdly, the "thermodynamic efficiency," although an essential feature in the prudent consideration of heat engines, is only one of many equally important factors in the whole industrial economy. Moreover, our text-books explain this efficiency only in terms of temperature conditions. Now, the differential calculus has taught us how to study the laws of Maxima, and this study shows us that the place of the maximum depends entirely upon the limiting conditions—upon which elements in the problem are taken as constant and which as variable. The maximum discovered in solving the problem is the maximum only in relation to the variation of the one thing—or two—which is allowed to vary in the solution worked out. But in an engine for a prescribed horse-power, many things may be varied in addition to the temperature reached in the cylinder. So that the temperature solution of the problem of maximum energy efficiency is only one of several solutions. The complete solution is found in the proper combination of the several partial solutions.

Heat is utilized for power production in three main ways: (1) For the sustenance of vital energy in living beings; (2) for the production of light; and (3) for the generation of mechanical power. It is with the third mode of utilization that this book is concerned.

There is no want of clearness about the idea of "mechanical work"; it is the transference of energy by the particular process of mechanical stress combined with motion of the stressed material. But it must not be confused with the idea of energy, although it is commonly so confused in the slipshod statement that "heat energy is converted into mechanical work."

Neither heat energy nor any other kind of energy can be converted into work. The work is the passage of energy from one to another mass, and the passage of the thing must not be confused with the thing itself. It is sufficiently correct to say that energy is spent in doing work, but this statement is quite different from that that any kind of energy is converted into work or that work is converted into any kind of energy.

Similarly the idea of mechanical horse-power is equally definite. It is the time-rate at which mechanical work is done.

There is, however, much greater difficulty in determining what is accurately meant by "mechanical energy" and what by "heat energy." According to one great authority, the term "heat" is used as a sort of refuse-bucket into which we throw all those odds and ends of energy with which we have not yet succeeded in making close acquaintance, and upon which we are still unable to fix a clear,

sharply graven label. Where are we to draw a clear line between heat and mechanical energy?

Kinetic energy is undoubtedly mechanical so long as it is that of "visible" motion, whether translatory or rotary. "Visible" has been gradually given the extended sense of "capable of being detected and identified as motion by any physical means whatever," not merely by eyesight or unaided by instruments. But so soon as the motion of kinetic energy becomes so minute as no longer to be capable of such identification, its existence becoming a matter of logical inference only, as in all molecular and atomic motion, then it is no longer classified as mechanical energy, except sometimes in purely theoretic explanations of the causes of things. Thus heat is theoretically explained to be wholly, or chiefly, due to molecular motion; but, nevertheless, heat is not reckoned mechanical energy.

If a steel spring, or a piece of any elastic solid, be strained, whether in compression, tension, torsion, or bending, no engineer will hesitate to say that its resilience in the strained condition is mechanical energy. Gravitational potential energy, whether terrestrial or solar or stellar, is universally classified as mechanical. "Water power" is energy of gravitation, and engineers would not tolerate any proposal to abandon the time-honoured custom of calling water power mechanical power. Steel spring elasticity and gravity are two out of a fairly long list of kinds of resilience. It is difficult to find any solid reason for including some, and excluding other, kinds of resilience from the classification mechanical energy. The criterion is that each acts by mechanical force, and does work by the method of mechanical work. For instance, it is not logical to call the resilience of a coiled steel spring mechanical energy, and to refuse this name to the expansive resilience of a vapour or of a gas. If a distinction be maintained between heat and mechanical energy, as it ought to be, then that part of the energy of a compressed gas which is resilient ought not to be included in the quantity of heat in the gas; and when, by thermal means, the resilience is increased, the phenomenon should be described as a conversion within the gas of heat energy into mechanical resilient energy. During this conversion there may, or may not, occur simultaneous transference of part of the mechanical energy to other substances by the process of mechanical work.

Adopting this view, we find that there are two main kinds of mechanical energy, viz., kinetic and resilient. The kinds of resilient mechanical energy may be sub-classified as (*a*) expansive or thermal, (*b*) tensive or molecular cohesive, (*c*) torsional or dependent on

molecular rigidity, (*d*) gravitational, (*e*) magnetic, referring to electro-magnetic as well as to "permanent" magnetism, and possibly (*f*) electrostatic. Magnetic and electrostatic attractions and repulsions are just as strictly mechanical forces causing mechanical mass-accelerations as are gravity attractions. These and gravity "act at a distance"; that is to say, the masses exerting these mechanical forces on each other act so in spite of their not touching each other, the action being transmitted through some intervening agent capable of such transmission. Except in case of the intervening space being filled by massive substance, this intervening agent is called ether; but attaching this or the other name to the agent does not alter the essentially mechanical character of the action.

In each case of resilience the energy is of the kind called "potential," and which would be better called "latent," because, as yet, we seldom feel sure as to whether the energy actually has its place in the matter said to possess it, or in the ether surrounding that matter. In each case the observed essential fact is that there is a strain accompanied by a stress. By mental analysis the strain, as also the stress, may be conceived as composed of two equal and opposite parts. For instance, a shear strain between horizontal layers may be thought of as, say, northward sliding of the upper over the lower layers, or otherwise of southward sliding of the lower under the upward layers. Similarly, a compressive stress may be thought of as, say, a northward push of the parts lying more to the south upon those lying more to the north, or otherwise as a southward push of the north parts against the south parts. Every physical phenomenon of whatever sort is capable of this dual analysis into equal and opposite factors; but the analysis is mental, and it is doubtful whether the parts have any separate physical existence. Certainly the one never does exist without the simultaneous existence of the other. A mechanical stress is thus analyzed into two equal and opposite mechanical forces across a section separating two portions of substance. Each force means the time-rate of transference of momentum across the section, the oppositeness of the two forces meaning that the material on one side of the section is losing while that on the other side is gaining momentum, and their equality meaning that no momentum is lost in the transference, the one gaining as much as the other loses. If the stresses across two separate sections be unequal, then the material between these sections is receiving more momentum through the one section than it is delivering through the other or *vice versa*, and

thus this material between the sections is either increasing or decreasing in the momentum and in the kinetic energy lodged in it. If A be the sectional area, δl the distance between the sections, m the density, p'_l the pressure gradient in respect of the distance between the sections, and v'_t the velocity gradient or acceleration in respect of time; then $A\ p'_l\ \delta l = - A\,\delta l \cdot m\, v'_t$,

$$\text{or } - p'_l = + m\, v'_t.$$

This is the time-rate of increase of momentum in unit volume, and the simultaneous time-rate of increase of kinetic energy in the mass occupying this unit volume is

$$K'_t = v \cdot m\, v'_t = - v\, p'_l.$$

The new kinetic energy so generated has come into this mass from outside, and is not the result of conversion inside this mass from another form of energy to the kinetic form.

In the first of these equations $A\,\delta l \cdot m$ is a definite mass in the volume $A\,\delta l$ inside which the density is m, and the faces of which are δl apart. The rates of increase of momentum and of kinetic energy here calculated are those of this particular mass, the front and back faces of which are exposed to different pressures. It is the difference of these pressures that results in the increase of momentum, and it is this pressure-difference multiplied by the movement of the whole which results in the increase of kinetic energy.

If one considers the variation of the kinetic energy $\frac{1}{2} M v^2$ of a mass M moving with velocity v, one finds that it may vary in two ways; (1) as the result of v varying, and (2) as the result of M varying. As to (2), if we consider the mass inside a certain definite volume with fixed faces, then the density inside this space may vary in consequence of more or less mass coming in through some faces than leaves through all the other faces; or again we may consider the gradual spread of kinetic energy throughout a mass, new portions of mass, which were previously motionless, one after the other being set in motion so that the whole moving mass moves at each instant of time with uniform or average velocity v. These are two ways in which the mass M involved in a varying quantity of kinetic energy may vary. If we wish to trace the time-rate of variation of this energy, we have to take it as the sum of the two parts (1) and (2) mentioned above. Using M'_t as the time rate of increase of the mass involved, then the time-rate of increase of Kinetic Energy:

$= M\,v\,v'_t + \frac{1}{2}\,M'_t\,v^2$

$=$ Momentum $\times$ acceleration of velocity.

$+$ Kinetic energy in new material into which the motion has spread per unit of time.

The K'_t of the previous and following equations is evidently the first part only of this last more general expression. It is so because in K'_t the mass involved does not change. Thus K'_t, although expressed per unit volume, is not a rate of change in any space divided by particular boundaries from the rest of space, but is a rate of change in a particular quantity of mass occupying, at the instant considered, unit volume.

The same equations are obtained for any kind of stress through continuous material, provided that the velocity v be measured in the same direction as the stress p.

The length δl, supposed normal to the sections A, and along which line the stress gradient p'_l is reckoned, lies in the direction in which the energy is transmitted, but not always in the direction of the velocity v. The direction of transmission is the same as that of the motion in the case of the work being done by compressive stress. It lies along the same line, but is oppositely directed to the motion if tension be the working stress ; and at right angles to the motion if the work is done by sheer stress.

Horse-power, or more shortly "power," is a time-rate either of transmission of energy or of conversion of energy, i.e. the destruction of one and the simultaneous generation of another kind of energy. The First Law of Mechanics is that in any exertion of power no energy is lost, the quantity leaving one mass, or disappearing in one form, equalling that reaching the other mass, or being generated in the other form. It would conduce to clearness of expression if we had two names for these two main kinds of power.[1]

In this book the word "transpower" will signify the time-

[1] For the time-rate of conversion or transmutation of one kind to another kind of energy, the names "metapower," "mutapower," "alterpower" and others have suggested themselves ; but none seem very satisfactory. The use of such terms involves the awkwardness of having to specify the two kinds of energy *from* which and *to* which the transmutation occurs. Engineers generally refer to such transmutation as the *generation* of the new kind of energy, as in the generation of heat by combustion or the generation of electricity by mechanical work. This generation takes place within a system without transference of anything out from or into that system ; and, therefore, the names "endopower" and "engenpower" have also been thought of, but these also appear clumsy. Of all these, Mutapower seems to be the most euphonious and suggestive term.

rate of transmission of energy. Examples of transpower are :—(1) Transmission by mechanical work ; (2) transmission of heat by conduction and radiation, called "heat-transpower" in what follows ; (3) transmission by electric current ; (4) transmission by radiation other than heat radiation. When the energy radiated is light the transpower goes also by the name of candle-power. Most mechanical-work transpower is the time rate of the exertion of resilience. But when in the above formula K'_t is negative, this means that the material is giving up kinetic energy, and that the energy so disappearing is being transmitted out of the mass from which it disappears ; it is not being converted to other kinds of energy within that mass. It is therefore a transpower, and may be called Kinetic Transpower. It is positive transpower when the velocity v is in the direction of an up-grade of compressive stress—as in the evasé discharge from centrifugal fans and pumps, and in the expanding discharge passage from a boiler injector—or in that of a down-grade of tensive stress. In the case of shear it is positive if, when looking at the strained material so that the shear strain appears as a right-handed twist, the velocity appears to have a right-handed moment round an axis placed at the upper end of the stress gradient—that is, placed at that side of the twisted layer where the stress is greater. In every case its magnitude is proportional, not to the stress, but to the gradient of the stress along the path of transmission, and also to the velocity. $-K'_t$ is the integral kinetic transpower out of the bounding surfaces of a unit volume ; and, if this unit volume be a cube, two of whose opposite surfaces alone are active, it is the excess of the transpower out of the front surface over the transpower into the back surface in virtue of the destruction of kinetic energy between these two surfaces, unit distance apart. $-K'_t \, \delta l$ is the kinetic transpower of the volume of unit section and of the short length δl.

It should be noted that the kinetic energy here spoken of is that technically termed "external," as calculated from the *average* velocity measured relative to an *external* base. In its formula $v \, p'_l \cdot \delta l$ no reference is made to differences of velocity within the two surfaces δl apart. The "internal" kinetic energy, calculated from these differences of velocity alone, has nothing to do with this transpower. If at any instant there be along the line l a uniform gradient of velocity, called v'_l so that there is $v'_l \cdot \delta l$ difference of velocities between two points distant δl from each other, then the internal kinetic energy in the volume of unit section and length δl is $\frac{1}{24} m \, \delta l \cdot (v'_l \, \delta l)^2$.

As seen below, such internal kinetic energy must exist wherever resilient transpower is being exerted, and a relation exists between the two ; but this relation is not significant for any purpose of the present investigation. The internal kinetic energy is proportional to the cube of δl, and, taken per unit of mass or per unit of volume, to the square of δl ; so that its relative importance decreases rapidly the smaller the dimensions within which we limit its calculation. Perhaps a better way of stating this would be to say that its comparative importance increases very rapidly with the quantity of matter within whose limits we compute the energy.

The exertion of resilience occurs during unstraining, the mechanical work done being measured by the stress and the decrease of strain conjointly, and the resilience is wholly exhausted when the material reaches the wholly unstrained condition, which has been called the "state of ease." This unstraining means a change of geometrical configuration, whether of size or of shape, and thus the time rate of this change at any instant is proportional to the differences of simultaneous velocity. In the case of pressure resilience the unstraining means expansion ; and if the expanding motion be along a parallel-sided tube, whose length we call l, then the difference of the two velocities at two points of l is the time-rate of expansion per unit cross-section. If v'_l signify the gradient of the velocity per unit length along l, so that $v'\delta l$ is the difference of simultaneous velocities at the two ends of δl, then the horse-power of the expansion in δl per unit cross-section is $p\, v'_l \cdot \delta l$. As the energy is transmitted through the front surface of the part δl long this horse-power is a transpower, and it constitutes an excess of transpower through the front surface over that through the back surface in virtue of resilient energy being destroyed within the length δl. Reckoned per unit volume, this resilient transpower is $p\, v'_l$. It is proportional, not to the velocity, but to the gradient of the velocity, and also to the stress. If R signifies resilience, and R'_t the time-rate of increase of resilience per unit volume, then $-R'_t = p\, v'_l$ is the resilient transpower. It is positive or negative according as the up-gradient of velocity is in the direction of, or opposed to, the transmission of energy, the direction of transmission being related to that of motion in the three cases of compression, tension, and torsion in the manners previously explained.

The diagram Fig. 38 helps to make these relations more obvious. Along the base line is measured l, the path of transmission of energy. At each point of this path is plotted upwards the stress p, in the diagram supposed to be pressure, and forward velocity v, existing at

any given instant; and the simultaneous values at all points of the path are joined up to form a stress curve and a velocity curve. The velocity is plotted upwards if it be in the same direction as that in which l is measured, that is, the same as that of the transmission of energy.

At any point of the path the transpower through unit cross-section is the product of the heights of the two curves. The difference of this product at any two points is partly due to kinetic energy being destroyed or generated—in the left-hand end of Fig. 38 generated because of the forward down grade of pressure—in the material between these points, and partly to the exertion or absorption of resilience by the same material. In the left-hand end of Fig. 38 resilience is being exerted because of the forward up-grade of velocity. These two variations of the trans-

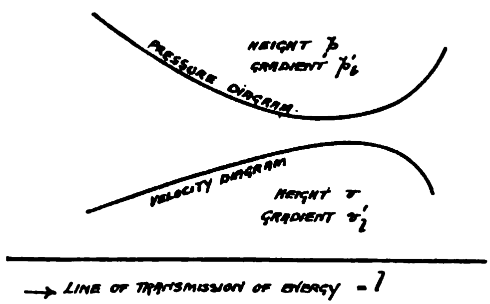


FIG. 38.—Velocity and Stress Diagrams along the path of Mechanical Power Transmission.

power from point to point are the kinetic and resilient transpowers respectively developed between the points compared. Taking these variations per unit section, and per unit length along the path, that is, the kinetic and resilient transpowers per unit volume of material in which they are developed, we have the kinetic transpower equal to the height of the v curve multiplied by the gradient of the p curve, and the resilient transpower equal to the product of the height of the p curve by the gradient of the v curve.

From the time-rate of straining and the modulus of elasticity, E = geometrical configuration multiplied by ratio of increase of stress to increase of strain, it is easy to calculate the time-rate of increase of stress in terms of E. It is $p'_t = -\mathrm{E}\, v'_l$.

Expressed algebraically, the facts of transpower already mentioned take shape in the following seven equations. The letter T signifies the mechanical transpower at any unit section, and T'_l signifies the gradient at which this varies from point to point along

the path l of transmission at any one instant of time. T'_t is the time-rate at which the transpower through any given section increases, or decreases if it be negative.

Equations of Mechanical Transpower.

I. $p'_l = -m\,v'_t \begin{cases}\text{Line-gradient of stress and}\\ \text{Time-gradient of velocity.}\end{cases}$

II. $p'_t = -Ev'_l \begin{cases}\text{Time-gradient of stress and}\\ \text{Line-gradient of velocity.}\end{cases}$

III. $T = p\,v$ Mechanical transpower per unit volume.

IV. $T'_l = p\,v'_l + v\,p'_l = -R'_t - K'_t \begin{cases}\text{Line-gradient of mechanical transpower and}\\ \text{Resilient and Kinetic transpowers.}\end{cases}$

V. $T'_t = p\,v'_t + v\,p'_t = E\frac{v}{p}R'_t + \frac{1}{m}\frac{p}{v}K'_t \begin{cases}\text{Time-gradient of mechanical transpower and}\\ \text{Resilient and Kinetic transpowers.}\end{cases}$

VI. $-R'_t = p\,v'_l$ Resilient transpower per unit volume.

VII. $-K'_t = v\,p'_l$ Kinetic transpower per unit volume.

Equations I. and II. give values for v'_t and p'_t, the time-rates of increase of velocity and of pressure at any point. These are proportional to $-p'_l$ and $-v'_l$, the line-gradients of the two curves in Fig. 38. Each of these two curves, at each point of it, rises or falls at a speed proportionate to the downward or upward slope of the other curve, the proportions being $\frac{1}{m}$ and E.

The value of E requires careful definition—given later on—as it depends on thermal circumstances.

In Figures 39 to 44 the physical meaning of these relations is illustrated in order to make clear their degree of importance. The example taken is that of a length of gaseous material of specific gravity nearly the same as that of thoroughly dry or super-heated steam, or about half that of air. In British mass units per cubic inch at 1 lb. per sq. inch absolute pressure and 62°Fahr., this density is taken as 5×10^{-8}. A length of 1,000 inches, or 83⅓ ft., of this is supposed to be moving in a straight line represented by the l ordinate of each of the six diagrams. On this base are plotted the values of p, v, p'_t, v'_t, T, and T'_t, at the end of each of five successive small intervals of time. The data from which this time history is worked out are — (1) the curve of pressures p all along the 80 ft. length at the first instant of time considered, this given by the wave-like line

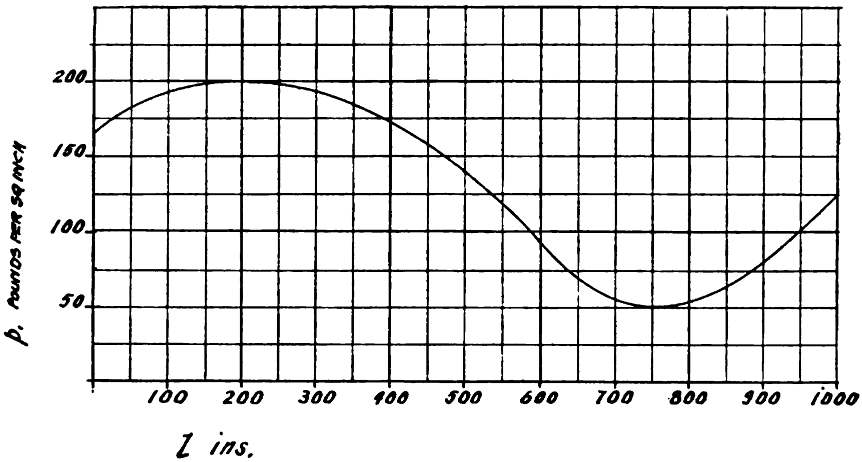


FIG. 39.—Mechanical Power Transmission. Curves of Pressure.

NOTE.—The pressure throughout the 1,000 *ins. of length remains appreciably steady during the time examined, the pressure changes being too small to plot to the scale of this diagram.*

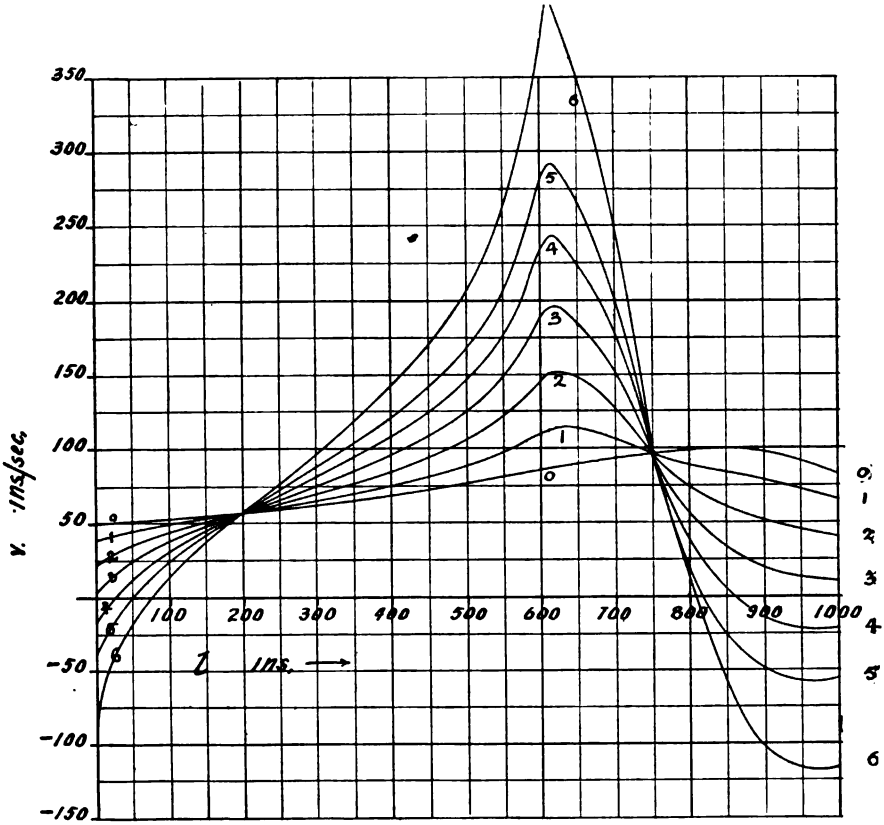


FIG. 40.—Mechanical Power Transmission. Curves of Velocity

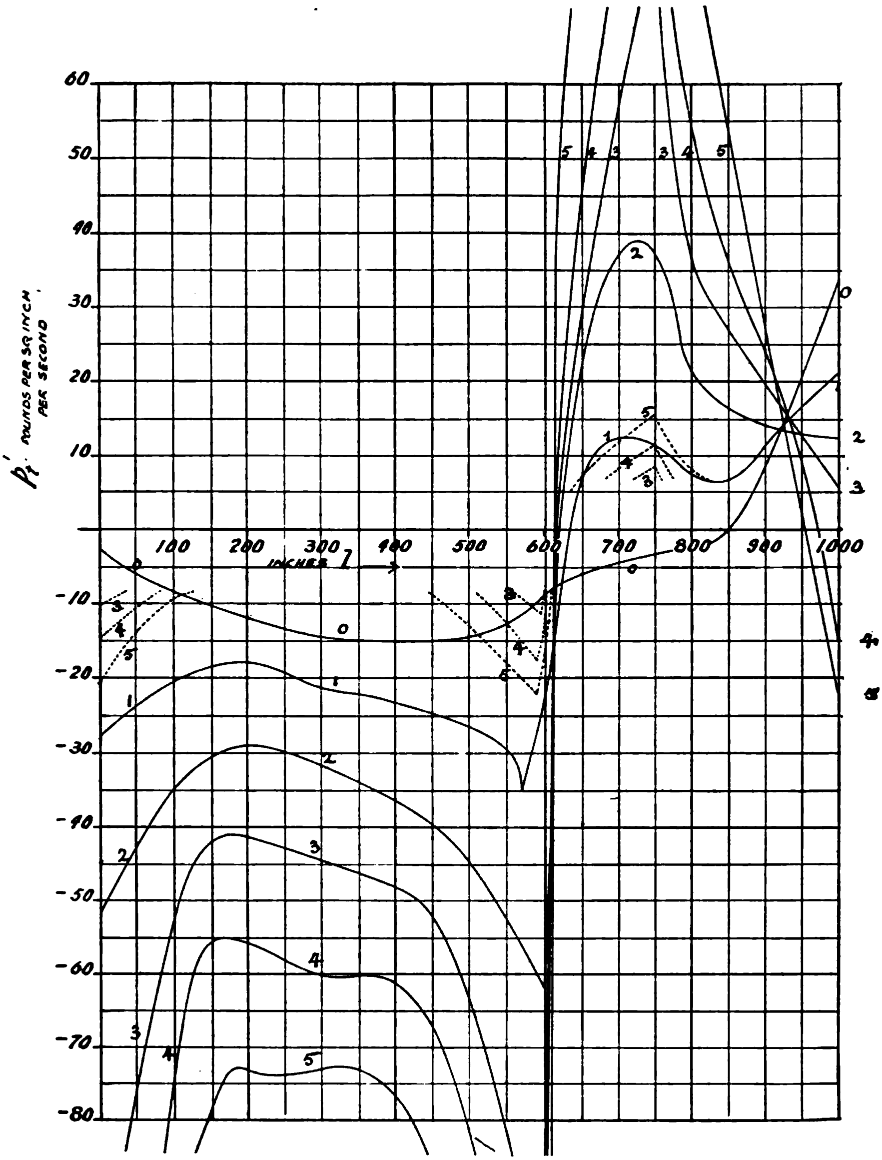


Fig. 41.—Mechanical Power Transmission. Time Rate of Change of Pressure.

NOTE.—The dotted parts of the curves are to a scale ten times closer than the rest, and are the continuations of the parts in full line where these run above and below the limits of the diagram.

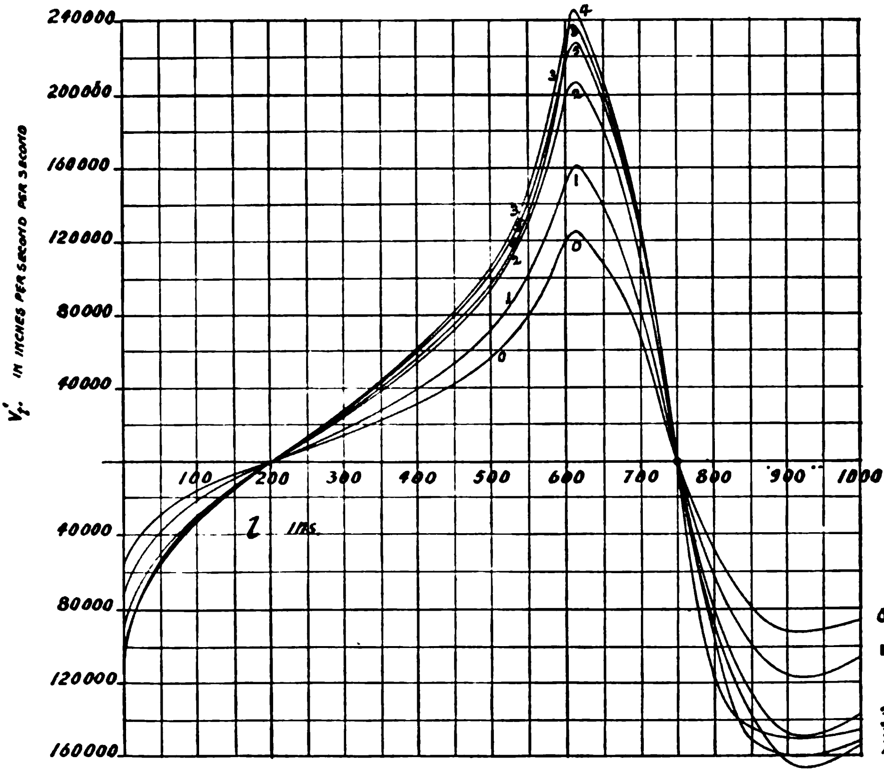


FIG. 42.—Mechanical Power Transmission. Time Rate of Change of Velocity.

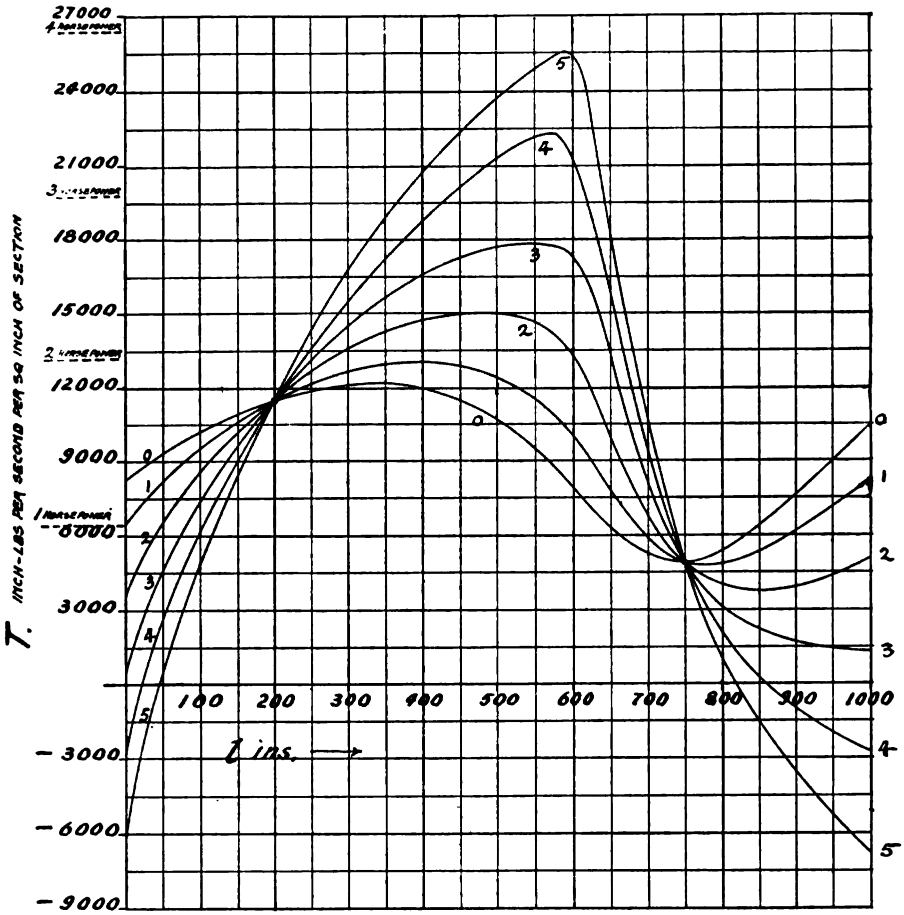


FIG. 43.—Mechanical Power Transmission. Curves of Mechanical Transpower.

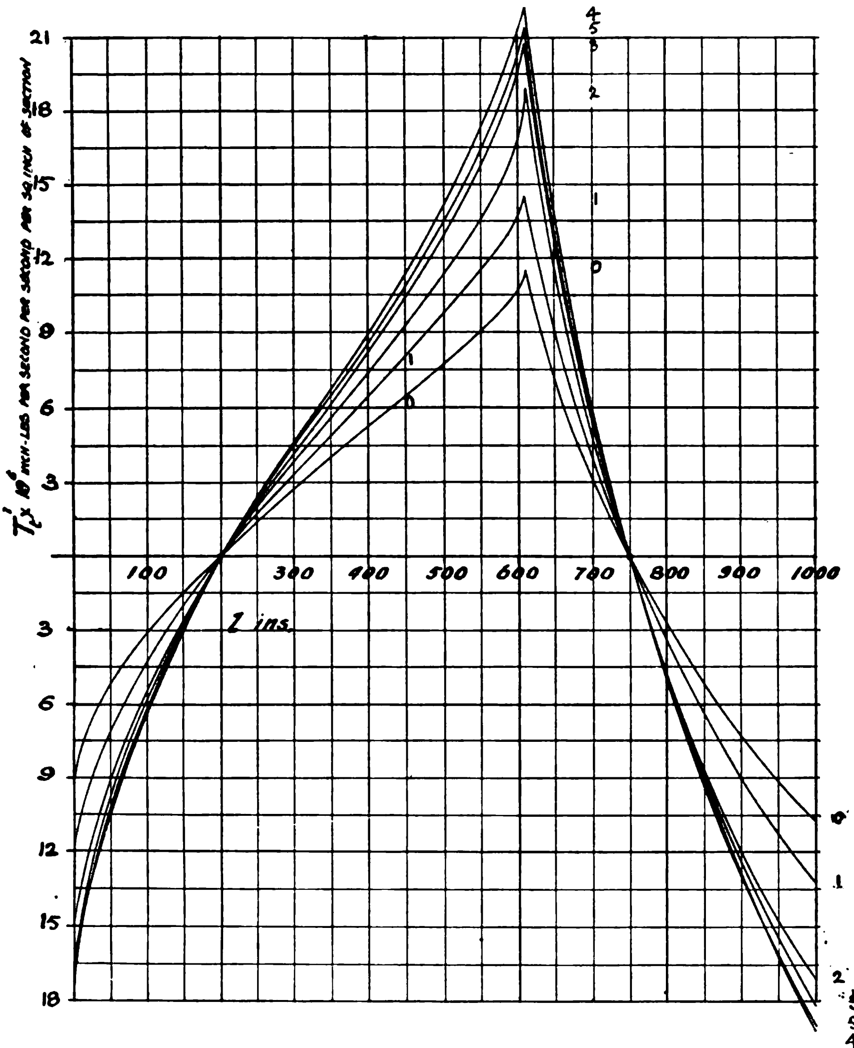


FIG. 44.—Mechanical Power Transmission. Time Rate of Change of Mechanical Transpower.

marked p; (2) next the simultaneous initial values of the velocity v all along this length, shown by curve marked v; (3) the density m supposed to vary directly as the pressure and equal to $\frac{5}{10^8} p$ in units of mass per cubic inch, while p is taken in lbs. per sq. inch; and lastly (4) the modulus of elasticity E, here taken as in constant proportion to the pressure and equal to $1{\cdot}2\, p$.

These last two data (3) and (4) would correspond to the pure gaseous condition kept always at one uniform temperature. The changes of pressure shown on the diagrams are so extremely rapid that it would be difficult or impossible to find means whereby uniform temperature could be maintained; but this is not of great consequence as the diagrams are introduced simply to illustrate the results of the above laws in effecting extraordinarily rapid changes of condition both from point to point of time and from point to point in space. This great rapidity results from the small density of the moving stuff, but it should be remembered that this density is about the same as that of steam, and that, therefore, steam, air and other like gases are subject to the like sudden changes of condition. It is probably safe to say that it is seldom recognized that the extremely small specific gravity of such substances renders them liable to enormous accelerations of velocity arising from very slight differences of pressure, and that these accelerations accompany enormous differences of simultaneous velocity at points of space quite close to each other. If the changes were adiabatic, or nearly so, instead of isothermal as assumed in this diagram, the suddenness of these changes in time and space would be even greater than appears here because the adiabatic E is greater than the isothermal E.

Considering now the results of the somewhat laborious calculation shown on these diagrams, it will be noted from Fig. 39 that the pressure remains measurably constant while the other quantities change almost violently. The time intervals are very minute, namely only $\frac{1}{5000}$th of a second, so that the whole series of curves cover only $\frac{1}{1000}$th second lapse of time. It is not surprising, therefore, that the pressure does not change much at any point: what is more surprising is that the other quantities should change so rapidly while the pressure curve remains unmoved. But reference to Fig. 41 shows that p'_t, the time-rate of change of pressure, is by no means steady. Its highest value occurs at the end of the $\frac{1}{1000}$th second, and is a rate of decrease of 22 lbs. per sq. inch per second. This multiplied by the time interval between successive curves is $\frac{22}{5000} = \frac{4{\cdot}4}{1000}$ lb. per sq. inch in one of these intervals, which on the

large scale to which the p diagram was originally drawn would be represented by less than $\frac{1}{1000}$th inch height on the paper. Of course, after the lapse of a second or two the pressure curve would become entirely altered in character; but the calculations and their plotting in diagram form occupy a skilful draughtsman a billion times longer than the actual physical change occupies, and the draughtsman who plotted these particular diagrams became fatigued when he got to the end of $\frac{1}{1000}$th of a second, and desired a holiday. The pressure curve ranges in height from 200 lbs/in.^{2} down to 50 lbs/in.^{2} in a length of 550 inches, the average gradient over this length being thus $\frac{150}{550} = {\cdot}37$ lbs/in.^{2} per inch length, and the steepest gradient being just $\frac{1}{2}$ lb/in.^{2} per inch of length. Immensely steeper gradients than this exist in the escape of steam from boiler safety-valves, and in the ports of steam engine-cylinders during the admission and exhaust of steam. In air or gas pipes and tanks into which air or gas is being pumped, steeper gradients than the above occur after every opening of the valves between pump and reservoir. The above-mentioned draughtsman tried to cut his task short by using $\frac{1}{10}$th second time intervals; but the other quantities involved change so rapidly that this method immediately produced results which were absurdities and physical impossibilities. It must, however, be clearly understood that this rapidity of change results from the combination of considerable pressure gradients like the above with small densities like those of steam and air.

The curve marked 000 in Fig. 40 gives the assumed variation of velocity throughout the 1,000-inch length at the beginning of the $\frac{1}{1000}$th second investigated. It is assumed positive or forwards at all points, and increases from 50 to 100 inches per second in a length of about 850 inches or 71 feet. The other curves marked 1, 2, 3, 4 and 5 show what the velocity has become at each point of the length at the ends of the 5 successive small time-intervals. At two points, namely at 200 and 750 inches, it does not change at all. These are the two points at which the pressure curve reaches its maximum and minimum heights, and where, therefore, there is zero pressure gradient and consequently no velocity acceleration, according to Equation I.

Where the forward pressure gradient is upwards the velocity curve falls, so that in the 4th time-interval it is reversed to a negative or backward velocity throughout the lengths 0–50 inches and 850–1,000 inches. After $\frac{1}{1000}$th second, *see* curve 5, it has risen to nearly 300 inches per second, or about 17 miles per hour, at the point 615 inches, while at the same instant only 25 feet further along the

path an opposite or backward velocity of about 3 miles per hour co-exists. In Fig. 42 are seen the peculiar curves showing the accelerations of velocity at each point of the path. As already stated, these show constant zero points, or cut through the base, at the points 200 and 750 inches. The general form of these curves somewhat resembles those of the velocity curves.

In Fig. 43 is seen how the mechanical transpower through each section varies. There are here again two nodes at which there is no variation, and these again are at the points 200 and 750 inches. The transpower remains positive wherever and whenever the velocity remains forward, and changes sign with the velocity. The nodes in this diagram are only nodes because the change of pressure throughout is immeasurably small. If the investigation were continued throughout a whole second of time, they would be seen not to be true nodes. The scale of this transpower diagram is marked in inch-lbs. per second per sq. inch of section, and most readers will understand it best by remembering that one horse-power is 33,000 ft.-lbs. per minute or 6,600 inch-lbs. per second. At the beginning of the $\frac{1}{1000}$th second considered, the greatest transpower shown by the diagram is 12,200 inch-lbs. per second, or rather less than 2 horse-power per sq. inch of section; while $\frac{1}{1000}$th second later it has increased to over 4 horse-power at 600 inches along the path, while *simultaneously* at 0 inches, or 50 feet back from this point, nearly 1 horse-power is being exerted in the *opposite* direction.

The time-rate at which the transpower varies at each point of the path, and at each instant of time is shown by the series of curves on Fig. 44. These follow the same general forms as the T curves, and show nodes on the base-line at the same points 200 and 750 inches. They are remarkable for their sharp points at their maximum heights.

The p'_t curves of Fig. 41, showing the time-rates of change of pressure are certainly the most curious in shape. They show sharp points at their minimum heights, or points of most rapid decrease of pressure (these points all lying in the negatives part of the diagram); and the excessively sharp gradients where they run from negative to positive values at about 620 inches are due to the very sharp curvature of the velocity (v) curves in this neighbourhood. This is recognized from Equation II where p'_t is seen to be proportional to v'_l. It is also proportional to E, which in this case is 1 2 p or proportional to the pressure. The pressure is low in this neighbourhood, but the sharpness of the v curvature gives p'_t a very steep path-gradient in spite of the small pressure.

Chapter V

Irreversibility and Heat Transpower.

Stress and Strain Specific Heats.

The actions described in the last chapter are *not* "*reversible*," except in one special case. Their leading characteristic is Irreversibility. This *Irreversibility* is a principle second in importance only to that of the conservation of energy expressed in the First Law of Mechanics. The fact of Irreversibility and its causes deserve to be styled the Second Law of heat-mechanics; but as the second law of the thermodynamic text-books is mathematically proved by *assuming* a physically impossible *reversibility*, there would arise an awkward confusion from adopting the same title for the doctrine of *irreversibility*. It will be seen immediately that the one only case in which there is reversibility is a case of pure dynamics unmixed with any thermal action; that is, it is distinctly not a case of thermodynamics.

The technical meaning in which the term "reversibility" is always used by scientists is the possibility of obtaining exactly contrary physical changes by reversing the kinematic conditions while leaving—that is, starting from—all other conditions unaltered. In the present case this means that in Fig. 38, while the measurement of l is reversed so as to be positive from right towards left, and the v curve is left unaltered except that its upward ordinates now mean velocities towards the left, the pressure, temperature, and all other conditions at each point stand unchanged. The slope of the v curve is the same as before on the paper, but it is now downward in the forward direction at the left-hand end of the diagram, so that the material is now contracting instead of expanding as before. The p curve stands unaltered, but its forward slope at the left hand is now upward, so that the material is now giving up kinetic energy instead of having it generated within it as before. The resilient transpower is now negative—that is, resilience is being created and stored in the material instead of being spent—and is of same magnitude as previously,

because—see Equation VI, see p. 159 chap. IV—the two factors p and v'_t remain the same everywhere, except that v'_t has its sign changed. The kinetic transpower is also of same magnitude but of reversed sign, because—see Equation VII—the sign of p'_t is changed. The total mechanical transpower T—see Equation III—is of same magnitude at each point of the path l, but is reversed in direction of transmission. Since both pressure and temperature are everywhere the same as before, so also is the density m; and, therefore, according to Equation I, the velocity acceleration v'_t is nowhere altered in magnitude, but is changed from a positive acceleration into a retardation by the change of sign in p'_t; so that now every part of the velocity curve is falling at the same time-rate as that at which it previously rose.

So far every physical change is precisely reversed. In the one case immediately to be mentioned, which is not a case of thermodynamics, but one of pure dynamics, this precise reversal is complete. The true history of the whole series of successive phenomena is exactly the opposite of what it was, and there is no reason why this exact oppositeness should not be maintained from moment to moment. It is as if the whole mechanical action lived backwards; and this results solely from changing the direction, without changing the magnitude, of all the initial velocities.

Looking at Equations II and V, it is seen that p'_t and T′ would also be exactly reversed if E were the same in the backward as in the forward action: that is, the pressure curve would at every point now rise at the same time-rate as that at which it previously fell, as would also the total mechanical transpower. But E depends upon the time-rate at which heat is being supplied or abstracted by conduction, radiation, or internal chemical or other generation, that is, upon the heat-transpower. In the one case in which this heat-transpower is kept at zero value, that is, under the condition of perfect adiabatic heat-insulation, and no internal generation, + or −, of heat, in this case alone is E the same in the backward as in the forward action. In such purely dynamic action, from which thermal action is entirely excluded, there is complete reversibility. It can be obtained by surrounding the active material by insulating material impervious to heat conduction and radiation, and at same time avoiding all internal chemical, electromagnetic, frictional, or viscous heating. Since, as seen below, it is the *time-rate* of the thermal action that is effective, this adiabatic action is also more or less closely approximated to in dynamic actions taking place at kinematic speeds very large as compared

with that of the heat-transpower. In sound waves through elastic material, the velocity of the vibratory motions constituting the alternate expansions and contractions is so great that they are very nearly the exact reverse of each other. The degree of ap-approximation to which they are exactly reversed is that from which is excluded consideration of dissipation of the energy of the mechanical vibration. Such dissipation is of two kinds; namely, (1) by lateral spread, which may be wholly mechanical, although it is seldom so, and (2) by "damping," which means the internal conversion of mechanical kinetic energy into other forms of energy, such as heat and sometimes probably electro-magnetic energy, by what is variously called internal solid friction, viscosity, and strain hysteresis. In all high-speed vibratory transmission a like approximation of greater or less degree exists. Such transmissions are almost purely dynamic, and hardly in any degree thermo-dynamic. It is a curious illustration of how sadly the science of dynamics has been neglected that the theory of sound in elastic material was only developed in connection with the new science of thermodynamics, it being the one case of the new equations in which heat action does not interfere, the one case of purely non-thermal resilience.

The heat-transpower depends upon temperature differences or temperature gradients between the active material and its surroundings. When the action is "reversed" in the orthodox sense of the term, which sense is essential in the ordinary proof of the second thermodynamic law by the reversal of Carnot's closed cycle, the temperature gradients are not reversed, but remain the same. If heat flows *in* during the forward action, it flows *in* at the same rate per second during the backward or reversed action. The influx or outflow per second depends upon the temperatures in the regions surrounding the active material, and these are, for the most part, independent of the temperature of this latter.

Let H' stand for flux of heat into unit volume or internal generation of heat caused as above mentioned. Let H'_t signify its time-rate per second. If H'_t be negative, it means outflow. Negative H'_t is pure thermal transpower exerted by the active material upon its surroundings in consequence of these surroundings being at lower temperature. In any small time δt, the heat flux is $H'_t\,\delta t$.

In diagram Fig. 45 the vertical ordinate p means stress, and the horizontal ordinate s the geometrical configuration, per unit mass, change in which constitutes the strain which is the elastic

accompaniment of the stress. The change δs is an increase of strain. But if p mean pressure and s specific volume, the increase of strain causing increase of pressure is a contraction or $-\delta s$. In this case to get a positive modulus of elasticity, we must write $E = -s p'_s$. Using this in Equation II, there is easily found for the time during which a change of strain δs develops, the measure $\delta t = \frac{1}{s v'_l} . \delta s$; and multiplying this by H'_t we find the simultaneous influx of heat into unit volume. Now let the

Strain specific heat $h_s =$ specific heat per unit volume per unit change in s when the strain alone changes without change of stress, and

Stress specific heat $h_p =$ specific heat per unit volume per unit change in p when the stress alone changes without change of strain.

In the Equations VIII and IX below for E and p'_t, only the ratio of h_s to h_p is involved in the first of the two parts of each expression, and in them, therefore, the specific heats per unit mass or per unit weight will serve equally as well as if taken per unit volume. The ratio of h_s to h_p is well known to equal the gradient p'_s of the adiabatic $s\ p$ expansion curve, so that $s \frac{h_s}{h_p}$ is the adiabatic modulus of elasticity, called E_a.

The second parts of VIII and IX depend upon the ratio of H'_t to h_p; and, if both are reckoned per unit of mass, this ratio, of course, is the same as if they are taken, as here, per unit of volume. H'_t is, however, a surface phenomenon, as transpower of all kinds is, and its meaning and value are more easily understood if it be reckoned volumetrically.

In any change involving the simultaneous strain and stress changes δs, and δp, the influx of heat per unit volume is evidently

$$h_s . \delta s + h_p . \delta p,$$

and equating this to the above measure of $H'_t . \delta t$, it is easy to deduce—

$$\text{VIII.} \quad E = s \frac{h_s}{h_p} - \frac{H'_t}{h_p v'_l} = E_a - \frac{H'_t}{h_p v'_l}$$

and

$$\text{IX.} \quad p'_t = -s \frac{h_s}{h_p} . v'_l + \frac{H'_t}{h_p} = -E_a v'_l + \frac{H'_t}{h_p}$$

Each of these two is the sum of two parts. The first part of each is the adiabatic or purely dynamic part. This first part of E remains the same in the forward and in the reverse action, while the second part is reversed in sign because H'_t does not change sign, while v'_t does do so. Thus the E's in forward and reverse action are the sum of, and the difference between, the adiabatic modulus and a thermal elastic addition proportionate to the ratio of the heat transpower to the velocity of expansion. If this ratio be small there is not much difference between the forward and backward actions.

When the stress p and the specific volume, or other geometrical configuration-measure, s, change together in obedience to the law "$p\,s^i$ remains constant," that is to say, the stress alters in inverse proportion to the constant power i of the volume, then $p'_s = -i\frac{p}{s}$; and therefore the modulus of elasticity, which is $-s\,p'_s$, becomes $E = i\,p$.

In the adiabatic expansion of "perfect" gases, this law is followed with i a little greater than 1·4. The index for adiabatic expansion will be called a, while i will indicate the index, or power, for any curve not adiabatic. But $E = i\,p$ is true of all expansion curves, or parts of curves, for which this law holds, and it holds with considerable approximation to accuracy for all gases and vapours through fairly long ranges of expansion under what may be termed uniform thermal, i.e. heating or cooling conditions, if only a suitable value of i be applied. For a short range of expansion a suitable value of i can always be found. For gases and vapours not near the condition of perfect gas, the adiabatic value a is more or less different from 1·4—always in direction of being less than 1·4.

To find the value of i suiting any short range of any curve given by experimental measurement, the simplest method is to measure, at the mean point of the range, the slope of the curve, p'_s, and also p and s, when

$$i = -\frac{s}{p}\,p'_s,$$

the minus sign only meaning that when the curve slopes to lower pressure for larger volume, then p'_s is negative, and this negative sign has to be changed in order to get i positive. When the slope of the curve is zero, that is, for isobaric expansion or any change of geometrical configuration without change of stress, the index

$i = 0$. Any curve lying between the isobaric and the gas isothermal has a positive i less than 1. It is worth while noting that curves of this kind may, under conditions of rapid heating and slow expansion, rise to higher pressures with larger volumes; and these cases may be brought under the law by using negative powers i. It is also well worth remembering, in comparing different curves passing through the same point p, s, on the diagram, that the index i is directly proportional to p'_s, the slope of the curve.

Another method more commonly found mentioned in text-books, is to measure $p_1\, s_1$ and $p_2\, s_2$ at the two ends of the range of expansion to be investigated. Then—

$$i = \frac{\log p_1 - \log p_2}{\log s_2 - \log s_1}$$

This latter method is more suited for long ranges, while the former one is best for very short ranges.

If, for any experimentally measured curve, i be found in this way, and if the power a for the adiabatic curve of the same substance be known, then, since $E = i\,p$ and $E_a = a\,p$, Equation VIII gives the means of calculating H'_t, or the heat-transpower into, or out of, the active substance, provided the time-rate of expansion be known. Thus—

$$\text{Equation X.} \quad H'_t = h_p\,(a - i)\,p\,v'_l.$$

This is the time-rate at which heat is being supplied by conduction, radiation or combustion, to unit volume of the substance when it is expanding at the time-rate v'_l. This heat-transpower is proportional to this v'_l, to the pressure p, and to the excess of the adiabatic over the actual index of expansion. This excess $(a - i)$ is positive when the actual curve falls less rapidly than the adiabatic curve, and the above heat-transpower is required to prevent it falling adiabatically. Since $i\,p = -s\,p'_s$, another form in which this may be written is

$$H'_t = h_p\,(p'_s - p'_a)\,s\,v'_l,$$

where p'_a means the slope of the adiabatic. This form shows more directly than the other that the heat-transpower is proportionate to the difference between the slopes of the actual and the adiabatic curves.

Conversely, one may calculate what curve, i.e. what i, will be followed in an expansion at a given velocity v'_l, accompanied by a given inward heat-transpower; thus—

$$i = a - \frac{H'_t}{h_p\, p\, v'_l}.$$

As a simple illustration, take the heat-transpower into a nearly "perfect" gas required to effect isothermal expansion at the given rate v'_l. It is, since $a = 1{\cdot}4$ and $i =$ nearly 1 for isothermal expansion,

$$H'_t = {\cdot}4\, h_p\, p\, v'_l.$$

With any less inward heat-transpower than this the temperature will fall as the result of expansion. If the heat-transpower be greater, then rise of temperature will accompany the expansion. If the slope of an actual gas expansion curve lie just midway between the isothermal and the adiabatic, the index i being $1{\cdot}2$, then the heat-transpower accompanying this expansion is

$$H'_t = {\cdot}2\, h_p\, p\, v'_l.$$

Here H'_t and v'_l must be taken per the same unit of time, and H'_t and h_p must be taken per the same unit either of volume or of mass or of weight.

For air at 15 lbs. per square inch pressure and at 500° Fahr. absolute temperature, h_p per cubic inch is $2{\cdot}63 \times 10^{-4}$ Brit. Th. Unit per 1 lb. per square inch rise of pressure.

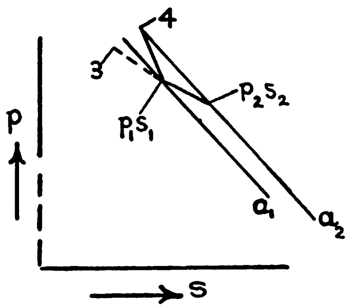
At the end of this chapter are given the values of h_p and h_s for a number of different gases used in the production of power and also for saturated steam.

The first part of p'_t, the time-rate at which the stress curve in Fig. 38 rises, is exactly reversed on the reversal of the motion, by reversal of v'_l. The second part remains unchanged, and is proportional to H'_t. The pressure changes at a rate compounded (1) of the rate at which it falls or rises along the adiabatic curve in accordance with the velocity of expansion or contraction, and (2) of the rate at which it rises by negative transpower of heat without expansion or contraction. The forward and reverse time-rates are the sum and difference of these two parts. They are thus different unless H'_t is zero, when the action becomes purely mechanic. In the reverse action the stress curve of Fig. 38 rises and changes shape at a different rate to that at which it falls in the forward action; and as the changes in the v curve depend, in ways already pointed out, upon the p curve, the v curve also, after an only momentary complete reversal, pursues a series of changes different to those preceding, in the forward action, the arrival at the condition of Fig. 38.

This irreversibility will be understood more completely by reference to Fig. 45. Here the forward action involving δs and δp is supposed to take the material from the condition, $p_1 s_1$ to $p_2 s_2$. The first point is on the adiabatic curve a_1, the second on a_2. During the change the heat influx δ H is measured by the area under the expansion curve 1, 2, and between the two adiabatics a_1, a_2 carried down to the zero isothermal. If a $\theta \phi$ diagram were used this area would be rectangular, but its shape is of no consequence, because in any case it cannot be used for practical measurement of δH owing to our not knowing where the zero isotherma l is.

The exact reverse of this change, starting again from 1, would be from 1 to point 3 on the backward continuation of the curve 2 1, and for the same time, δt, the length 13 would equal the length 12.

FIG 45



DEMONSTRATION OF IRREVERSIBILITY

But this would lead to an adiabatic a_3 on the side of a_1 opposite to a_2, because, if the curve 2 1 3 do not coincide with the adiabatic a_1, it must cut through it. The two changes, 1 2 and 1 3, would thus involve heat fluxes, δ H, of opposite signs. But H'_t and, therefore, δ H, are unchanged in the actual backward action; which, therefore, carries the material from 1 to a point 4, which, so long as the changes are very small ones, must lie on the same adiabatic a_2 as the point 2. If the change be not very minute, the point 4 falls a little short of the adiabatic a_2, but is still on that same side of a_1. Thus, the gradient of the reverse rise of pressure 1 4 is necessarily steeper than that of the adiabatic curves, and still more so than that of the curve 2 1 3. The modulus E being proportionate to this gradient, the difference between the forward and reverse

moduli is measured by the difference in steepness between 1 3 and 1 4; and this last difference also (multiplied by $s\ v'_l$) equals the difference between the forward p'_t, or fall of pressure per second, and the reverse p'_t, or rise of pressure per second. Thus, for the same small intervals of time, 4 would be higher above 1 than 2 is below it.

The forward and reverse paths necessarily differ in slope from the adiabatic in opposite directions: the one slope is greater and the other is less than the adiabatic slope. Thus the more the forward path differs from adiabaticity, the greater is the difference between the forward path and its reversal: the greater is the *irreversibility*.

The ratio of the thermodynamic to the adiabatic or purely dynamic part of E, as also the similar ratio for p'_t, is that of $\frac{H'_t}{v'_l}$ to $-\ s\,h_s$. This ratio does not involve h_p, the "stress specific heat." $\frac{H'_t}{v'_l}$ is positive if heat influx accompanies expansion. If s mean specific volume, then $s\ h_s$ is the strain specific heat per unit of mass taken per unit increase of strain. These two parts, when of opposite sign, may exactly neutralize each other, and must do so whenever the stress is kept steady during straining. In this case the modulus $E = 0$. The most important example is the evaporation of a liquid into saturated vapour under constant pressure, such as water to steam. Here $s\ h_s$ is the latent heat of evaporation reckoned per unit mass of the steam produced and taken per unit volume of expansion, that is, divided by the excess of the steam over the water volume. If, however, H'_t be taken per unit mass of the steam, this means multiplying its volumetric value by s, and $s\ h_s$ must be similarly multiplied. When so multiplied $s\ h_s$ becomes the latent heat L per lb. of the ordinary steam tables. In this case, therefore, the ratio of the thermodynamic to the adiabatic part of E and of p'_t is $\frac{H'_t}{L\,v'_l}$.

In materials in which a very small influx of heat under constant stress produces a large strain, there is little adiabatic elasticity, and any approximation to "reversibility" must always be very difficult.

On the other hand, in such materials as require a very large heat influx to effect, under constant stress, any considerable change of strain, the resilience is chiefly adiabatic, and the modulus of elasticity deviates little from the adiabatic modulus when either the heat influx or outflow is slow or the velocity of change of strain is

rapid. In such cases a fair approximation to "reversibility" may be obtained.

Another important case is that in which v'_l is extremely slow. The extreme limit is reached when it is zero, or when change of stress occurs without any change of strain. Here $H'_t \div v'_l$, and, therefore, also E, is "infinite," the line of action on Fig. 45 being vertical. The second terms alone of VIII and IX are now effective, the first terms being relatively small and becoming zero in the limit. h_p is now the sole governing specific heat, the influence of h_s disappearing. The transpower is now wholly thermal, there being no dynamic action.

These three cases (*a*) the purely thermal, or heat-transpower without straining, (*b*) the purely dynamic or adiabatic, or dynamic without thermal transpower, and (*c*) the isobaric, or thermodynamic action without change of stress, are undoubtedly those of chief theoretical importance to mechanical engineering. (*b*) alone is "reversible."

In practice all intermediate varieties of change of condition occur. One of these intermediate varieties is the isothermal. The present writer has not been able to discover that the isothermal is of any greater practical importance than any other intermediate variety. The isobarics of evaporation happen to be isothermals—not by reason of anything that can fairly be called physical accident, because it is the conjunction of corresponding critical pressures and temperatures that determines the evaporation—but in engineering we obtain isobaric evaporation always by controlling and steadying the pressure, and never by controlling and steadying the temperature. If any one has the idea that evaporation necessarily takes place at constant temperature, he can at once get rid of so mistaken a notion by considering the adiabatic expansion of a mixture mostly of water and partly of steam, during which evaporation occurs along with rapidly falling temperature and pressure; while if the steam largely predominates in the mixture, condensation takes place along with the same rapid fall of both. Evaporation can take place along any expansion curve at a rate dependent at each stage mainly apon the momentary H'_t.

In all cases of thermodynamic activity it is this factor H'_t, or heat-transpower, that introduces "irreversibility." Thus, whenever we use heat conduction or generation for the purpose of obtaining mechanical power, we are sure that the process is irreversible. And the faster we use it, that is, the greater the heat-transpower we employ per unit volume of active material, the more

hopeless the irreversibility. The conditions of life require rapid work, so that the sooner we give up worshipping reversibility as a fetish worthy to be aspired after and approximated to, the better will we succeed in engineering.

Regarding the practical, as distinguished from the theoretical, possibility of reversibility in purely dynamic or adiabatic transpower, there is this very important consideration to note. The heat-transpower H'_t depends as much upon the temperature of the surrounding regions as upon that of the working material; it depends on the difference. Now in all adiabatic straining or unstraining of material, its temperature changes. It would involve arrangements so superlatively difficult as to be humanly impossible, to change also the temperature of the surroundings so as to follow simultaneously those of the straining or unstraining material, and thus keep the temperature differences constant during the action. Thus H'_t infallibly changes during the action; although it may be zero at starting it cannot remain so, the adiabatic action cannot be maintained, and "irreversibility" must be the characteristic of most of the action. That is, the maintenance of purely mechanic and reversible transpower is an impossibility, although it is rigorously possible for an instant. To obtain approximation to such maintenance, the only method is to make H'_t slow, and to do this not by attempting to restrict temperature differences—because we cannot do so effectively—but, firstly, by increasing the heat insulation between the active material and its surroundings; and, secondly, by increasing the velocity of the straining. By increasing working speed, we give H'_t, whatever its magnitude, less relative influence; we diminish the ratio $H'_t \div v'_l$. If we wish adiabatic expansion in our engine cylinders, we can approximate thereto by using high piston speed.

An apt illustration of this difference between theoretical, or, more properly, academic, reversibility and actual reversibility in engineering operations, is found in the use of compressed air for the transmission of power. Compressed air has proved itself a highly useful medium for such transmission under special industrial conditions. These are (1) that high thermodynamic efficiency in the transmission is not of great importance, and (2) that the transmission is required for the *distribution* of power from a central power generator to a large number of small tools or instruments each of which takes only a very small power and takes it intermittently. Central power stations distributing large quantities of air power to machines regularly absorbing large horse-power have never been a

commercial success. The Paris pneumatic-clock system, and workshop percussion tools driven by compressed air, are wholly successful: engineering work could hardly now go on economically without the latter.

For high thermodynamic efficiency in compressed-air power transmission the air should be compressed adiabatically. During this compression its temperature rises, and the high temperature so attained should not be lost before the air enters the cylinders of the engines which it drives. But it is found impracticable to prevent this cooling between the compressor and the distant driven engine. Between these it always cools down to atmospheric temperature, and in such cooling loses a proportionate amount of resilience because the volume decreases along with the temperature. In the compression, therefore, the extra work done in obtaining the volume and resilience corresponding to adiabatic compression instead of isothermal compression is wholly wasted, because the heat so generated cannot be preserved. Consequently in the compressors every effort is made to keep down the compression curve by various cooling means, such as water-sprays, cold-water jackets, etc. In large compressors the utmost that can be done is to keep down the compression index to about 1·2, as the author found from indicator cards taken at the large air-power station at Birmingham. In the driven engine the utmost power that can be extracted from the contained resilience is by adiabatic expansion with index 1·4. In these engines the back-pressure is necessarily above atmospheric, while in the compressor the air is sucked in at somewhat below atmospheric pressure. The expansion in the driven engine cannot be economically carried so far as the back pressure, while in the compressor the compression is necessarily started from the suction pressure.

If the compression be carried with index 1·2 to five times the suction pressure, say, from 14 lbs./in.^{2} absolute to 70; if then cooling to atmospheric temperature take place; and if, in the driven engine the expansion be down to ·4 of the initial pressure, say, from 70 to 28 lbs./in.^{2} absolute, with index 1·4, and the back pressure be ·3 of the initial or 21 lbs./in.^{2}; then a not very laborious calculation shows that the work done in the compressor is 1·85 $p_1 v_1$, and that recovered on the piston of the driven engine is 1·00 p_1 v_1, where p_1 and v_1 are the pressure at which the suction into the compressor takes place (or 14 lbs./in.^{2}) and v_1 is the volume so sucked in. Thus ·85 p_1 v_1 is lost, and the efficiency is only $\frac{100}{185}$ =

54 per cent., the loss being mainly due to the *irreversibility* explained in this chapter.

As this case of power transmission by compressed gas is of special engineering interest, the formulas by which the above calculations have been made may be stated here ; but the student must be left to prove them correct for himself.

TABLE XLVI

FORMULAS FOR POWER TRANSMISSION BY COMPRESSED GAS

Initial volume sucked into Compressor	v_1
Absolute initial pressure at which v_1 is sucked in	p_1
Compression curve index	i
Compression to pressure, absolute	p_2
Cooled at p_2 to atmospheric temperature.	
Driven Engine Expansion curve index	a
Terminal pressure in ditto	p_3
Back pressure in ditto	p_4

Work done in Compressor—

$$p_1 \; v_1 \quad \frac{i}{i-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{i-1}{i}} - 1\right\}$$

Indicator Diagram Work done in driven engine—

$$p_1 \; v_1\left\{\frac{a - \left(\frac{p_4}{p_2}\right)^{\frac{a-1}{a}}}{a-1} - \left(\frac{p_4}{p_2}\right)^{\frac{a-1}{a}} \left(\frac{p_4}{p_3}\right)^{\frac{1}{a}}\right\}$$

In some of the quantities dealt with here it is, as already mentioned, immaterial whether h_p and h_s be reckoned per unit volume, or per unit mass, or per unit weight ; namely, in those cases where only the ratio of these to each other, or that of either to H or some other quantity of like physical dimensions, is involved. But generally throughout this book, h_p and h_s are to be understood as taken per unit volume in accordance with the strict definitions given on page 171 of this chapter. It follows that, whenever a change of density occurs, the h_p or h_s does not remain calculable from the same mass as before, but becomes applicable to a changed mass. Thus in dry gas the relation between h_p per unit volume and the same per unit mass, varies as $\frac{\text{pressure}}{\text{absol. temperature}}$, because the density varies inversely in this proportion.

It may be well to state exactly the relation between these specific heats and the two specific heats found commonly mentioned in physical text-books. These latter are all reckoned per unit mass

(1 lb. or 1 kilogramme) *per unit rise of temperature.* They are generally, in the text-books, symbolized by the letter k with a suffix to indicate whether the co-efficient is for change of temperature at "constant volume" or at "constant pressure." The reader should note that in these text-books the suffix is used differently to the use of them made here. In them k_p means k *with pressure kept constant*; whereas h_p in this book is a contraction for $\frac{d\,H}{d\,p}$ or H'_p, namely the rate at which heat has to be added per unit rise of pressure *with volume kept constant.* The convention y'_x has long been used by mathematicians as a convenient short method of writing $\frac{d\,y}{d\,x}$, which is the same method as is used in writing $\frac{d\,H}{d\,p}$ shortly H'_p. That is the suffix x in y'_x indicates the variable, and in H'_p the suffix p indicates the variable. It seems to the author unfortunate that the same mathematicians who habitually use this method in writing generalizations, should, immediately they apply their equations to the concrete investigation of heat, proceed to use the suffix to indicate *what is kept constant* instead of the *variable.*

If k_v be the specific heat of the text-books per 1 lb. per 1° rise of temperature at constant volume; and if p'_t be the rise of pressure, or other stress, per 1° rise of temperature at constant volume; then $\frac{k_v}{p'_t}$ is the specific heat per 1 lb. per unit rise of pressure at constant volume; and

$$\frac{k_v}{s\,p'_t} = h_p$$

is the specific heat per unit volume (or other measure of geometrical configuration) per unit rise of pressure (or other stress) without change of volume. This is the relation of the h_p per unit volume of this treatise and the k_v per lb. of the physical text-books.

Similarly, k_p being the specific heat per lb. per 1° rise of temperature at constant pressure, the specific heat per lb. per unit increase of volume at constant pressure is $\frac{k_p}{s'_t}$, where s'_t is the change in s per 1° rise of temperature when the pressure is kept unchanged. Now the heat required per 1 cubic inch of expansion in 1 lb. is just say, one-sixth that required per 6 cubic inches expansion in 6 lbs. of the same substance, and this equals the heat needed for 1 cubic inch expansion in the same 6 lbs. That is to say, the heat needed per 1 cubic inch of expansion is the same whatever the quantity be.

Thus the above expression must not be divided by s in order to pass from the quantity 1 lb. to the quantity 1 cubic inch. Thus

$$\frac{k_p}{s'_t} = h_s$$

the specific heat per unit volume per unit increase of volume at constant stress.

When unit volume of substance is expanded one unit volume, the volume is evidently doubled, and this is a very large ratio of expansion, during which all, or many, of the properties of the substance may change greatly. Therefore, care must be taken not to apply this coefficient to an actual expansion to double volume, but only to small expansions, in spite of its being expressed as above. In this respect, the commonly used coefficients of thermal expansion, the modulus of elasticity and many others, are exactly similar.

For dry gases following the law $p\,s = \mathrm{R}\,t$, the value $p'_t = \frac{\mathrm{R}}{s}$; so that $h_p = \frac{k_v}{\mathrm{R}}$, and is constant for all pressures and densities to the same degree of approximation as k_v and R are constant. It is known, however, that in most gases k_v increases with temperature to a small extent; but the law of this increase has not been as yet accurately determined.

For the same gases, $s'_t = \frac{\mathrm{R}}{p}$; so that—

$$h_s = \frac{k_p}{\mathrm{R}}\,p.$$

This is not constant, but increases in simple proportion to the pressure when variation in k_p and R is negligeably small.

It will be seen later that, in the analysis employed in this book, h_s appears very seldom. Neither it, nor its counterpart k_p, is really a fundamental physical characteristic. On the other hand h_p has this fundamental character, as will appear later, and it will be found to appear frequently in subsequent calculations. This being so, it is well to remember that h_p per unit volume is approximately a constant for each dry gas in all conditions. Its variation in vapours, like steam, is a measure of their departure from the physical condition of a "perfect dry gas." h_p is a pure number; and, in fact, the ratio $\frac{k_v}{\mathrm{R}}$ is shown in treatises on thermodynamics to be $\frac{1}{a-1}$ for "perfect" gases, where a is the index for adiabatic

expansion. This index is about 1·4, so that for these gases this ratio is about 2·5, or more exactly 2·45. This is the true value of h_p whatever volume be adopted as unit, so long as a consistent system of units be adopted. For instance, h_p is 2½ inch-lbs. of heat energy per cubic inch per 1 lb. per square inch rise of pressure: it is also 2½ ft.-lbs. of heat energy per cubic foot per 1 lb. per square foot rise of pressure. But if a mixed system of units be employed this simplicity of value is lost sight of. If it be measured in Br. Heat Units per cubic inch per lb. per square inch rise of pressure, then the measure 2·45 inch-lbs. of heat is to be reduced to Br. Heat Units by dividing by 9,300 the number of inch-lbs. in 1 Br. Heat Unit. This divisor has been variously estimated at from 9,264 to 9,336 inch-lbs. = 778 feet-lbs.; and as it is not accurately known, the mean of these or 9,300 may be used. This gives $2{\cdot}63 \div 10^4$.

The values of h_p and h_s are given below in Table XLVII for several substances used in heating and refrigerating plant, taking it in English Heat Units per cubic inch of substance.

TABLE XLVII

Stress and Strain Specific Heats in Br. Thermal Units per Cubic Inch of substance at 15 lbs. per sq. inch absol. pressure and 500° Fahr. absolute temperature.

Substance.	Stress Sp. Ht. h_p Per 1 lb./in² Rise of Pressure.	Strain Sp. Ht. h_s Per 1 cub. in. Expansion.
Dry Air	$2{\cdot}62 \times 10^{-4}$	·0055
Coal Gas Mixture. 1 Gas to 9 Air	$2{\cdot}77 \times 10^{-4}$	·00567
Mond Gas Mixture. 4 Gas to 6 Air	$2{\cdot}74 \times 10^{-4}$	·00561
Ammonia	$4{\cdot}0 \times 10^{-4}$	·00782
Oxygen	$2{\cdot}98 \times 10^{-4}$	·0063
Water	$4{\cdot}27 \times 10^{-4}$	137
Wrought Iron	$\cdot 773 \times 10^{-4}$	1630
Copper	$\cdot 779 \times 10^{-4}$	1560

The h_p for dry saturated steam, that is, the heat conduction needed to raise 1 cubic inch of such steam 1 lb. per square inch in pressure above the pressure of saturation without expansion or contraction, has been carefully calculated through a large range of pressures (from 15 to 300 lbs. per square inch) by Mr. H. M. Hodson. Table XLVIII and Fig. 48 give its value and show how this varies with the pressure. At atmospheric pressure it is seen

to be about $1\frac{1}{4}$ times that of air or of oxygen; but at 300 lbs./in.^{2} pressure it rises to nearly $1\frac{3}{4}$ times that of these gases.

As this quantity has not been given in other publications, it may be well to explain here how it has been calculated. In order to lead 1 lb. of steam up the "saturated steam curve" so as to increase its pressure by 1 lb. per square inch, it is necessary to conduct heat out of it. The amount of this heat conduction equals the difference between the two "total heats per lb." at the two pressures minus the area of the indicator card between the two horizontal pressure lines. This area equals the mean volume per pound of saturated steam multiplied by 1 lb. per square inch. The heat conduction along the saturated curve can thus be easily calculated from any accurate table of "properties" of steam. If now the saturated steam be raised in pressure at constant volume instead of along the saturated curve, the heat conduction needed is the same as that required to first travel up this curve and then expand at constant pressure, because the work done represented by the area of the small triangular area is negligeably small. This work is 1 lb. per square inch multiplied by half the contraction of volume of saturated steam for 1 lb. rise in pressure. The correction (neglected in Table XLVIII) may be measurable at very low pressures, but is not so at medium and high pressures. The heat needed for constant-pressure expansion is taken as ·48 multiplied by the rise of temperature, ·48 being the specific heat per lb. per 1° Fahr. of steam-gas; while the rise of temperature from the saturated condition at the lower pressure is taken as proportional to the rise of pressure, that is, in the proportion $\frac{p+1}{p}$. The assumption is that the quantities involved are the same as if the pressure and temperature varied according to the "perfect gas" law. This is, of course, not an accurate assumption. It gives, however, the nearest approximation at present available to the h_p of steam just above the saturation curve. The heat so calculated per pound is, of course, divided by the specific volume to obtain the value per cubic inch given in the table. The following formula is the symbolic statement of the above calculation. δ L is the *decrease* of latent heat per 1 lb./in^2 rise of pressure, taken from a table such as that at page 206 of F. R. Hutton's excellent book, *Heat and Heat Engines*; δt the corresponding rise of temperature of saturated steam; t, s and p the temperature, the specific volume, and the pressure (all "absolute"), midway in the 1 lb. range of pressure; then

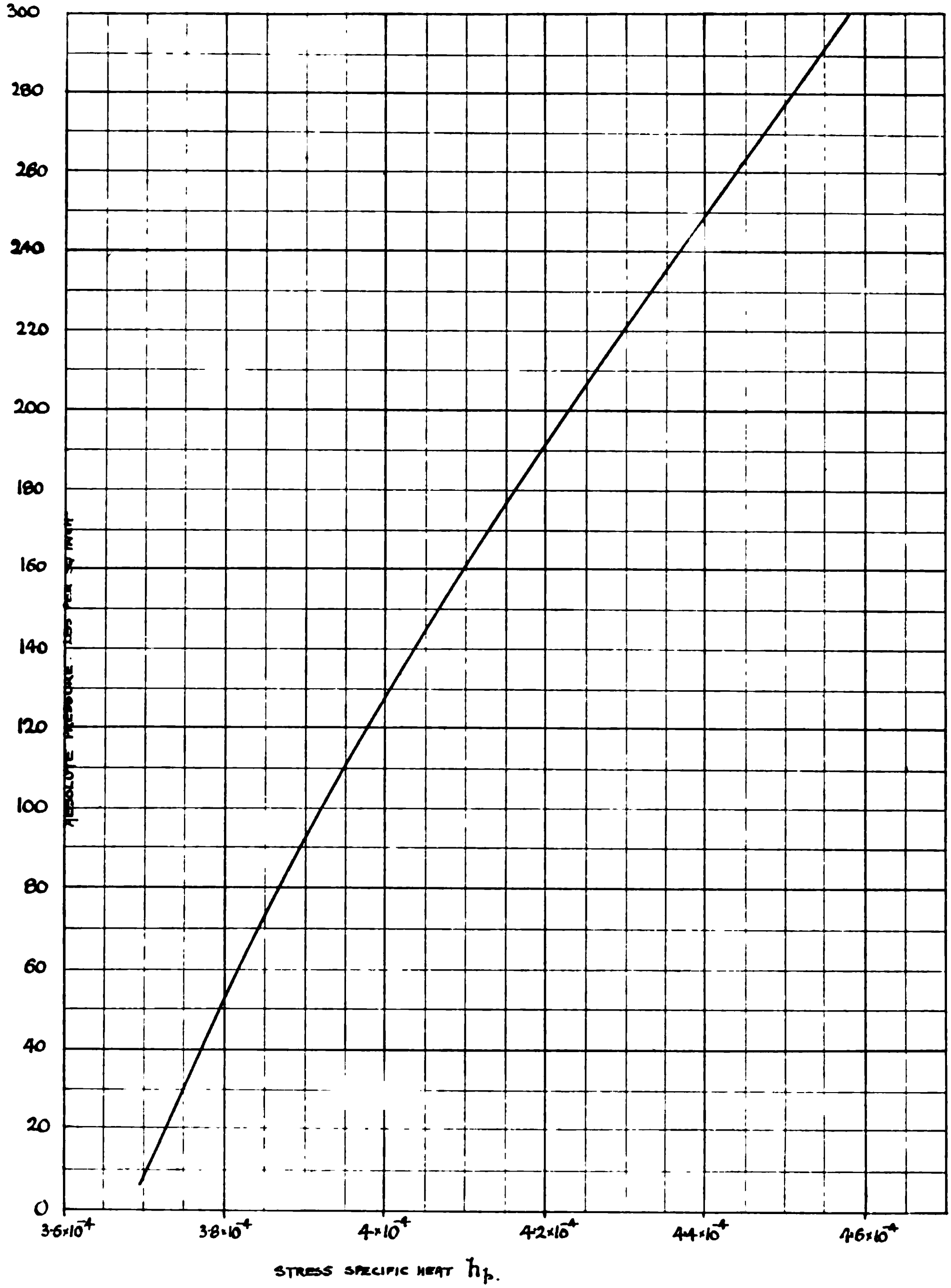


Fig. 48.—Stress Specific Heat of Steam (h_p.) per cubic inch, and per 1 lb. sq. in. rise of pressure immediately above the saturated condition.

N.B.—For Table XLVIII see next page.

$$h_p = \frac{1}{s}\left\{ \cdot 52\,\delta t - \delta L + \cdot 48\frac{t}{p} \right\} - \frac{1}{J},$$

where J is the "mechanical equivalent" of the Br. Thermal Unit taken in inch-lbs. when s is taken in cubic inches per pound. The term $\frac{1}{J}$ is the work $s \times 1$ reduced to heat units and to per cubic inch of steam. From the diagram Fig. 48 it will be seen that h_p rises rather more slowly with the pressure at low pressures than at higher pressures; but the curve deviates very little from a straight line. Between 50 and 250 lbs. per square inch absolute, its value is given very closely by the straight-line formula—

$$h_p = (3{\cdot}65 + {\cdot}003\,p)\,10^{-4}$$

per cubic inch and per 1 lb. per square inch rise of pressure.[1]

TABLE XLVIII

Stress Specific Heat h_p in British Thermal Units per cubic inch of Steam and per 1 lb. per sq. inch rise of pressure immediately above the saturated condition.

Absolute Pressure lbs. per sq. in.	Stress Specific Heat h_p
10	$3{\cdot}70 \times 10^{-4}$
20	$3{\cdot}73 \times 10^{-4}$
40	$3{\cdot}77 \times 10^{-4}$
60	$3{\cdot}82 \times 10^{-4}$
80	$3{\cdot}87 \times 10^{-4}$
100	$3{\cdot}92 \times 10^{-4}$
120	$3{\cdot}98 \times 10^{-4}$
140	$4{\cdot}04 \times 10^{-4}$
160	$4{\cdot}10 \times 10^{-4}$
180	$4{\cdot}16 \times 10^{-4}$
200	$4{\cdot}23 \times 10^{-4}$
220	$4{\cdot}30 \times 10^{-4}$
240	$4{\cdot}37 \times 10^{-4}$
260	$4{\cdot}44 \times 10^{-4}$
280	$4{\cdot}51 \times 10^{-4}$
300	$4{\cdot}58 \times 10^{-4}$

[1] *The specific volume s of saturated steam has never been determined very accurately, and recently it has been demonstrated that what Regnault supposed to be dry saturated steam was slightly wet. Also it is now known that the specific heat of superheated steam is by no means constant. The latest measurements make it vary from about ·46 to above ·68. The variation only slightly affects the results in Table XLVIII, and does not affect those in Table XLIX. See the author's chart of the Specific Heat of Superheated Steam in "The Engineer," 8th July, 1904, and Mr. S. A. Reeves' article in Jour. Worcester Poly. Inst., Nov., 1904.*

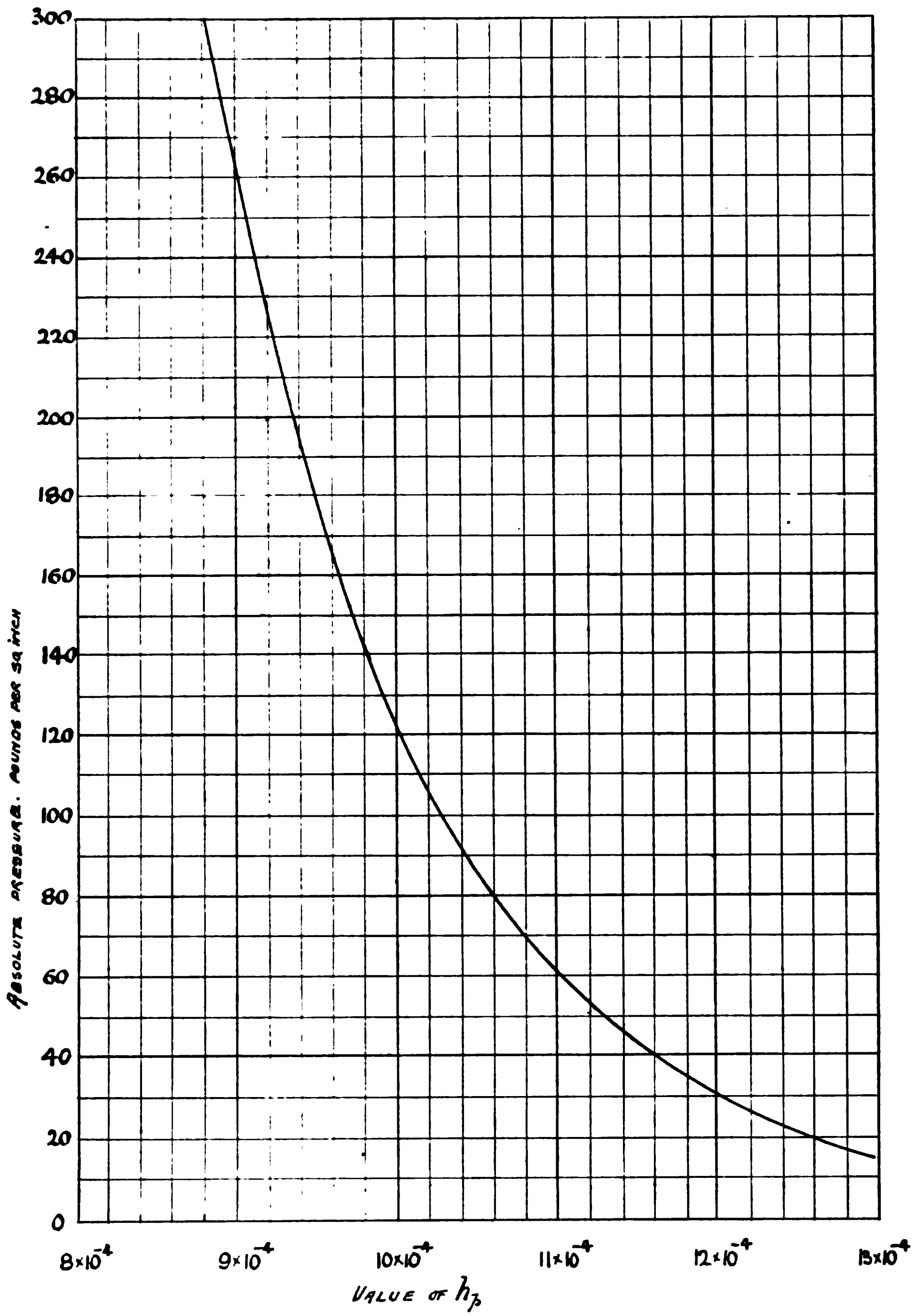


FIG. 49.—Stress Specific Heat of Wet Steam in final stage of drying by heating at constant volume. Per cubic inch and per 1 lb. per sq. inch rise of pressure.

N.B.—For Table XLIX see next page.

A similar h_p can be calculated for the same substance for a rise of pressure *immediately below* the saturated condition; but here we are not dealing with pure steam, but with a mixture of water and steam in the final process of being dried to pure dry saturated steam by heating at constant volume. In this drying, the major portion of the heat absorbed is spent in evaporating the water mixed with the steam. The calculation is more difficult than the above, although not involving any elements of approximation. It has also been made by Mr. H. M. Hodson with the results tabulated in the following Table XLIX and Fig. 49. In this case it may be noted that the quantity decreases very considerably as the pressure rises; but at its lowest value it is still over twice as large as the h_p of the previous table.

TABLE XLIX

Stress Specific Heat of Wet Steam in the final stage of drying by heating at constant volume in Brit. Heat Units per cubic inch of steam per 1 lb. per sq. inch rise of pressure.

Absolute Pressure lbs. per sq. in.	h_p
20	$12{\cdot}6 \times 10^{-4}$
40	$11{\cdot}6 \times 10^{-4}$
60	$11{\cdot}0 \times 10^{-4}$
80	$10{\cdot}6 \times 10^{-4}$
100	$10{\cdot}3 \times 10^{-4}$
120	$10{\cdot}0 \times 10^{-4}$
140	$9{\cdot}8 \times 10^{-4}$
160	$9{\cdot}65 \times 10^{-4}$
180	$9{\cdot}5 \times 10^{-4}$
200	$9{\cdot}35 \times 10^{-4}$
220	$9{\cdot}25 \times 10^{-4}$
240	$9{\cdot}1 \times 10^{-4}$
260	$9{\cdot}0 \times 10^{-4}$
280	$8{\cdot}9 \times 10^{-4}$
300	$8{\cdot}8 \times 10^{-4}$

The next Table L and Fig. 50 give the calculation of h_s, the "strain" or "expansion" specific heat of wet steam being finally dried to the dry saturated condition by heating at constant pressure—therefore at constant temperature. This process is the ordinary one referred to as evaporation of water into steam; although, as has been previously remarked, water may be evaporated into steam under an infinite variety of heating and even cooling conditions. In this case the heat conduction per pound evaporated is the ordinary tabulated "Latent Heat"—the whole of this without deduction

of the mechanical work done externally. Taken per pound and per cubic inch of the expansion, it is

$$\frac{\text{L per lb.}}{s-\text{vol. per lb. of water}}$$

where s is the specific volume of saturated steam in cubic inches per lb. The heat required per 1 cubic inch of expansion is the same whatever be the quantity as already explained. The above is, therefore, the expansion specific heat per cubic inch of steam per cubic inch of expansion.

TABLE L

Expansion Specific Heat of Wet Steam or Water in being dried or evaporated by heating at constant pressure in Brit. Heat Units for any quantity of Wet Steam or Water per 1 cub. inch of expansion.

Absolute Pressure lbs. per sq. in.	h_s
10	·0148
20	·0277
40	·049
60	·072
80	·093
100	·116
120	·136
140	·156
160	·175
180	·194
200	·213
220	·232
240	·251
260	·268
280	·286
300	·304

N.B.—For Fig. 50 *see page* 191.

After passing the saturation curve the expansion at constant pressure changes character since evaporation of water is no longer going on. We have now to expand dry steam instead of a mixture of steam and water. Up to the saturation limit no real expansion of steam occurs ; the expansion is entirely that of water into steam by evaporation ; the heat absorbed being entirely devoted to this evaporation and not to the heating or other change in the already generated steam. Beyond the limit all the heat is spent on the steam—in expanding it and raising its temperature. Since it is close to the saturated condition, the steam does certainly not behave as a "perfect gas," the demarcation between wet steam

and " steam-gas " being a very blurred one ; still an approximation may be obtained by assuming ·48 as the specific heat per pound per 1° rise of temperature, and that the temperature (absolute) and the volume increase in the same proportion. Thus s being the volume in cubic inches and t the absolute temperature at saturation, then with expansion by 1 cubic inch the increased temperature will be found from $\frac{t+\delta t}{t}=\frac{s+1}{s}$, or $\delta t=\frac{t}{s}$; also $\cdot 48\ \delta t=\cdot 48\frac{t}{s}$. This is the h_s at constant pressure per cubic inch of expansion for any quantity of steam. It is given in the following table. Comparing the values of this table with those in the last table for wet steam, it will be seen that for dry steam at low pressures h_s is about one-third those for wet steam, while at high pressures the ratio is about one-half. h_s for dry steam is seen to increase approximately in proportion to the pressure, and to differ little from h_s for air. It is very closely given by the formula

$$5\cdot 2\times 10^{-4}p\ ;$$

while the h_s for wet steam being dried between the pressures 50 and 250 lbs. per square inch does not deviate much from the formula

$$(\cdot 01+\cdot 001\ p).$$

TABLE LI

Expansion Specific Heat of Dry Steam in Brit. Heat Units for any quantity of Steam per 1 cub. inch of expansion at constant pressure immediately beyond the saturated condition.

Absolute Pressure lbs. per sq. in.	h_s
10	·0047
20	·0096
40	·019
60	·029
80	·039
100	·05
120	·06
140	·07
160	·08
180	·091
200	·102
220	·113
240	·124
260	·135
280	·146
300	·157

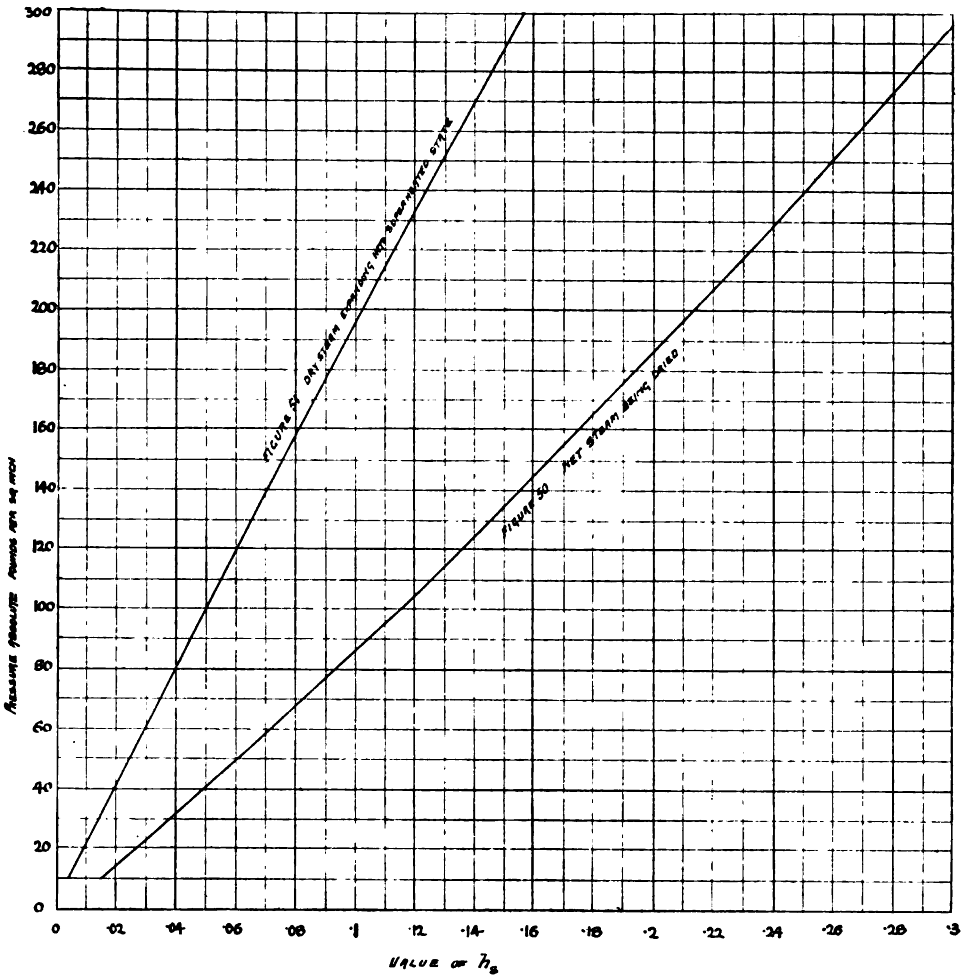


Fig. 50.—Expansion Specific Heat of Wet Steam, or Water being dried or evaporated by heating at constant pressure in Brit. Heat Units for any quantity of wet steam or water per 1 cubic inch of expansion.

Fig. 51.—Expansion Specific Heat of Dry Steam in Brit. Heat Units per 1 cubic inch of expansion at constant pressure immediately beyond the saturated condition, for any quantity of steam.

It has been seen in Table XLVII, page 183 of this chapter, that for the coal gas mixtures used in gas engines the value of h_p is $2{\cdot}77 \times 10^{-4}$; while for other gases it is for some a little above 3×10^{-4} and for others somewhat below this, the value for the ideal "perfect" gas being $2{\cdot}63 \times 10^{-4}$. In the diagram given below, Fig. 52, this rounded-off value, 3×10^{-4}, is used in calculating a series of heat-transpowers needed for gas expansions along curves with various indices i. The adiabatic index a is taken as 1·4; and the standard velocity of expansion v'_t is taken as 10,000 cubic inches per second. This velocity of expansion is that effected in an engine cylinder whose piston area is 100 square inches (diameter 11·28 inches) and whose piston speed is 100 inches per second = 6,000 inches per minute = 500 feet per minute. According to Equation X, page 173 of this chapter, the heat inward transpower required in heat units per second per cubic inch of expanding gas, in order to keep the expansion to the curve of index i, is

$$H'_t = (a - i)\, h_p\, v'_t\, p = 3\,(1{\cdot}4 - i)\, p \;=\; (4{\cdot}2 - 3\,i)\, p$$

for the above standard values of a, h_p, and v'_t. This is to be read directly from the diagram Fig. 52 to the horizontal scale at the proper height p as read on the vertical scale and to the proper oblique straight line for the given value of i. The series of oblique straight lines are for the values of $i = 0$, ·1, ·2, ·3, ·4, ·5, ·6, ·7, ·8, ·9, 1·0, 1·1, 1·2, and 1·3; and each line is marked with its value of i.

All the lines radiate from the origin, giving $H'_t = o$ when $p = o$; and at $p = 100$, the horizontal ordinate of each line is

$$(4{\cdot}2 - 3\,i)\,100 = 420 - 300\,i.$$

The vertical scale takes the pressure up to 300 lbs. per square inch.

This is the requisite heat-transpower per cubic inch. If the expanding substance occupy at any instant a total volume V cubic inches, then the total heat-transpower is $H'_t\,V$; and, if the surface through which the heat is conducted (or radiated) to this volume be S square inches; then the heat-transpower required through each square inch of this heating surface is $H'_t\,\frac{V}{S}$. In this $\frac{V}{S}$ is what is called the "mean hydraulic radius" of the surface. This is, of course, the average transpower, and it may not be uniformly distributed over the surface. During the expansion p falls while V increases, so that the product p V, and therefore H'_t V, falls in inverse proportion to $V^{(i-1)}$. The transpower per square inch of surface

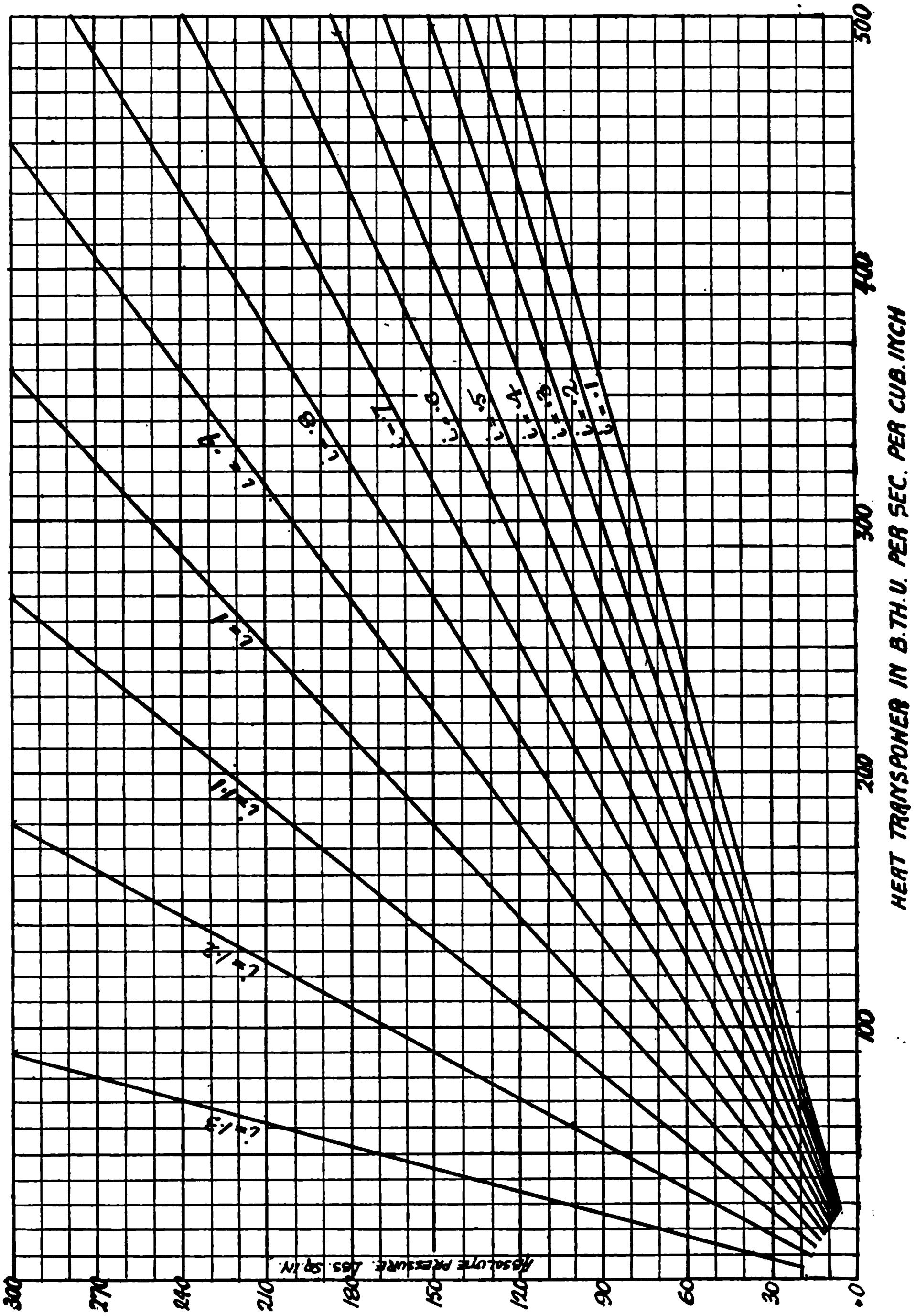


FIG. 52.—Heat Transpower into a gas expanding along curve pe^i = const. for i = ·1, ·2, ·3 ... ... 1·2, 1·3.
Rate of Expansion. 10,000 cub. ins. per sec.

$H'_t \frac{V}{S}$ falls still more rapidly if S increase along with V, as it always does in working engines.

In the diagram Fig. 53 is given the similar calculation for dry saturated steam expanding along the "saturated steam" curve. If such steam expand without receiving heat, part of it condenses. In order to prevent condensation and so keep the steam dry and only just saturated, heat needs to be conducted to it at a rate proportionate to the velocity of expansion and given by Equation X. The h_p to be used is that for wet steam being dried, see Table XLIX and Fig. 49. It varies along the curve, that is, as the pressure varies. The adiabatic index a is assumed uniform at Rankine's value 1·13, while the index i for the saturation curve is also assumed uniform $= 1{\cdot}06$ which is Rankine's value. The normal value $v'_t = 10{,}000$ cubic inches per second is taken for the velocity of expansion, and for any other velocity of expansion the heat-transpower read from the diagram must be increased in proportion. Here $v'_t\,(a-i) = 700$ and is constant throughout the range of pressures, which in the diagram is carried up to 300 lbs. per square inch absolute. Thus Equation X reduces for this diagram to

$$H'_t = 700\ h_p\ p.$$

This does not increase according to a straight line with the pressure, as h_p is smaller at higher pressures; so that the diagram of H'_t is a curve whose rise with pressure is somewhat less rapid at the higher than at the lower pressures. The curve, however, is a very flat one, and throughout its length above 20 lbs. per square inch deviates little from the straight line

$$9 + {\cdot}6\,p.$$

This H'_t read from the diagram and corrected for the given volumetric piston speed, must be multiplied by the total volume of expanding steam in the cylinder in order to get the total heat-transpower needed; and to find this transpower per square inch of the heating surface, H'_t must be multiplied by $\frac{V}{S}$ as in the case of the gas expansion.

If S were the *total* surface enclosing the expanding steam or gas, then $\frac{V}{S}$ would be the *mean hydraulic depth* of the space occupied; and, therefore, the *mean* heat-transpower per square inch over the *whole* of this surface, is always

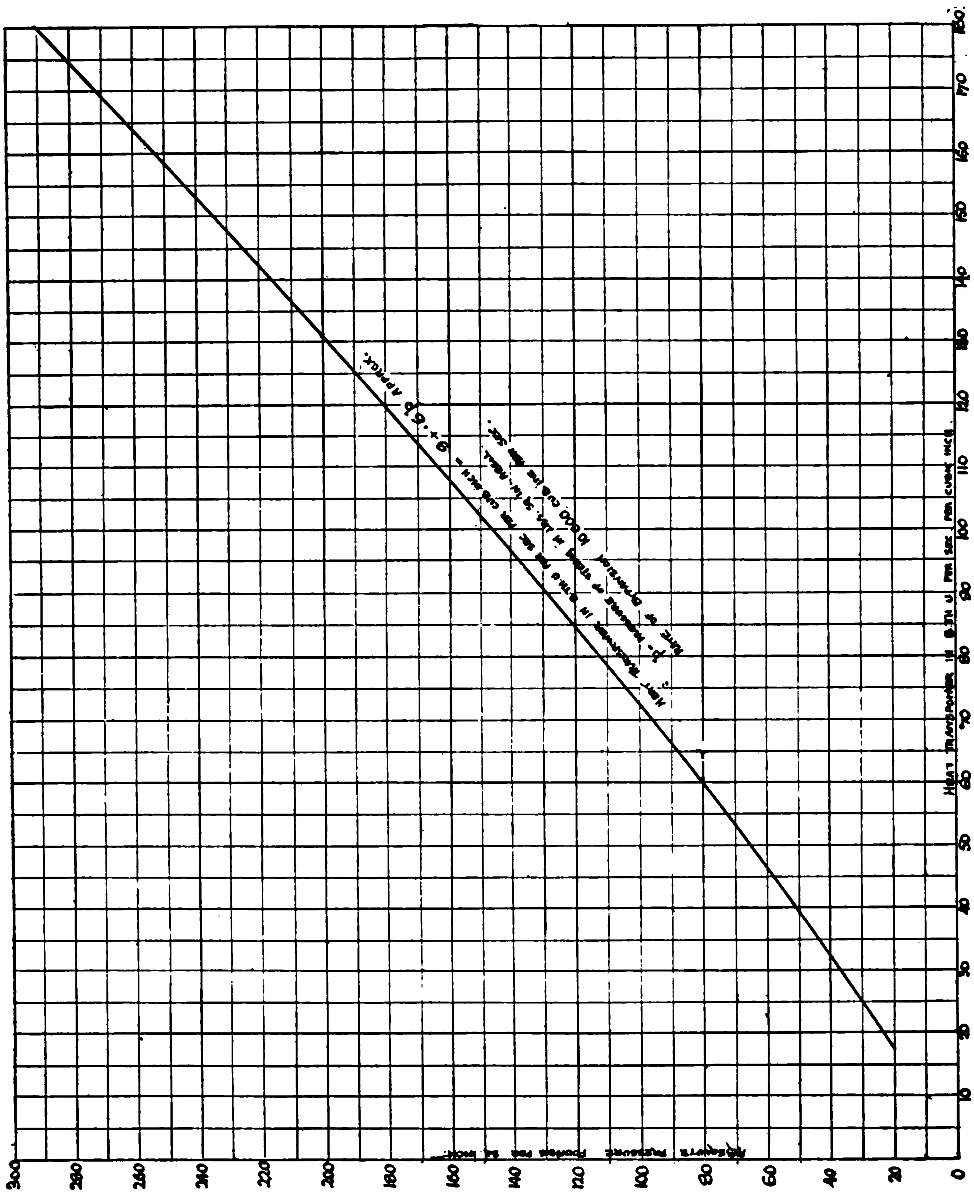


FIG. 53.—Heat Transpower in expansion of saturated steam along the saturation curve.
Rate of expansion, 10,000 cub. ins. per second.

H'_t × Mean Hydraulic Depth.

A cylindric volume of length l and diameter d, has the mean hydraulic depth—

$$\frac{1}{\frac{2}{l}+\frac{4}{d}}$$

if the two flat end surfaces be included in the total surface.

Chapter VI

Analysis into Thermal and Mechanic Elements. Dynothermic Coefficient.

We have seen that mechanical energy, like each other kind of energy, may be divided into two great classes, the first of which is active and is known in one form only, namely kinetic energy, while the second is quiescent, or " potential," or " latent," or " static." The author himself prefers the term " latent " because it postulates nothing about the particular form in which the energy exists except that it is *not visible*. Kinetic energy is often described as energy of *visible* motion, so that latent, which means " hidden," not only offers a correct contrast to kinetic, but also correctly describes the fact that, as yet, we have not discovered either the form or, for certain, the place of the latent or potential energy. The name " potential " was originally applied to indicate a *power* of producing active energy. But it has long since been appropriated by mathematical physicists to a much more definitely restricted sense, according to which potential energy is energy which has what mathematicians call a " potential " with respect to a given set of " co-ordinates " ; that is, which, after undergoing any variations whatever, resumes its original amount as soon as a certain mathematical function, called the " potential," of these co-ordinates is restored to its original value—the same value as it had before these variations.

All mechanical energy of the potential or latent order is of the resilient kind, and the co-ordinates involved in the calculation of its potential are strain and stress values. There are many kinds of resilience. It is difficult to say whether electro-magnetic and electrostatic resilience should, or should not, be called mechanical, or electrical, or magnetic : the propriety of doing so depends upon a consensus of opinion in the scientific world, to arrive at which no attempt has yet been made. What is certain is that these potential energies involve electro-magnetic strains and stresses in the ether

and probably also in gross matter, and furthermore that these result in mechanical forces acting on gross matter, and thus in mechanical stresses and strains in gross matter. Any objections to calling them *mechanical* resilience would in the same degree apply to calling the potential energy of gravitation a mechanical energy. In so far as engineers have to consider the production of mechanical energy and power from *heat*, however, the main resilience to be dealt with is the *resilience of stress and strain in gross matter*.

The total resilience, whose time-rate of exertion is mechanical transpower, and which alone is of any *ultimate* interest to the mechanical-power engineer, is measured by the total mechanical work done in unstraining to the limit of complete freedom from strain and stress. The resilient energy in any specified elastic condition is evidently measured by that portion of the work so done accomplished in virtue solely of that specified starting elastic condition, without aid from external sources during the unstraining; that is, the work done by adiabatic unstraining from the initial condition. The area underneath the adiabatic curve through the initial point on the $p\ s$ diagram carried down to the intersection, or junction, of this curve with the base line of zero stress, is the mechanic resilience in that starting condition. If the unstraining follow any other curve, the excess or deficiency of the resilient-work area under the new curve over that under the adiabatic curve is due to external aid supplied during the process. During each small part of the process a portion of the effect of this external aid is immediately and simultaneously spent in doing mechanical work. This portion is the area of the strip between the actual and the adiabatic curves included between the two vertical ordinates, positive if the actual lie at higher stress than the adiabatic curve, otherwise negative. The remainder of the resilient effect of this external aid results in the adiabatic resilience at the end of the change being greater or less than at the beginning.

There are two means of producing resilience in material, which two may be combined in any proportions. The first is purely dynamic, and consists in adiabatic stressing, the accompanying straining being governed by the special adiabatic elastic law of the material dealt with. In any subsequent unstraining the resilience so developed is wholly recovered in work done, without loss if the unstraining be at higher stresses than was the straining; and if the unstraining stresses be less, the deficiency of work done is due to the operation of external thermal transpower, or of internal generation or destruction of heat by conversion of energy.

According to what has been just written and to what appeared in the previous chapter upon "irreversibility," it would appear that work done in producing resilience by adiabatic straining is completely and exactly recovered in the reverse process of adiabatic unstraining. It is evidently so if the adiabatic unstraining curve be identical with the adiabatic straining curve. To make the law exact and incontrovertible it is necessary to pay close attention to the definition of adiabaticity in reference to *hysteresis*. Without any external transpower of heat either inwards or outwards, the unstraining curve formed by co-ordinating strain with *external* stress will differ from the similar straining curve whenever there is hysteresis : the external stress will be lower and not so much external work is done. To make the straining and unstraining curves identical and so make the reversibility of adiabatic straining a completely true proposition, it is needful to follow one of three courses : (1) to restrict the proposition to "perfectly elastic" substances, i.e. such as show no mechanical hysteresis ; or (2) to make it apply, not to external stress alone, but to the sum of this and of the internal mechanical hysteresis stress—but this course is not satisfactory because it is by no means certain that such stress exists as a mechanical stress ; or (3) to take account of the internal conversion by hysteresis of the mechanical energy of resilience into other forms of energy such as heat, in the definition of adiabaticity, declaring that by adiabatic change is meant change unaccompanied by any such internal conversion, as also unaccompanied by external heat-transpower. The third alternative appears to the author to be preferable ; because it harmonizes with the manner in which one is practically forced to deal with heat gained by internal combustion. Heat so gained must be treated in exactly the same way as if it had been received by heat-transpower or conduction from without. The expansion of the gas charge in a gas engine during the generation of heat by the combustion of this charge could not usefully be considered an adiabatic expansion, apart from all question of the action of the water cooling jacket. This being so with respect to heat internally generated in one manner, it seems convenient to treat all heat or other energy internally generated or destroyed in the same manner. The work lost in a hysteresis cycle in consequence of "imperfect" elasticity is of course spent in producing some one or more forms of internal energy.

The question of mechanical or resilient hysteresis has, so far as physical research has yet gone, arisen only in connection with solids ; solids of semi-elastic semi-plastic character. It is intimately con-

nected with the question of plasticity. The existing paucity of exact knowledge on the subject is due to lack of much observation (1) in regard to the time-rate at which tests of elastic solids are made and the influence of variation of these time-rates, and (2) in regard to change of temperature in the solid during test. Thousands of tests are made, from which thousands of values of the elastic moduli are, or may be, determined; but it is never known *which modulus* it is that reigns in any such test, whether the adiabatic, the isothermal, or what intermediate modulus. A research on the changes of temperature occurring during such tests would be very difficult, because in engineering tests of materials it is well known to physicists that the stressing is very far from being uniformly distributed. It is well-known that heat is "*generated*" by the test, just as heat is "*generated*" by the adiabatic or other compression of a gas—by which loose phrase all that is meant is that rise of temperature is easily noticed. This rise of temperature due to the test-straining must be very far from uniform throughout the test-bar. Those parts in which it is most rapid will lose heat by conduction to those other parts where it is less rapid, so that neither part is strained adiabatically. The author does not know of any attempt ever having been made to overcome these difficulties in the exact experimental investigation of the thermodynamics of solids.

Hysteresis has not been experimentally observed in vapours, gases, and other fluids. It probably exists, because these fluids suffer viscosity, while all compression and expansion of them normally involve shear strains. Its amount is, however, probably small.

Adiabatic straining being the first method of producing resilience, the second is by the operation of external thermal transpower; or by internal heat mutapower, that is, generation of heat by combustion or like means.

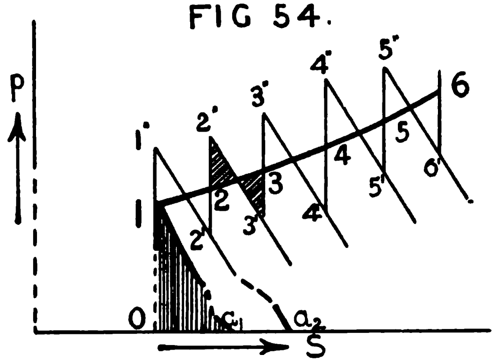
These two means of producing resilience correspond to the two additive parts of the elastic modulus E and to the two like parts of p'_t in Equations VIII and IX of Chapter V, page 171.

The clearest method of following all thermodynamic actions is to analyse them into two parts corresponding to the purely dynamic and purely thermal methods of creating and of spending resilience. The analysis must be made continuous—that is, by small pairs of steps continuously proportioned so as to follow closely the actual complete thermodynamic operation.

In Fig. 54 the curve 1, 2, 3, 4, 5, 6 shows an expansion with rise

of pressure. Divide it into a sufficiently large number of small parts, and through the middle of each part draw adiabatic curves to meet the verticals through the points of division. If the points be taken close enough, all the physical results of the actual path will be, to a sufficient degree of approximation, the same as those obtained from the zig-zag path 1 1″ 2′ 2″ 3′ 3″ . . . . 5′ 5″ 6′ 6. In this latter, for example, the excess of mechanical work measured by the small triangular area hatched over to right of 2 2″ is balanced by the similar and equal deficiency of the hatched triangle to left of 3′ 3. The adiabatic unstraining 2″ 3′ is half above and half below

FIG 54.



THERMAL AND DYNAMIC ANALYSIS OF THERMODYNAMIC ACTION

the actual path, and its mean elastic modulus is therefore that at the point where it coincides with or crosses this path. The two isometric rises of stress 2 2″ and 3′ 3, practically equal in amount, are one above and one below the curve, so that their mean specific heat h_p is the same as the mean h_p along the actual path 2 3. Along the adiabatic 2″ 3′ there is no heat-transpower—nothing but pure mechanic transpower by expenditure of resilience. During the rises of pressure 2 2″ and 3′ 3 there is no mechanical work done, that is, no resilient transpower—nothing but pure heat-transpower. The rise of pressure along the actual path 2 3 equals the sum of these two rises 2 2″ and 3′ 3 with the adiabatic drop of pressure from 2″ to 3′ subtracted. It is a good exercise for a student to work out in detail this analysis in terms of the δs and δp from 2 to 3 and of h_p, E_a, H'_t, and v'_l. There is no need to put it out here in symbols,

because the result is precisely the same as we have already in Equation VIII, that is, the elastic modulus along the actual path equals $\left(\frac{H'_t}{h_p v'_l} - E_a\right)$.

This analysis is given here in order to suggest that there is only one real specific heat, namely h_p, that for increase of stress without straining, The adiabatic specific heat is, of course, zero essentially by definition. The curve 1...3...6 of Fig. 54 has been drawn at random ; it may be taken of any shape and direction whatever. Whatever resilient change and action be effected, it is always the result of pure heat-transpower with specific heat h_p, whereby the stress is changed without change of strain and therefore without expenditure of resilience by mechanical work, accompanied by pure dynamic or adiabatic unstraining—or straining—that is, pure mechanical transpower with the elastic modulus E_a.

Each of these transpowers, $-H'_t$ and $-R'_t$, is a *time-rate* of gain or loss of energy, and their combination results in effects entirely dependent upon their comparative *rapidities*. The resultant elastic modulus E is the elastic modulus E_a superimposed upon $\frac{H'_t}{h_p v'_l}$ in which H'_t is the rapidity of heat transpower and v'_l is the velocity of the mechanical transpower.

These two rapidities result as much from outside surrounding conditions as from those of the active materials, and the two result from entirely separate and independent surrounding conditions. Therefore no true understanding of any thermodynamic action can be obtained by any other method than that of considering and comparing *time-rates*, except in the two special cases in which one of the two time-rates, or one of the two parts of p'_t, is zero. Neither of these two special cases is really thermodynamic ; the one is purely thermal, the other purely dynamic.

Isobaric expansion—that under constant stress—is only obtained when the increase of stress by heat transpower is so *timed* as to exactly counterbalance the decrease of stress by adiabatic mechanic transpower.

Isothermal expansion is only obtained when the increase of temperature by heat transpower is so *timed* as to exactly counterbalance the decrease of temperature by adiabatic mechanical transpower.

When the indicator diagram keeps at a slope less steep than the adiabatic, the mechanical work is still done wholly adiabatically. New resilience is being created by heat transpower, while at the

same time part of the resilience, new and old together, is being spent in doing adiabatic external work. There is no other way of doing external mechanical work than by adiabatic unstraining. In the creation of resilience by heat the only specific heat involved is h_p, the stress-without-straining specific heat. In the doing of mechanical work the only elastic modulus involved is E_a, the adiabatic, or straining-without-heat-transmission-or-generation modulus.

The creation of resilience by mechanical work—that is, by adiabatic stressing-and-straining—is perfectly *reversible* ; in subsequent unstraining the whole of such work is recovered, the " efficiency " between the charging with resilient energy and the discharge being perfect, or 1.

In this statement it must be remembered that the word " reversible " is used in its academic sense, previously fully explained. If the rise of temperature caused by adiabatic straining be lost by conductive or radiative cooling before the reversal and unstraining takes place, then the work recovered is less than the work absorbed. It is also to be understood that adiabatic changes exclude hysteresis effects.

The creation of resilience by heat transpower is *irreversible* ; the process can be reversed only by changing the temperatures in the regions surrounding the material operated on. No reverse thermal discharge is possible except by such change of external temperatures. This does not simply mean that a low efficiency is obtainable, but that the process is entirely impossible.

The same is true of the creation of resilience by internal heat generation or mutapower. Among the modes of such internal heat generation are chemical combustion, electro-magnetic hysteresis, superficial friction, internal friction—constituting at least part of resilient hysteresis—and internal viscosity. As these processes do not depend upon external or internal temperatures or their differences, they are entirely irreversible even by alterations of the temperature conditions.

The resilience so created by heat can always, however, be discharged mechanically by adiabatic unstraining. If the efficiency of such complete discharge were measured by the ratio of the mechanical work done during discharge to the resilience created during the charge, this efficiency would again be perfect, or unity. But if it be measured by the ratio of the discharge mechanical work to the heat employed in the charging process, then the calculation given below shows the result. Again, if the adiabatic discharge be interfered with by thermal transpower or internal heat generation *or destruction*

during discharge in such manner as to lessen the work done, then another " efficiency " could be calculated as the ratio of work done, down to complete freedom from stress and strain, to the full initial adiabatic resilience. Since, however, heat transpower during discharge may increase, instead of diminishing, the work done, such a measure of efficiency consistently carried out would sometimes give a measure greater than unity.

To engineers the ratio of the new adiabatic resilience created by heat transpower to the heat spent in this creation is the thermodynamic ratio of the greatest importance. It cannot be well called an efficiency, because, as will be seen from what follows, it may, in liquids and solids, be greater than unity. It must not be called the thermodynamic coefficient, because this name would be confused with the thermodynamic function which has, ever since Rankine's days, had a definite and useful scientific signification. It ought to have a distinct name, and some fourteen years ago the author adopted for it the name

DYNOTHERMIC COEFFICIENT,

which he has found extremely convenient.

As already stated, the only resilience possessed by any mass in any specified condition in virtue solely of that condition is the adiabatic resilience. Therefore in referring to this resilience the adjective " adiabatic " may be omitted as superfluous. Also in the view taken above of heat-resilient action, there is one only real or fundamental specific heat, namely h_p or that per unit increase stress at constant volume or more generally at constant unchanged strain. Therefore, in most of what follows in this volume the suffix p will be omitted as superfluous ; and, except when special reference is made to the other specific heats for increase of volume or increase of temperature, the simple symbol h will be used for what has been hitherto designated by h_p.

The whole of the resilience possessed by material in any specified condition may have been produced by heat-transpower or internal heat generation. In Fig. 54 the process of such creation of the resilience in the condition 1 is described by the gradual rise of stress from point 0, the base on the axis of zero stress of the vertical through 1, up this vertical to 1, there being no change of strain during the rise. If h_p or h were constant from 0 to 1, the whole of the operative heat supply per unit volume would be $p\ h$; and, in any case, if h be the average value between 0 and 1, $p\ h$ will measure this heat supply. The resilience at 1 is the thatched area above the

zero-stress axis to right of the vertical 0 1, and under the adiabatic through 1 carried down to zero stress at a_1. In gases under pressure the adiabatic runs tangentially into this axis, but this results in no indefiniteness in the measure of the adiabatic resilience. In solids, both pressure, tension and torsion adiabatics run sharply into and through this axis. If the process of stressing without straining be carried further from 1 to 1″, the increase of resilience is the area $1a_1\,a_2\,1''$. Call this δ R and $1\ 1'' = \delta\,p$. Then, following the notation hitherto used, we call $\frac{\delta\,R}{\delta\,p}$ the stress-gradient of the resilience, and write it R'_p; so that $\delta\,R = R'_p\,.\,\delta\,p$.

R is the resilience for the volume, or other appropriate geometrical configuration, s; and if we divide R'_p by s, and call the quotient r_p, or simply r, we have a stress specific resilience exactly analogous to our stress specific heat h, giving $\delta\,R = s\,r\,.\,\delta\,p$. If h be the stress specific heat at 1 1″, the heat supply is $s\,h\,.\,\delta\,p$. The ratio between the resilience created and the heat transmitted is $r \div h$.

THIS RATIO r/h IS THE DYNOTHERMIC COEFFICIENT.

If the adiabatic curve be correctly described by the equation $p\,s^a =$ a constant, then $R = \frac{p\,s}{a-1}$. If so, a and the "constant" are, of course, the same for all simultaneous pairs of p and s on the same curve; but it does not follow that they are the same for the different adiabatics a_1, a_2, a_3, etc. So far as R is concerned, variation of the "constant" is not involved in the expression $\frac{p\,s}{a-1}$. As for variation of a with p, we know that it is at least small so long as no "change of state" is involved. When it is permissible to neglect its variation throughout the range over which our heat transpower is exerted, we find—

$$R'_p = \frac{s}{a-1},\ \text{and}\ r = \frac{1}{a-1}.$$

In this case we have the

$$\text{Dynothermic Coefficient} = \frac{r}{h} = \frac{1}{(a-1)\,h}$$

where it must be remembered that h is taken per unit volume. To correct this accurately for the neglect of the variation a'_p, we

should multiply it by $\left(1 - \frac{pa'_p}{a-1}\right)$, which may be very different from 1 if a be close to 1 and a'_p not very small.

At first the "dimensions" of this ratio $\frac{1}{(a-1)h}$ may seem to be wrong, as the dynothermic coefficient is a pure number, as is also $(a-1)$. But it may be noted that h is also a numerical ratio : it is heat energy per volume per increase of pressure per area, that is, it is energy per (length $\times$ force).

The resilience in any condition specified by ps being $\frac{p\,s}{a-1}$, where a is the index of the adiabatic unstraining curve supposed to remain the same down to the foot of this curve on the axis of no stress, evidently this resilience has the same value at all points of the diagram where the product $p\,s$ and the index a are the same. If a be the same throughout the diagram, then hyperbolic curves (curves of simple reciprocals) give, each of them, the locus of equal values of $p\ s$ and equal values of the resilience. Each such curve is a resilience equipotential, and might be called an "iso-resilient," or an "isoresalic," or, more simply, an "isosalic" because the "re" is not an essential, or even a proper element in the word "resilience." Resilience is the power to spring *back* when the strain has been produced dynamically ; but we have just considered the production of resilience by heat-transpower, in which case there is no springing back, the power to spring being produced *de novo* by heat conduction. In "perfect" gases these isosalics coincide with the isothermals.

Wherever the adiabatic holds to a constant index a throughout its length down to zero stress, the dynothermic coefficient is $\frac{1}{(a-1)\,h}$. Thus this coefficient varies only with a in passing from one to another adiabatic and with h.

For perfect gases, $h = \frac{1}{a-1}$; so that for these the dynothermic coefficient is unity throughout, and is the same for all such gases. For them, $a = 1{\cdot}4$ very nearly.

If the adiabatic equation be $p\,s^{-a} =$ a constant, s being the strain and not the volume, then $\mathrm{R} = \frac{p\,s}{a+1}$ and $\mathrm{R}'_p = \frac{s}{a+1}$, if a remain the same for the different adiabatic curves.

Under pressure h is generally positive. That is, both solids,

liquids, vapours and gases usually have the pressure-stress increased by influx of heat when the volume is held unchanged.

Under tension h is generally negative—that is, solids usually slacken their tension-stress if heat flows into them when held to constant volume. Here the resilience is decreased by influx of heat, and increased by loss of heat by conduction or radiation.

This proves that it is not essentially negative heat transpower that produces the potentiality of positive mechanical transpower—that is, inward flux of heat energy does not necessarily produce the power to do mechanical work.

In some pressure conditions h is negative, as in melting ice ; and it is positive in some tension conditions, as in some states of india-rubber. These cases are rare, but their existence makes it clear that neither in compression nor in tension is there any universal and invariable law connecting the sign of the heat transpower with the development of mechanical power.

In pure mechanical action without heat transpower or generation, increase of stress always accompanies increase of strain, strain being deviation from the size or shape in which there is zero stress. Opposite deviations produce opposite stresses, such as tension and compression, or right and left-handed twists. Inward mechanical transpower always produces increased stress, and *vice versâ*. In stress-volume diagrams, all adiabatics slope down towards the right if pressure be measured upwards, tension downwards, and volume towards the right.

In pure thermal action without mechanical transpower, inward flow, or internal generation of heat, always produces rise of temperature, and *vice versâ*. In $\theta\ \phi$ diagrams, all equal-strain curves —which are those of no mechanical transpower—slope upwards to the right, if temperature be measured upwards and entropy towards the right.

Thus, when h is negative, as it usually is in tension and sometimes in compression, the stress decreases when temperature is increased by thermal action without dynamic action.

When a surplus of positive or outward mechanical transpower is obtained from a series of pairs of opposite actions, which leave no permanent change in the active material, as in all three diagrams (A), (B), (C), of Fig. 55, the unstraining must take place at higher stresses and the straining at lower stresses. As straining and unstraining through the same range along the same adiabatic gives no area to the indicator diagram, in consequence of the complete reversibility of pure dynamic action, and since thermal transpower

or generation is needed to get away from the one adiabatic, therefore, in order to obtain mechanical power from " closed " diagrams, it is necessary to make use of irreversible heat transpower or generation. There must be a surplus of negative, or inward heat transpower, in two separated heat transpowers.

The pair of opposite heat fluxes must be such as to raise stress at larger strains and lower it at smaller strains. This is the com-

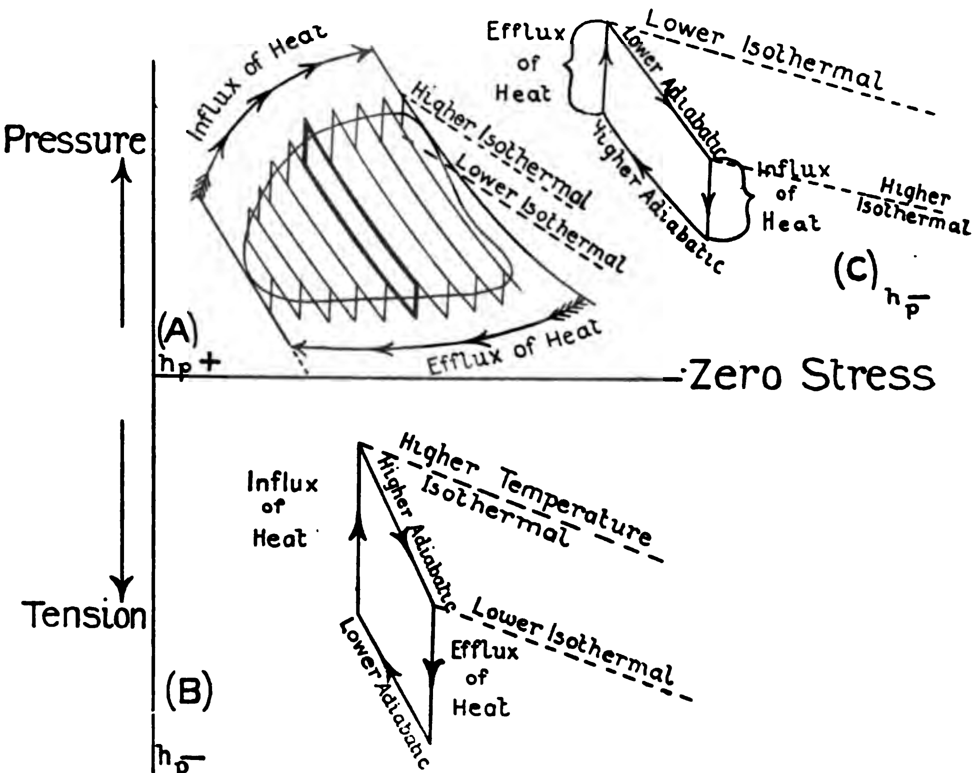


FIG 55.

plete and only criterion, whether the stress be compressive, tensive, or torsional. When h is negative, as in (B) and (C) of Fig. 55, the heat influx acts at the lower strains, and unstresses the material, and *vice versâ* for the heat outflow. In all cases the heat influx carries the material to higher temperatures, and *vice versâ* for the heat outflow; the rise or fall being, however, sometimes possibly masked by simultaneous greater fall or rise of temperature by mechanical action.

Thus, although it is essentially the rise of stress at larger and its fall at lower strains that make the heat fluxes effective in forming a closed diagram to give outward mechanical work, still this efficacy may be stated in terms of temperature.

It appears to be a universal law that in order to prevent change of temperature during unstraining (that is, during the doing of mechanical work), a heat flux or generation must take place such as increases stress. Otherwise stated, adiabatic unstraining involves greater fall of stress than does isothermal unstraining, as is seen on all the three typical diagrams of Fig. 55. As seen by comparing these diagrams, the necessary heat flux may be inwards or outwards; it may be such as increases temperature, or *vice versâ*, but it must always be such as increases stress. If any diagram, closed or unclosed, cross any adiabatic at two places, as in the series of such lines in (A), Fig. 55, the two crossings must be in opposite directions, and at one there is inflow and at the other outflow of heat. The result of the invariable relative inclinations of the adiabatics and isothermals above mentioned is that the inflow always takes place at higher temperature, and the outflow at lower temperature, if the crossings be in the directions giving a positive balance of outward mechanical work. If a pair of adiabatics be drawn near each other, as the pair drawn in heavy lines in (A), then the "conservation of energy" shows that the difference between the influx and the efflux between this pair equals the work area between them and the two crossing boundaries of the diagram. The choice that has been made of the "thermodynamic" measure of temperature makes the same difference proportional to the difference of the two temperatures, and the flux at each end proportional to the "absolute" temperature there. This method of measurement of temperature is a matter of choice, but the physical fact that it gives the same temperature scale under all circumstances and in all materials is the essence of the second law of orthodox thermodynamics. The restricted meaning of "conservation of energy" in this statement is the specification that there is to be no generation or destruction of energy of any kind inside the active material, and no transpower into or out of it, except by thermal and dynamic processes.

The measure of the difference of entropy per unit mass between the two adiabatics is taken as equal to the heat flux into or out of unit mass divided by the absolute temperature of that mass. Since the heat flux is from one mass at higher temperature to another at lower temperature, it follows that the entropy lost by the mass from which heat flows is less than the entropy gained by the mass into

which it flows; or the sum of the entropies of the two masses is increased by the heat-transpower between them. It falls outside the scope of this book to discuss the theory of entropy. Mr. James Swinburne has recently (1903, pub. A. Constable & Co.) written a full and instructive discussion of this theory, in which he demonstrates the dangerous misconceptions to which students are exposed who have no thorough knowledge of physics to guide them in this matter. The difficulty arises from the question, What is the temperature of the mass by which the heat flux inward or outwards is to be divided? There cannot be uniformity of temperature where there is heat flux; and, therefore, if average temperature is to be used as the divisor, no other method of procedure in the calculation can be satisfactory than that of cutting up into extremely thin slices the material through which the flux takes place, by plain or curved surfaces perpendicular to the flow. The slices must be so thin that the average temperature of each does not appreciably differ from its maximum and minimum temperatures. If the heat transpower per unit cross-section be H'_t; and if the temperature gradient along the path l normal to this section be τ'_l; and if δl be the thickness of the slices; then $\tau'_l \, \delta l$ is the drop of temperature from the face of one slice where there is influx of heat to its face where heat is flowing out, and is also the temperature drop from middle of one to the middle of the next slice, that is, appreciably the difference of average temperatures between the two contiguous slices. If H'_t be the same through the two slices, which means that inside these slices there is no accumulation of heat and no heat mutapower or conversion of heat to other forms of energy, then each slice is receiving as much heat as it loses, its temperature does not change, and its entropy also does not change. Thus it is only where there is change of H'_t from point to point along the path l that there is any increase or decrease of entropy. Now if c be the ordinary coefficient of heat conductivity, $H'_t = c\,\tau'_l$; and the above H'_t gradient along l is

$$H''_{tl} = c\,\tau''_l + c'_l\,\tau'_l.$$

In homogeneous material $c'_l = o$, and in this case the time-rate of accumulation of heat in a slice δl thick is $c\,\tau''_l\,\delta l$ and time-rate of the increase of entropy is $c\,\frac{\tau''_l}{\tau} \cdot \delta l$ on the assumption that there is no heat mutapower. The integral of this within any limits of l and within any time-limits gives the total increase of entropy within these limits. Mr. Swinburne states that there is growth of entropy where the temperature gradient τ'_l is uniform. It is theoretically important to note that this is not so unless the conductivity c varies

along l. Where materials of different conductivity are in contact, then there will be accumulation of heat on one side of the dividing surface unless the change of c is just balanced by a complementary change in τ'_l. But in this case of contact of differing substances and heat-flux across the contact, it is known that there is electric potential generated and electric current started as soon as any circuit for its passage is closed; so that there is here mutapower or conversion of heat to electric energy.

In this treatise no specific use of the quantity entropy is made, and there is, therefore, no need to discuss the matter further. Adiabatic curves are used largely; but an understanding of the actual physical existence of these curves and of their nature does not in any degree depend upon the definition of any quantitative measure of the distance between any pair of such curves.

The material active in engineering power-machines is sometimes carried through a closed cycle, but very rarely so within the machine and guided in any way by the machine. The nearest approach to a closed cycle within the power plant is in the case of the water and steam used in marine steam boilers and engines, where the expense of supplying fresh water justifies the cost of guiding the material round the cycle as fully as practicable in order to use it over and over again. In non-condensing engines the steam eventually condenses, to a large extent, to water of about the feed temperature, but the process is accomplished uncontrolled in the outside atmosphere. The engineer throws it away as waste as quickly as possible after it has expanded to terminal pressure. In all cases the cylinder indicator card has only its admission and expansion parts even remotely approximating to the thermodynamic cycle through which the material runs. In gas and oil engines there is never any attempt to cool the exhaust and to separate it into its original pre-combustion elements of unburnt air and fuel. In steam-boiler plant no effort is made to resolve the chimney gases into air and unburnt coal and to cool them to atmospheric temperature. What we do is to utilize natural sources of thermal power as far as is profitable in getting resilience from them, and then to abandon the material products of the process as waste. All that industry demands of science, therefore, is how to get as large a ratio r/h as practicable, under the various conditions of profitable industrial activity, and how to utilize the resilience R thus obtained with as little waste as possible. The latter part of the problem is purely dynamic, except in so far as it consists in avoiding a tendency to convert useful dynamic into wasteful thermal action.

Now, in "perfect gases," r/h is unity, under all circumstances, and in operating with these the problem of energy-utilization concentrates itself in its second half, i.e. in avoiding waste of resilience. In internal combustion gas engines effective means are deliberately taken to waste a very large portion of it by water-jacket cooling of the cylinder ; and this is a very clear proof that the value of energy-efficiency is often completely overborne by other more important items in commercial economy.

The equations of these "perfect gases" are adiabatic $p\,s^a = $ a constant for each curve, and isothermal $p\,s \propto$ absolute temperature.

For any substance whose similar equations are $p\,s^a =$ adiabatic constant, and $p\,s^\theta \propto (\text{temperature})^\rho$ for each isothermal, it is not difficult to show that, while r remains $\frac{1}{a-1}$, the specific heat $h = \frac{\rho}{a-\theta}$. In this case then r/h would be $\frac{a-\theta}{(a-1)\rho}$, which would retain the same value throughout those parts of the diagram in which the powers a, θ and ρ are unchanged. This formula is suggested in order to show how the coefficient r/h depends mainly on the difference of the slopes of the adiabatic and of the isothermal. According to it, these slopes are respectively, $a\frac{p}{s}$ and $\theta\frac{p}{s}$, while the temperature gradient of the pressure up the constant volume line is $\rho\frac{p}{\tau}$, where τ is the temperature.

In that degree in which the adiabatics and isothermals of any substance diverge less from each other than those of perfect gases, the divergence being measured by the difference of the tangents of inclination, and in that degree in which the pressure increases more rapidly with the temperature than in perfect gases, the dynothermic coefficient of the substance when used as a thermodynamic medium becomes less.

Adiabatic and isothermal curves, according to any such formulas, in which s means the volume or other whole configuration, never cross the zero stress axis, thus permitting only one kind of stress. The formulas $p\,s^a$ and $p\,s^\theta$, with s meaning the volume, are unsuitable for tension conditions. With a positive, they reduce the stress to zero only at an indefinitely large volume. For solids which can pass from compressive to tensive stress a modified formula of the same shape, with $(p+\pi)$ substituted for p, may serve for long ranges of the curves. Here p is pressure stress if positive, and tension if

negative, and π is a maximum tension that cannot be borne without breakage. The mathematical analysis of such curves is the same as for the previous ones, and $h = \frac{\rho}{a-\theta}$, as before. If the equation for any one adiabatic be $(p+\pi)\, s^a = \mathrm{A}$, then the volume at which the stress becomes zero is $\sqrt[a]{\frac{\mathrm{A}}{\pi}}$; and the resilience at any point of this adiabatic is $\frac{(p+\pi)s}{a-1} - \mathrm{A}^{\frac{1}{a}}\pi^{\frac{a-1}{a}}$. In this formula p is to be taken positive, whether it be a compression or a tension. Of course, A is different for the successive adiabatics.

If π and a were the same for all of these, then a simple calculation gives $r = \frac{1}{a-1} - \frac{1}{a}\left(\frac{\pi}{p+\pi}\right)^{\frac{a-1}{a}}$, and the ratio of this to $\frac{\rho}{a-\theta}$ is the dynothermic coefficient. So long as p is only small as compared with π, this formula gives, without much error, $r = \frac{1}{a(a-1)}$ and $r/h = \frac{a-\theta}{a\,\rho\,(a-1)}$. r increases from this value as the stress becomes greater.

We have, however, no experimental proof as yet that π is the same for the different adiabatics for one substance. But as the breakage of the material in compression and in tension prevents the curves actually running to near their mathematical limits, very likely the real variation of π does not much affect the approximation to accuracy in the above formulas.

In view of the fact that resilience is produced sometimes by inflow and sometimes by outflow of heat energy, it seems clear that, if heat energy can be converted into resilience, then there must be two opposite kinds of resilience, the one compressive increased by addition of heat, the other due to conservative molecular forces of cohesion and rigidity. However these latter operate, and whether the energy associated with them be in the material or in the ether surrounding it, the resilience due to them is wholly or partly masked by the heat resilience. Sensible zero stress results from the two exactly counterbalancing each other, and according to the overbalance of one or the other, we have tension or compression.

Chapter VII

First Adjustment of Size for Maximum Economy.

Dynothermic Coefficients of Steam.

Having examined the proportion in which resilience is produced by heat transpower and generation, it remains to examine the conditions under which the resilience so produced can be economically used in giving utilizable mechanical work.

Any volume s at pressure p contains resilience whose full amount is $\frac{p\,s}{a-1}$, if it be capable of adiabatic expansion down to zero stress and if this expansion curve have a constant index a. This is the amount of external work done in its expansion down to zero stress, to effect which, however, expansion to an excessively large volume is needed.

In expansion down to the pressure p_2, the amount of work done is—

$$\frac{p\,s}{a-1}\left\{1-\left(\frac{p_2}{p}\right)^{\frac{a-1}{a}}\right\}.$$

In expansion to the volume s_2, the amount of work done is—

$$\frac{p\,s}{a-1}\left\{1-\left(\frac{s}{s_2}\right)^{a-1}\right\}.$$

Here no back-pressure allowance is made, the back pressure on the other side of the piston having nothing to do with the work done as above, except that part of this work is spent in overcoming and driving back this back pressure.

In the first case the ratio of the work done to the total initial resilience is—

$$1-\left(\frac{p_2}{p}\right)^{\frac{a-1}{a}},$$

which ratio may be properly called the efficiency of the utilization of the resilience by expansion from p down to p_2. Note that this efficiency depends solely on the ratio between final and initial pressures, and not upon the absolute value of either. It would not depend solely on this ratio if a were not constant along the curve, and also the same for different adiabatic curves.

In the second case, the ratio of work done to total initial resilience is—

$$1-\left(\frac{s}{s_2}\right)^{a-1}$$

which is the efficiency of the utilization of the resilience by expansion from volume s to s_2. It depends solely on the ratio of these volumes, and not on the absolute amount of either of them.

Call this resilience-efficiency e; write e equal to this last ratio; and transform the equation into one giving directly the ratio of final to initial volume in terms of e; thus—

$$\frac{s_2}{s}=(1-e)^{-\frac{1}{a-1}}.$$

This is the ratio of volumetric expansion needed in order to utilize any specified proportion e of the whole resilience.

This extremely useful formula is described graphically by the curves on Fig. 56, where the co-ordinates are e and $\frac{s_2}{s}$. There are four curves for the values of $a=1{\cdot}1$, $1{\cdot}2$, $1{\cdot}3$, and $1{\cdot}4$. The law will not apply unless a be greater than 1, but is applicable however small the excess over 1 may be. The same results are set out numerically in Table LVI., the curves being plotted from the calculations recorded in this table. The table is in four parts corresponding to the values of a. For these four values the total resilience is 10, 5, $3\frac{1}{3}$ and $2\frac{1}{2}$ times the initial $p\,s$ respectively.

TABLE LVI

Ratio of Expansion	Proportion of Total Resilience e Utilized in Expansion.			
	$ps^{1\cdot1}$	$ps^{1\cdot2}$	$ps^{1\cdot3}$	$ps^{1\cdot4}$
1·5	·039	·078	·114	·148
2	·067	·129	·187	·24
3	·104	·20	·28	·35
4	·129	·242	·34	·424
6	·164	·30	·414	·510
8	·187	·34	·46	·562
10	·205	·369	·498	·596
12	·217	·387	·522	·625
14	·227	·405	·542	·647
16	·237	·422	·562	·667
18	·245	·437	·58	·685
20	·253	·452	·594	·70

In expansion $ps^{1\cdot1}$ total resilience $= 10\ ps$
" " $ps^{1\cdot2}$ " " $= 5\ ps$
" " $ps^{1\cdot3}$ " " $= \frac{10}{3}\ ps$
" " $ps^{1\cdot4}$ " " $= 2\frac{1}{2}\ ps$

From the table and diagrams it will be seen that for any one ratio of expansion the larger a gives the larger ratio of utilization. For each a this ratio of utilization at first increases rapidly with further expansion, and then much more slowly.

Similarly the ratio in which the pressure must be reduced by adiabatic expansion in order to utilize any specified proportion e of the total resilience is—

$$\frac{p_2}{p} = (1 - e)^{\frac{a}{a-1}}.$$

This law is also described graphically by four curves on Fig. 57 for the same four values of a, and numerically in Table LVII. In these the ordinates are e, as in the previous diagram, but the vertical co-ordinate is $\frac{p}{p_2}$, the reciprocal of the pressure ratio in the above formula. This reciprocal is the ratio of the greater initial to the smaller terminal pressure, and its use as co-ordinate gives the curves the same general character as in the ratio of expansion diagram. The curves, are, however, steeper, and curve upwards more rapidly because the pressure ratio is greater than the reciprocal volume ratio since $a > 1$.

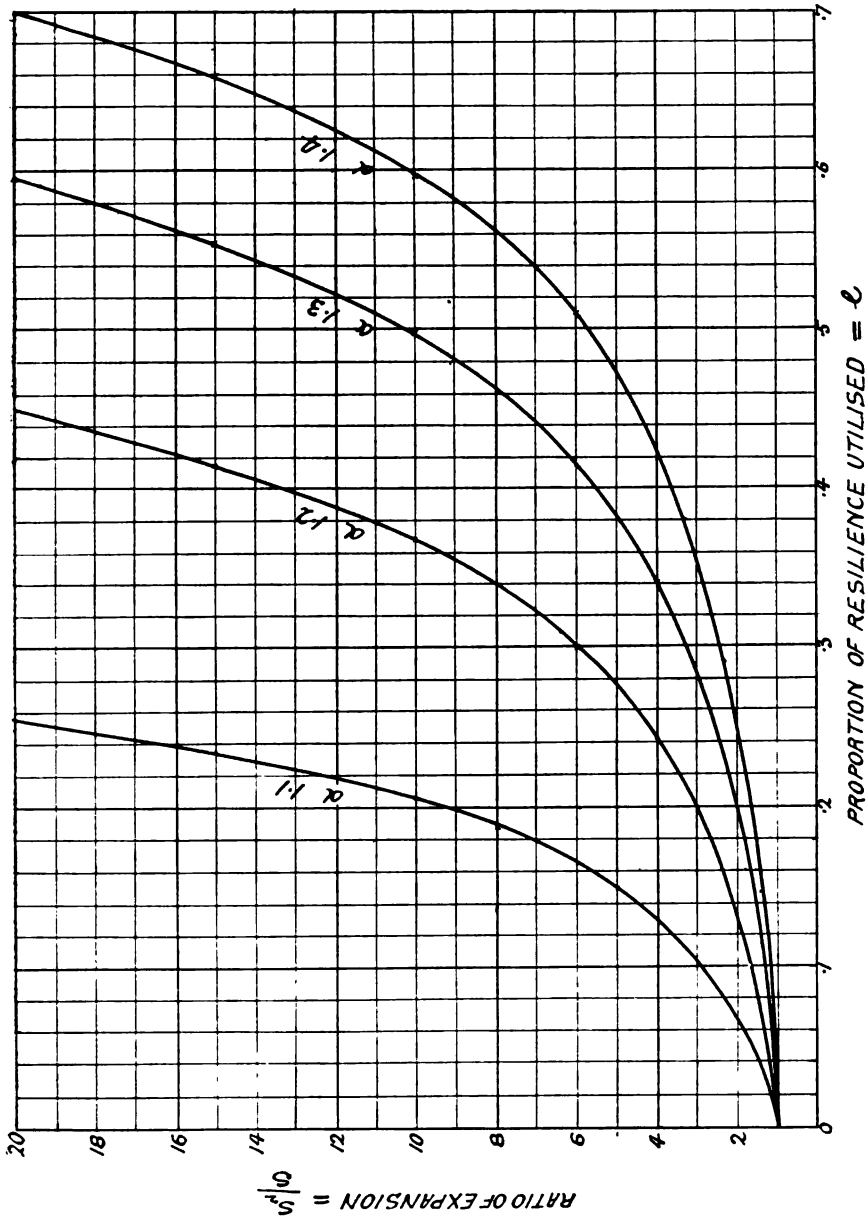


FIG. 56.—Ratios of Expansion required to utilise various proportions of total adiabatic resilience.

TABLE LVII

Ratio $\frac{p_2}{p}$ in which Pressure is Reduced.	Proportion of Resilience e Utilized in Expansion.			
	$ps^{1\cdot 1}$	$ps^{1\cdot 2}$	$ps^{1\cdot 3}$	$ps^{1\cdot 4}$
$\frac{1}{5}$	·135	·235	·31	·37
$\frac{1}{10}$	·19	·317	·415	·465
$\frac{1}{15}$	·22	·365	·465	·535
$\frac{1}{20}$	·24	·392	·5	·57
$\frac{1}{25}$	·255	·415	·525	·595
$\frac{1}{30}$	·265	·43	·544	·613
$\frac{1}{35}$	·275	·445	·56	·628
$\frac{1}{40}$	·283	·46	·573	·642
$\frac{1}{45}$	·29	·475	·585	·655
$\frac{1}{50}$	·297	·485	·595	·667

In expansion $ps^{1\cdot 1}$ total resilience $= 10\ ps$
,, ,, $ps^{1\cdot 2}$,, ,, $= 5\ ps$
,, ,, $ps^{1\cdot 3}$,, ,, $= \frac{10}{3}\ ps$
,, ,, $ps^{1\cdot 4}$,, ,, $= 2\frac{1}{2}\ ps$

Confining, in the first place, attention to the first of these diagrams, it gives the volume s_2, which the steam, or other vapour or gas, must finally occupy in order to secure this degree of resilient efficiency : that is, the volume it occupies when it is still confined in the working cylinder or other vessel, and still acting on the piston or other driven machine part. This volume may be taken as a measure of the bulk, and, to a rough first approximation, of the prime cost of the power plant. The difference between this and the initial bulk s is active and profitable ; the initial bulk is inert and unprofitable. The most unprofitable feature of gas-engine design is the unavoidable large dead initial volume occupied by the charge before heating. If this resilience so utilized—without deduction for back pressure—be divided by this final bulk s_2, we have—

$$\frac{p\,s}{a-1}\cdot e\cdot\frac{(1-e)^{\frac{1}{a-1}}}{s} = \frac{p}{a-1}\cdot e\,(1-e)^{\frac{1}{a-1}}$$

as the measure of the work done per unit bulk of the power plant employed, and, very roughly, per unit of prime cost. This must not be confused with the absolute mean working pressure, which is the same work divided by (s_2-s), and which is, therefore, greater than the above economy factor. The mean working pressure decreases continuously as the grade of expansion is made greater.

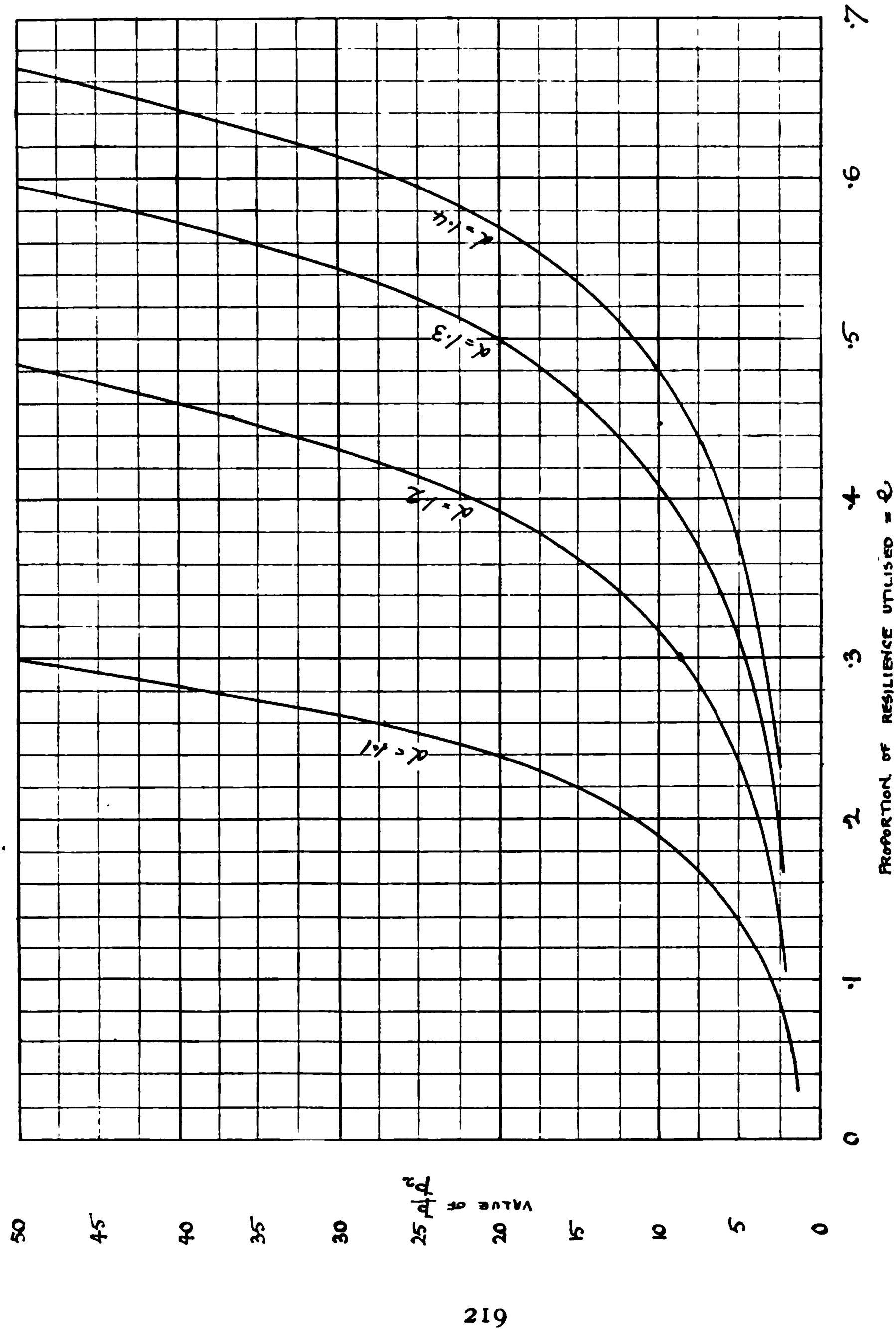


FIG. 57.—Ratio in which pressure must be reduced to utilise various proportions of whole resilience.

But the above measure of economy starts with zero value, increases to a maximum, and then decreases again.

The value of e which gives this maximum is—

$$e = \frac{a-1}{a};$$

and this is reached when the ratio of expansion is—

$$\frac{s_2}{s} = a^{\frac{1}{a-1}}.$$

At this maximum limit the measure of economy itself is—

Maximum Ratio of Utilized Resilience to Final Bulk $= p \div a^{\frac{a}{a-1}}$,

or the initial pressure divided by the power $\frac{a}{a-1}$ of a. Thus if $a = 1{\cdot}4$, then $\frac{a}{a-1} = \frac{14}{4} = 3\frac{1}{2}$, and the above maximum economy factor is $p \div 1{\cdot}4^{3\frac{1}{2}} = 0{\cdot}308\ p$.

In making use of all such calculated limits giving maximum values of any kind, it should be remembered that at this limit the gradient is zero, so that small deviations either way from the exact limit make no change of consequence in the value obtained. Thus the above value $p \div 1{\cdot}4^{3\frac{1}{2}}$ would be very nearly obtained whether the ratio of expansion were somewhat less or somewhat greater than $a^{\frac{1}{a-1}} = 1{\cdot}4^{2\frac{1}{2}} = 2{\cdot}32$. Therefore since a greater ratio of expansion gives the greater utilization of the resilience, it is clearly correct practice to use a greater ratio of expansion than the limit theoretically calculated as above, because increased resilient efficiency is economically desirable in fuel and other ways irrespective of bulk of plant.

Many other items of economy have to be allowed for in the complete solution of the industrial problem of power supply. Meantime, in respect of this one item of bulk and weight of plant, it is most important to note that this economy-factor at its maximum and best value is proportional to p, the working pressure.

To show the general meaning of these first results in the search after economy, and their relative importance, the subjoined Table LVIII. is given. The same results are plotted out as continuous curves in Fig. 58. In this diagram the ordinate is a ranging from just over 1 to $1{\cdot}4$; while the co-ordinates to the three curves are $a^{\frac{1}{a-1}}$, the ratio of expansion; $\frac{a-1}{a}$, the resilience efficiency; and

$a^{-\frac{a}{a-1}}$, the ratio of utilized resilience to final bulk for unit initial pressure, all at the above limit of maximum value of this last ratio.

TABLE LVIII

TABLE SHOWING MAXIMUM UTILIZATION OF RESILIENCE WITHOUT BACK PRESSURE PER UNIT BULK OF POWER PLANT.

Index of adiabatic expansion curve	$a=$	1·01	1·05	1·1	1·2	1·3	1·4
Ratio of expansion giving maximum	$a^{\frac{1}{a-1}}=$	2·70	2·65	2·59	2·49	2·40	2·32
Resilience efficiency or proportion of resilience utilized	$e=\frac{a-1}{a}=$	·0099	·0476	·091	·167	·231	·286
Ratio to initial pressure of foot-pounds work per cubic foot final volume	$a^{-\frac{a}{a-1}}=$	·366	·359	·35	·335	·32	·31

Looking now at the diagram, it will be noted that the maximum value of the economy-factor always lies between ·37 and ·31, changes only slowly with a, and that its curve is nearly a straight line. The straight-line approximation is $\{\cdot 49-\cdot 13\ a\}$.

Again it may be pointed out that the ratio of expansion needed to reach this maximum economy-factor decreases rapidly as a increases. There is here more deviation from a straight line. But a very fair approximation is the straight line $(3{\cdot}71-a)$. This requisite ratio of expansion does not go above 2·7 for the lowest values of a, and is as little as 2·32 for $a=1{\cdot}4$ as in perfect gases.

The resilience efficiency attained at this limit is very low, ranging between nearly 0 and ·28. It increases with a rather rapidly. The curve deviates considerably from a straight line, but its general trend is along the straight line $\cdot 7\ (a-\cdot 1)$.

The remarkable features of these calculations are the low ratios of expansion and the low proportions of the total resilience utilized when the above economic limit is reached.

It is not possible to obtain so simple general rules as these for

maximum utilization of resilience per unit of prime cost when account is taken of the back pressure deduction from absolute mean pressure to give effective pressure, and of the fact that the total prime cost does not increase in simple proportion to the final bulk. But the rule is well worth giving here in the form of an

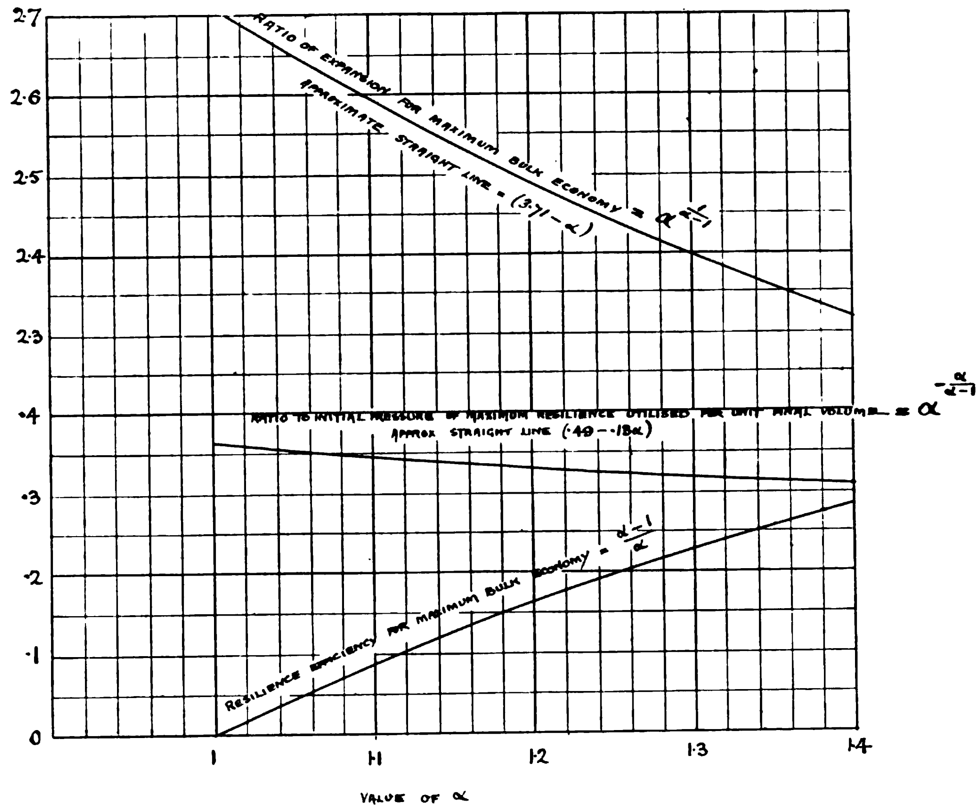


FIG. 58.—Maximum Utilisation of Resilience without back pressure per unit bulk of power plant.

N.B.—The title of the middle curve is printed by error along a horizontal line instead of along the curve.

equation for e. It is quite easy, with the help of a book of logarithms and two or three trials, to solve it for any particular case, and tables and diagrams of the results are given below. The back pressure is called b, and the prime cost is supposed to vary in proportion to the sum of the final volume, $s_2 = s(1-e)^{-\frac{1}{\alpha-1}}$, and a constant, here called S. Fig. 59 explains the exact meaning of this constant S.

It is unnecessary here to detail the mathematical steps by which the solution is reached. Suffice it to say that the equation below, when solved to obtain e in terms of $\frac{b}{p}$, the ratio of back-

STRAIGHT-LINE LAW ASSUMED FOR PRIME-COST OF POWER PLANT IN CALCULATION OF MAXIMUM BULK ECONOMY

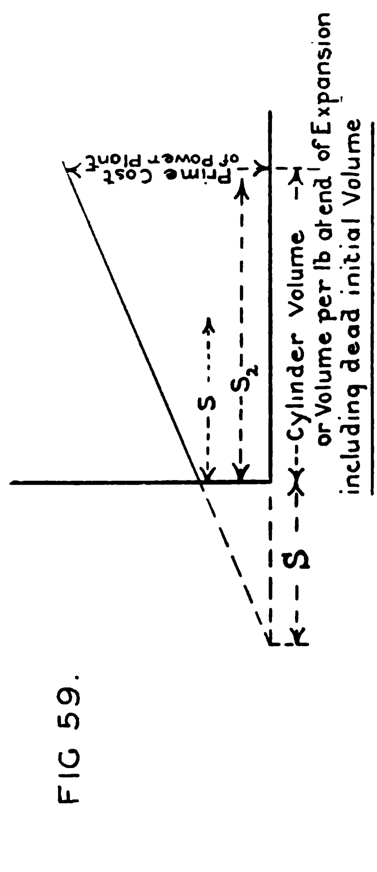


FIG 59.

N.B. S is the Volume at which the resilience is generated by heat transpower or internal conversion. And S_2 the Terminal Volume. s is a constant special to each style of plant.

Numerous examples of this law are given in the diagrams of Chapter III.

pressure to initial pressure, of $\frac{s}{S}$, the ratio of the volume at which the resilience is created to the additive initial term of the prime cost, the prime cost of plant being taken proportional to $(S + s_2)$, and of a, gives the proportion of total resilience utilized at the limit at which this utilized work with deduction of the work done on back-pressure bears its maximum ratio to prime cost of plant.

$$(1-e)^{\frac{a}{a-1}} - \frac{a}{a-1} \cdot \frac{s}{S} \cdot e = \frac{b}{p} - \frac{s}{S}\left(1 - \frac{b}{p}\right).$$

In solving this equation, which is of the form $(1-e)^m - l\,e = k$, use it in the shape m log. $(1-e_1) =$ log. $(k + l\,e_2)$. First guess e_1 from the tables here given, and calculate e_2 from this guess. Next guess e_3, differing from e_2 in the same direction as e_2 differs from e_1. Insert e_3 in place of e_1, and calculate a new value of e in same way as e_2 was found. If this new value differs too much from e_3, repeat the process once more; but ordinarily the two calculations will give a sufficiently close result.

The corresponding ratio of expansion is—

$$\frac{1}{\sqrt[a]{\frac{b}{p}\left(1 + \frac{s}{S}\right) - \frac{s}{S}\left(1 - \frac{a}{a-1} \cdot e\right)}}.$$

As an example of this calculation, take $b = \frac{1}{10}\,p$, and $s = \frac{1}{5}\,S$, Then the equation for e is—

$$(1-e)^{\frac{a}{a-1}} - \frac{a}{a-1} \cdot \frac{e}{5} = -\cdot 08,$$

and the ratio of expansion is the reciprocal of

$$\sqrt[a]{\frac{a}{a-1} \cdot \frac{e}{5} - \cdot 08}.$$

For the above four values of a this gives—

	$a = $ 1·1	1·2	1·3	1·4
	$e = $ ·132	·235	·315	·380
Ratio of expansion	= 4·1	3·8	3·55	3·33

In this example S has been taken extremely large. To further illustrate these results, take $S = s$, or only one-fifth as large as in the last example, and take $b = \frac{1}{10}\,p$, as before. The equation for e then becomes—

$$(1-e)^{\frac{a}{a-1}} - \frac{a}{a-1} \cdot e = -\cdot 8,$$

and the ratio of expansion becomes the reciprocal of

$$\sqrt[a]{\frac{a}{a-1}\,e - \cdot 8}.$$

TIO OF
L VOLUME
TO
COST INITIAL
TERM

$\frac{S}{\bar{S}}$

·2

·4

·6
·8

·2

·4

·6
·8

·2

·4
·6
·8

·2
·4
·6
·8

a	$\frac{s}{S}$	Section 1. $\frac{b}{p}=0$		Section 2. $\frac{b}{p}=\cdot05$		Section 3. $\frac{b}{p}=\cdot1$		Section 4. $\frac{b}{p}=\cdot15$		Section 5. $\frac{b}{p}=\cdot2$		Section 6. $\frac{b}{p}=\cdot3$	
		e	Ratio of Expansion	e	Ratio of Expansion	e	Ratio of Expansion	e	Ratio of Expansion	e	Ratio of Expansion	e	Ratio of Expansion
1·1	·2	·163	5·2	·148	4·6	·132	4·1	·118	3·6	·106	3·2	·088	2·6
	·4	·147	4·15	·132	3·8	·119	3·5	·108	3·15	·097	2·9	·082	2.4
	·6	·133	3·7	·121	3·4	·110	3·1	·101	2·9	·093	2·7	·079	2·3
	·8	·124	3·4	·114	3·2	·105	2·9	·096	2·7	·09	2·6	·076	2·2
1·2	·2	·286	4·9	·26	4·3	·235	3·8	·212	3·4	·194	3·0	·16	2·5
	·4	·245	4·0	·227	3·6	·21	3·2	·194	2·9	·178	2·7	·15	2·3
	·6	·23	3·6	·212	3·3	·198	3·0	·184	2·7	·170	2·5	·144	2·2
	·8	·215	3·4	·202	3·1	·188	2·8	·176	2·6	·164	2·4	·14	2·1
1·3	·2	·38	4·8	·346	4·1	·315	3·6	·286	3·1	·262	2·85	·218	2·3
	·4	·334	3·9	·308	3·5	·284	3·1	·262	2·8	·24	2·5	·204	2·2
	·6	·31	3·5	·288	3·15	·268	2·85	·248	2·6	·23	2·4	·196	2·15
	·8	·296	3·3	·277	2·95	·258	2·7	·239	2·5	·222	2·3	·19	2·1
1·4	·2	·44	4·5	·416	3·85	·38	3·4	·346	3·0	·316	2·6	·262	2·15
	·4	·40	3·6	·372	3·2	·344	2·9	·318	2·6	·295	2·4	·25	2·05
	·6	·376	3·3	·352	3·0	·326	2·7	·304	2·5	·282	2·3	·24	2·0
	·8	·364	3·1	·34	2·8	·318	2·55	·296	2·4	·276	2·2	·236	1·95

The results are now—

a	=1·1	1·2	1·3	1·4
e	= ·10	·183	·250	·307
Ratio of expansion	=2·99	2·74	2·64	2·52

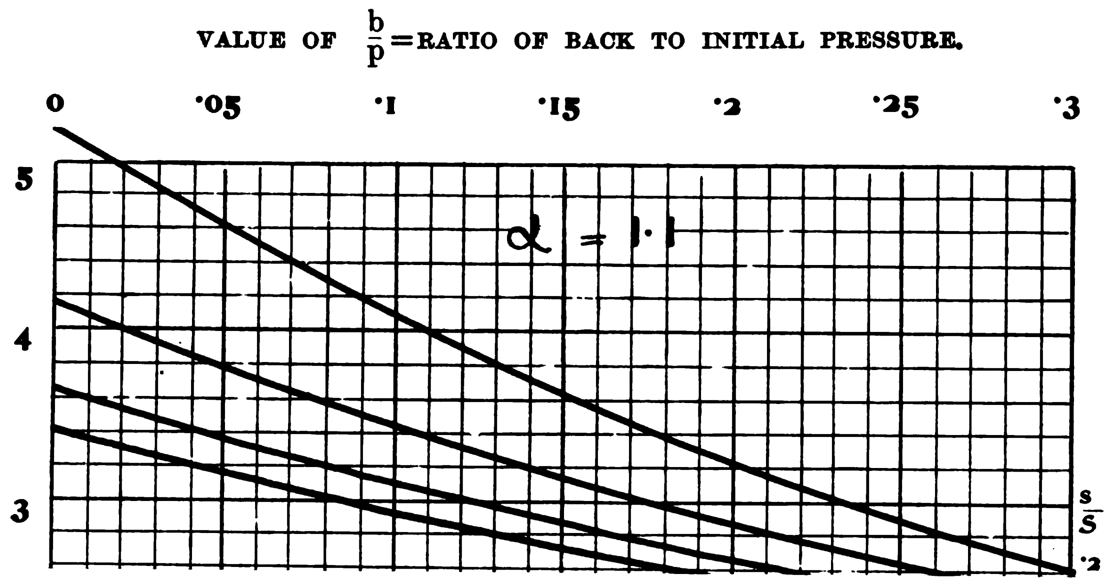
By the method illustrated in these two examples, all the results recorded in Tables LX. and LXI. have been calculated by Mr. H. M. Hodson, who has also plotted these results in the series of curves seen in the two Figs. 60 and 61.

The table is set out in 6 sections, corresponding to ratios of back-pressure to initial pressure 0, 0·5, ·1, ·15, ·2, and ·3. Each section is divided into 4 sub-sections corresponding to values of a=1·1, 1·2, 1·3 and 1·4. In each subsection the calculations are made for 4 values of $\frac{s}{S}$, namely ·2, ·4, ·6, and ·8. The table thus contains the final results in values of e and of grade of expansion $\frac{s_2}{s}$ of 96 sets of calculation.

These 96 × 2 = 192 numerical results are plotted in the two diagrams. In each of them the horizontal ordinate is $\frac{b}{p}$, the ratio of back to initial pressure ranging from 0 to ·3. In Fig. 60 the vertical ordinate is e, the proportion of resilience utilized at the limit of maximum economy. In Fig. 61 the vertical ordinate is the ratio of expansion at which this limit is reached. Each diagram contains four groups of curves, each group corresponding to one value of a. Each group comprises four curves, each curve for one particular value of $\frac{s}{S}$. In all there are 32 curves on the two diagrams.

Examining now the general nature of the results shown in Fig. 60, it may be first noted that the ratio of resilience utilized at the limit of maximum prime-cost economy, decreases rapidly as the back pressure rises. Also it is very much larger for the higher indices a, that is for the steeper adiabatic curves or those most approximating to the perfect gas adiabatic. Nextly this proportion of utilized resilience falls fairly regularly as $\frac{s}{S}$ rises; or otherwise stated, it rises with S, the initial additive term in the straight-line prime-cost-bulk law assumed.

With larger S it becomes profitable to utilize a greater percentage



VALUE OF $\frac{b}{p}$ = RATIO OF BACK TO INITIAL PRESSURE.
0
·05
·1
·15
·2
·25
·3
5
4
3
$\frac{s}{S}$
·2
α = 1·1

of the created resilience by enlarging the size of the plant to obtain greater expansion, the increase of cost involved being less important in proportion to the whole cost. Both b and S have decisive influence in settling the proportions giving maximum economy.

The curvature of these curves is much greater for small $\frac{s}{S}$, that is, for large S, than for relatively small S. The lower two curves of each group are very nearly straight lines, and the third deviates not very much from straightness. The steepness of the curve in all cases increases with S (that is, increases as $\frac{s}{S}$ decreases). This means that the influence of back pressure in reducing the profitable utilization of resilience is more marked the greater S is. This steepness is also greater for the greater values of a.

The fall of any one of these curves from $\frac{b}{p} = 0$ to $\frac{b}{p} = \cdot 3$ is approximately equal to—

$$\cdot 25\ a - \cdot 000267\ \frac{S}{s} + \cdot 0133\ a\ \frac{S}{s} - \cdot 245,$$

this formula having been deduced empirically from a close examination of the curves. Dividing this by $\cdot 3$, the difference between the two values of $\frac{b}{p}$, there is deduced

Fall of Resilience Efficiency at Maximum Economy Limit per unit of ratio of Back to Initial Pressure

$$= \cdot 83\ a + \cdot 043\ a\frac{S}{s} - \cdot 82 - \cdot 00089\ \frac{S}{s},$$

which, if a rougher degree of accuracy be admitted, may be simplified to—

$$\cdot 83\ \left\{a - 1 + \frac{a}{20}\ \frac{S}{s}\right\}$$

For practical use, however, it is much better to read directly off the diagram any particular values desired.

Turning now to the next diagram, Fig. 61, which gives the ratio of expansion for this economy limit, the four groups of curves have the same general character but do not fall so rapidly with increase of back pressure. The influence of large index a on the ratio of expansion is opposite to its effect upon the utilization of resilience, the larger expansions being for the smaller indices. This is explained by the fact that with the steeper adiabatics a much smaller

ratio of expansion is required to develop a given proportion of the total resilience. The difference between the results for different a's is, however, extremely small; so small that in the diagram the different groups of curves have to be separated by repeating the vertical scale for each group to avoid confusion of lines. When $a = 1{\cdot}1$, the profitable grade of expansion ranges from $3\frac{1}{4}$ to 5 when the back pressure is small, down to from $2\frac{1}{4}$ to $2\frac{1}{2}$ when it is large; while when $a = 1{\cdot}4$, it ranges from 3 to $4\frac{1}{2}$ for small back pressure down to from 2 to $2\frac{1}{4}$ for large back pressure.

The fall of any one of the curves of this diagram from $\frac{b}{p} = 0$ to $\frac{b}{p} = {\cdot}3$ is approximately equal to

$$a + 1{\cdot}36\,\frac{S}{s} - {\cdot}8\,a\,\frac{S}{s} - {\cdot}5,$$

and this divided by ${\cdot}3$ gives the

Mean Fall of Ratio of Expansion at Maximum Economy Limit per unit of ratio of Back to Initial Pressure

$$= 3{\cdot}33\,a + 4{\cdot}53\,\frac{S}{s} - 2{\cdot}73\,a\,\frac{S}{s} - 1{\cdot}67.$$

For example, if an average $a = 1{\cdot}2$ be inserted in this formula it reduces to—

$$2\tfrac{1}{3} + 1\tfrac{1}{4}\,\frac{S}{s}.$$

Here again, however, it is recommended that in making use of these calculations the particular desired results should be read off the diagram, and that these approximative formulas should be regarded as simplifications framed only to make the general nature of the variations more easily intelligible and apparent. The results for all cases arising in practice can be read from the diagrams with sufficient approximation by aid of simple interpolation between the plotted curves.

The proportion of resilience utilized and the ratio of expansion here calculated are very small. They are calculated, of course, upon a restricted view of economy. When other considerations are also taken into account, as is done below, these results are modified so as to obtain larger ratios. But this aspect of the economic question is that most commonly overlooked, while it is generally the most, or, at least, among the most, important. It is, therefore, well to see plainly how small the efficiency ratios are which this consideration of bulk and prime cost alone would make most profitable.

The resilience is here supposed created by heat transpower at one constant volume. If it be actually created at various volumes or gradually changing volumes, as in Fig. 54 and Fig. 55A, then the calculation applies to each portion separately with the appropriate p and s for each part inserted. The best ratio of expansion for each part would be $a^{\frac{1}{a-1}}$. Here a may possibly be different for the various volumes at which the parts of the resilience is created. But as there is necessarily the same terminal volume for all, those parts created at the larger volumes are inevitably utilized in a smaller ratio than those created at small volume. Either the former are utilized in too small a ratio, or the latter in too great a ratio. To obtain the best economy all of the resilience should be created at one volume, and at as small volume as possible. Not only is the initial volume inert and useless, but the most profitable terminal volume is large and expensive in proportion to it. As already mentioned, this large initial volume is the worst feature of gas and air heat engines, while steam obtains a large proportion of its heat resilience in the very condensed form of water before evaporation. On the other hand, in internal combustion gas engines a larger proportion of the heat resilience is created at the minimum volume, while steam has the major portion of it created at gradually increasing volume during evaporation.

It is necessary to caution readers against the misconception that the initial ps of this chapter corresponds with the product of pressure by volume of steam at the point of cut-off in a steam engine cylinder. As just stated, during evaporation in the boiler, the mixture of water and steam is receiving heat at constant pressure and new resilience is being created all along this level evaporation line. According to our analysis, each small part of the evaporative heat conduction must be regarded as raising the pressure at constant volume to a minute extent, this being followed by a correspondingly minute adiabatic expansion which brings down the pressure again to its original level.

Equal small amounts of evaporation effected in this way correspond to equal amounts of heat conduction. If the index for adiabatic expansion were the same for all proportions in the mixture of water and steam, these would also correspond to equal extra amounts of newly created resilience. This index, however, does not remain constant in this way; and, besides, it has to be remembered that a large proportion of the heat is received before the evaporation begins. The general definition of the Dynothermic Coefficient is

"ratio of resilience created to heat transmission creating it." During the evaporation the resilience is being created and simultaneously being partly spent in doing external work, and at the completion of the evaporation the part already spent is the $p\,s$, where s is the volume of the dry saturated steam. The total resilience created is thus $p\,s$ plus the resilience still held at the completion of the evaporation. This latter is $\frac{p\,s}{a-1}$, if a be the adiabatic index for dry saturated steam—following the only roughly correct but commonly used assumption that this a remains the same along the curve to its extremity at $p = 0$. Thus the total resilience created is

$$p\,s\left(1 + \frac{1}{a-1}\right) = p\,s\,\frac{a}{a-1}$$

and the ratio of this to the "total heat" spent in producing it, is the Dynothermic Coefficient suitable to the process of producing dry saturated steam. The index a usually assumed for the adiabatic expansion of such steam is 1·135, and with this $\frac{a}{a-1} = 8{\cdot}4$.

In the two graphic tables of "useful properties of saturated steam," Figs. 9 and 10, Chapter II, one of the curves in each diagram gives the ratio of "$p\,s$ to Total Heat from 60° Fahr." In the one diagram this can be directly read off for any temperature, and in the other for any pressure, used in engineering practice. As 60° is sufficiently near the average feed temperature, 8·4 times the height of this curve is, for all practical purposes, the Dynothermic Coefficient for steam produced by this normal method.

This curve on Fig. 9 is very nearly a straight line between 280°F. or 50 lbs./in.^{2} pressure and 432° or 350 lbs./in.,^{2} giving

$$\frac{p\,s}{\text{Total Heat}} = {\cdot}0578 + {\cdot}000\,038\ t^\circ \text{ Fahr.}$$

Multiplying this by 8·4, gives the
Dynothermic Coefficient

$$= {\cdot}4855 + {\cdot}000323\ t^\circ \text{ Fahr.}$$
$$= \text{nearly } {\cdot}49 + {\cdot}0003\ t^\circ,$$

the temperature being measured from the ordinary Fahr. zero.

In diagram Fig. 10 the curve is not nearly so straight; but between 100 lbs./in.^{2} (absolute or 85 gauge pressure) and 250 lbs./in.,^{2} its deviation from straightness is not great, and it gives for this range with considerable accuracy—

$$\frac{p\,s}{\text{Total Heat}} = {\cdot}0705 + {\cdot}000\,010\,7\ p,$$

where p is absolute pressure in lbs. per square inch. Multiplying this by 8·4, we have the

$$\text{Dynothmeric Coefficient} = \cdot 592 + \cdot 000\,09\,p$$

very nearly for the range of pressures in common use.

This method of calculating the total resilience when generated (1) through a great rise of pressure at constant volume, followed by (2) expansion at constant pressure, is applicable whatever the working fluid may be, each fluid having its own particular a. The total resilience generated after completion of the constant pressure expansion to volume s, is $p\,s\,\frac{a}{a-1}$, of which total $p\,s$ has already been spent as external work when the volume s is reached.

The diagram Fig. 8, and Table VIII, page 55 of Chapter II, give the values of $\frac{a}{a-1}$ for all indices actually occurring. That portion of this total resilience $p\,s\,\frac{a}{a-1}$, which is utilized in doing external work with any grade of expansion beyond the volume s, is shown by the curves marked W on the two diagrams, Figs. 13 and 14, in Chapter II. The first of these diagrams gives it for particular values of a for steam and gas; the second gives it for the standard values 1·1, 1·2, 1·3, and 1·4. This work divided by the final volume occupied is in this case the "mean absolute pressure," and may be read from the same two diagrams on the curves marked M. Now it is seen, from these diagrams that, for all values of a, this work done per unit of final bulk goes down continuously from the beginning of the expansion. Thus there is, in this case, no limit of expansion, and no limiting proportion of total resilience utilized, corresponding to maximum bulk economy. The maximum bulk economy corresponds to the use of the engine without any expansion, that is, to full admission up to the end of the stroke.

This does not mean that bulk economy counts for nothing in these cases. There are several other factors in the resultant economy factor. The fact just pointed out means only that if all these other factors were neglected or were, in any possible circumstances, really inoperative, then this one consideration of bulk economy would dictate absence of expansion in steam engines and other engines producing resilience at constant pressure.

The case of these engines is included in the investigation detailed in the next two chapters, where the economical limit of high pressure and other restrictive limits are explained.

Chapter VIII

Work, Heat and Costs of Actual Indicator Diagrams.

The fact that in the generation of steam-resilience a large portion of the required heat flux is at constant pressure leads to the consideration of the next fundamental limiting condition imposed by industrial necessities. This is the pressure limit. The resilience produced at constant volume is $\frac{p\,s}{a-1}$; and, when utilized to the extent e, the utilized amount per unit of final bulk is—

$$\frac{p}{a-1}.e\,(1-e)^{\frac{1}{a-1}},$$

while the maximum found above for this latter is $p \div a^{\frac{a}{a-1}}.$ Each of these quantities increases with the stress p. When r/h is affected by p, it increases with p. For any specified back pressure b, the profitable efficiency e of resilience utilization increases with p. For all these reasons, high pressure is desirable. The reasons against high pressure are the increased prime cost of boilers, engines, etc., and the increased difficulty in keeping joints tight. These objections operate against increase of pressure in a similar manner to that in which the increasing costs of bulk operate against high-grade expansion. If the cost went up with the pressure by a straight-line law, say, proportionately to $(P + c\,p)$, then the resilience obtained per unit of cost would increase indefinitely with the pressure, no maximum limit being reached. The fraction $\frac{p}{P + c\,p}$ never becomes so great as $1/c$ but rises near this limit rapidly at first and then approaches it very slowly. The constructive difficulties, however, go on increasing in an accumulative ratio. If they follow the curve $(P + c_1\,p + c_2\,p^2)$ then the maximum resilience per unit of cost is

reached at the pressure $p = \sqrt{P \div c_2}$. Whatever the law be, if it give a diagram with upward curvature, p being plotted horizontally and the cost vertically, the maximum is easily found graphically by drawing from the origin a tangent to touch the curve.

It is worth pointing out here that if the law of cost as affected by pressure be $\left(P + c_1 p + \frac{c_2}{p}\right)$, then there is no pressure maximum limit to the ratio of resilience to cost, the ratio increasing continuously with the pressure if c_2 be positive. The increase at first is relatively rapid and gradually becomes very slow. If, on the other hand, c_2 were negative, there would be a minimum limit, which is, however, of no commercial interest. In a formula of this shape, c_2 would probably be always positive.

But the problem as to most profitable initial pressure may be looked on in another way. A full resilience indicator diagram, which runs up a vertical line at as small volume as is practicable and then runs down an adiabatic curve to as small back pressure as is possible, has elongated sharp top and right-hand corners, which add comparatively little to the area but add greatly to the prime and annual costs. Therefore it is economical to cut off the extreme right-hand portion at a limit already investigated, in order to save cost of bulk at the expense of loss of mechanical efficiency. For the same reason, it is economical to cut off the extreme top corner, in order to save cost of extra high pressure at the expense of sacrificing, in a certain degree, the advantages obtained from creating the resilience by transpower all at one volume. The resilience utilized is, of course, lessened in a greater ratio than is the heat flux employed; but the deduction from the resilience just equals the decrease of heat flux. In Fig. 62 the area so cut off above the pressure p_1 is hatched over vertically. The law of energy conservation shows that this area equals the excess of the heat influx up the vertical from point $p_1\, s_o$ to point $p_o\, s_o$ over the heat influx along the horizontal from $p_1\, s_o$ to point $p_1\, s_1$. By following this horizontal instead of going up to $p_o\, s_o$ and thence down the adiabatic to $p_1\, s_1$, we save as much heat flux as we lose resilience and work done. In perfect gases the total resilience generated equals the heat flux, but in all other cases it is less.[1] In all common cases, as the resilience utilized

[1] Except very probably in solids at very high tensions and very low temperatures, when the increase of tensive resilience is likely to be greater than the efflux of heat, the withdrawal of heat energy setting free a relatively large amount of energy of molecular attraction latent in the ether or elsewhere.

is always much smaller than the total generated, these two equal deductions lessen the utilized resilience in much greater ratio than the heat flux, and thus the thermodynamic efficiency is much lowered. Even if the reductions were in equal ratio, that of the resilience would be commercially the more important, because it lessens the earning power more than the other lessens the fuel cost

There is, therefore, to be sought a limit at which the loss in thermodynamic efficiency balances, in money value, the gain due to saving in the intrinsic costliness of high pressure. In passing it is most interesting to observe that the history of the development of the Diesel engine followed precisely the line of argument here submitted. Diesel originally aimed at a quadrilateral isothermal-adiabatic indicator diagram, probably because of belief in the strange academic legendary myth that there is peculiar virtue in isothermal expansion. This, of course, gave (1) a bottom left-hand corner rounded to an extremely flat compression curve, and (2 and 3) very elongated and sharp right-hand and top corners. When this engine was taken in hand by practical engineers, the first item of reform was to sharpen up corner No. 1 to as near a right angle as the mode of action of the engine allowed. The second reform was to cut off the unprofitable right-hand tail; and the final reform—giving at last real commercial merit and success to the engine—was to cut off the unprofitable top peak. In the course of this historical development all traces of isothermal expansion gradually disappear from the indicator diagrams. The legend was forgotten, and the myth gently, but not reverently, buried. It ought to be specially and carefully noted that the third step in this Diesel reform reduced very greatly the maximum pressure in the engine, but *did not reduce at all the maximum temperature*, and that in this engine high temperature has never occasioned difficulties such as to erect temperature into one of the limiting conditions governing the design.

One element in the great practical success of steam-power plant is clearly due to the fact that steady production of steam naturally leads to evaporation at constant pressure, and that steam engineers have thus been, so to speak, *compelled* to cut off the unprofitable top peak of their adiabatic expansion card. Another element in this success has, no doubt, been due to a large proportion of the heat being necessarily supplied at practically constant volume—that portion, namely, spent in heating the water to the evaporation point. Thus physical necessity has shaped the left-hand portion of the steam diagram approximately to the form seen now to be scientifically the most commercially profitable.

The complete problem of adjusting the conditions—so far as physically practicable—for maximum industrial economy must take into account—

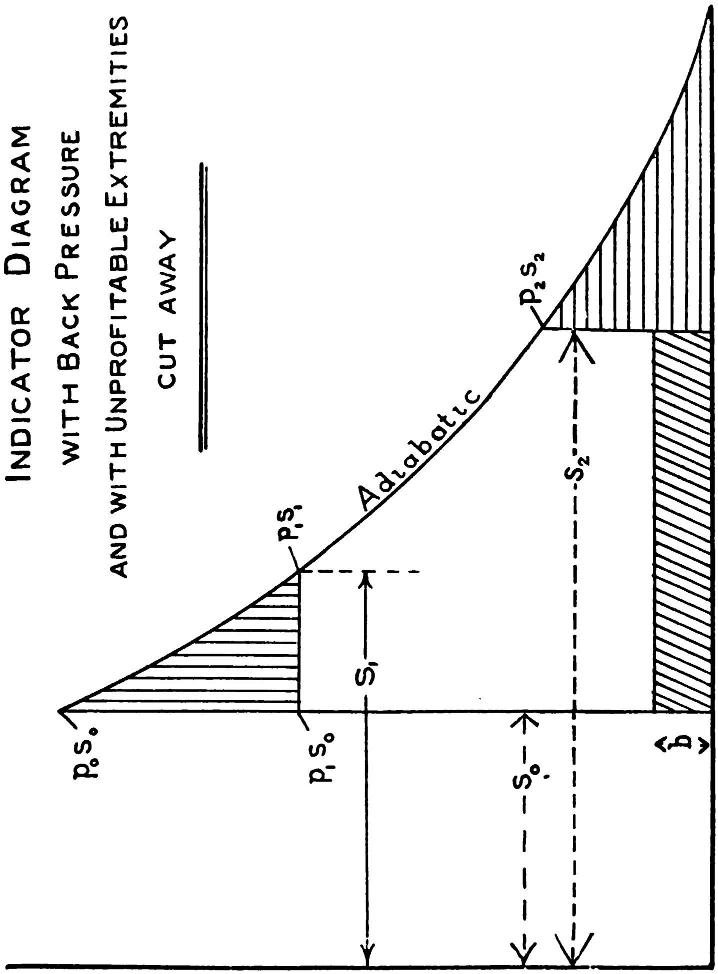


FIG. 62.

First, cost of Fuel to obtain the heat flux or heat generation, along with other accompanying working expenses;
Secondly, cost of Bulk and Weight of the power plant;
Thirdly, cost of High Stress; and,
Fourthly, profitableness of High Speed.

In Fig. 62 the working expansion is adiabatic, stopped at s_2 to avoid excessive bulk costs; and the resilience is created by heat flux

or generation at constant volume s_0 stopped at pressure p_1, short of the point $p_0\ s_0$ on the adiabatic expansion curve, in order to avoid excessive high-stress costs, the creation of resilience and the heat flux being continued from $p_1\ s_0$ to $p_1\ s_1$ at constant pressure. The back pressure is b.

The whole area under the adiabatic from $p_0\ s_0$ down to zero stress is $\frac{p_0 s_0}{a-1}$.

The top part cut off above the pressure level p_1 is

$$\frac{p_0 s_0}{a-1} - \frac{a}{a-1}\ p_1 s_1 + p_1 s_0.$$

The right-hand corner cut off beyond the volume s_2 is $\frac{p_2 s_2}{a-1}$.

The part cut off by back pressure b is $b\ (s_2 - s_0)$.

After making these deductions the area of the effective indicator diagram may be computed from any one of the four subjoined formulas. These are identical in value, each one being transformable into any other by help of the relation $p_1 s_1^a = p_2 s_2^a$. The first (1) separates the independent parts most clearly from each other; the second (2) is the most symmetrical; in (3) p_2 has been eliminated, being expressed in terms of p_1, s_1 and s_2; and in (4) s_1 is eliminated by expressing it in terms of $p_2\ s_2$ and p_1.

Equation XI—

$$\begin{cases}
(1) & W = \dfrac{a}{a-1} p_1 s_1 - \dfrac{p_2 s_2}{a-1} - p_1 s_0 - b\ (s_2 - s_0) \\[2ex]
(2) & \quad = \dfrac{p_1 s_1 - p_2 s_2}{a-1} + p_1 (s_1 - s_0) - b\ (s_2 - s_0) \\[2ex]
(3) & \quad = \dfrac{p_1 s_1}{a-1}\left\{1 - \left(\dfrac{s_1}{s_2}\right)^{a-1}\right\} + p_1 (s_1 - s_0) - b(s_2 - s_0) \\[2ex]
(4) & \quad = \dfrac{p_2 s_2}{a-1}\left\{a \left(\dfrac{p_1}{p_2}\right)^{\frac{a-1}{a}} - 1\right\} - p_1 s_0 - b\ (s_2 - s_0).
\end{cases}$$

In each of these four forms the last term $b\ (s_2 - s_0)$ is the deduction by back-pressure. In (1) $\frac{p_2 s_2}{a-1}$ is the deduction by non-completion of resilient expansion beyond s_2.

It is desired to examine how this card area can be enlarged.

The diagrams in Figs. 19, 20, 21 and 22, Chapter II, show how this indicator diagram is varied in area per unit of boiler power by extending expansion and by raising the initial pressure in both

condensing and non-condensing steam engines. Its enlargement has now to be considered in relation to costs.

By pushing the right-hand end outwards, that is by enlarging s_2 an amount δs_2, the increase of area is evidently $(p_2 - b) \cdot \delta s$.

By pushing the upper horizontal boundary higher by δp_1 — without shifting the curve or altering s_o — the enlargement is $(s_1 - s_o) \cdot \delta p_1$.

By diminishing the back-pressure by $-\delta b$, the enlargement of area is $-(s_2 - s_o) \cdot \delta b$, the negative sign being inserted because the δb needed for enlargement is a decrease of " absolute " back pressure.

By pushing the left-hand vertical boundary outwards—that is, by diminishing s_o without shifting the curve or altering either p_1 or b—the enlargement is $-(p_1 - b) \cdot \delta s_o$.

Recapitulating, and calling the respective rates of augmentation W'_{s_2}, W'_{p_2}, W'_b, and W'_{s_0};

Equation XII—

$$\begin{cases} (1) & W'_{s_2} = p_2 - b = p_1 \left(\dfrac{s_1}{s_2}\right)^a - b \\ (2) & W'_{p_1} = s_1 - s_o = s_2 \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{a}} - s_o \\ (3) & W'_b = -(s_2 - s_o) = -s_1 \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{a}} + s_o \\ (4) & W'_{s_0} = -(p_1 - b). \end{cases}$$

As regards heat flux, it is not quite fair to reckon it as starting from zero stress, nor is it accurate to measure it from back-pressure; but the latter reckoning is much nearer the actual conditions than the former, and leads to no material error in commercial calculations. It is here taken as starting from the condition $b\, s_o$.

It may be measured in two ways. First as a uniform heat flux at constant volume up to point $p_o\, s_o$ with deduction of the hatched triangular area above the pressure p_1. Naming h the average stress-specific heat per unit volume and per unit increase of pressure between $b\, s_o$ and $p_o\, s_o$, this measure gives

Equation XIIIA—

$$\text{Heat Flux per lb.} = h\,(p_o - b)\, s_o + \frac{a}{a-1}\, p_1 s_1 - \frac{p_o s_o}{a-1} - p_1 s_o.$$

According to the other method of measurement, the specific heat per unit increase of volume at constant pressure is used for the heat flux from point $p_1 s_o$ to $p_1 s_1$. If the diagram be taken per lb. of

working fluid, and h_p and h_s be *taken per lb* (instead of per cubic inch), h_p at specific volume s_o and h_s at pressure p_1, this measurement gives

Equation XIIIB—

$$\text{Heat Flux per lb.} = h_p\,(p_1 - b) + h_s\,(s_1 - s_o).$$

These two modes of measurement should, of course, in each particular case, give identical results if accurately carried out.

In whichever of the two ways the heat flux be measured, it is evident that it is unaffected by the grade of expansion. It is affected only by change in p_1, b and s_o, so long as the expansion curve is not shifted into a new position. Calling the total heat flux H, and its rates of variation with p_1, b and s_o by the symbols H'_{p_1}, H'_b and H'_{s_0}; and using the first form A of Equation XIII;

Equation XIVA—

$$\text{(1)}\quad H'_{s_2} = 0$$

$$\text{(2)}\quad H'_{p_1} = (s_1 - s_o) = s_o\left[\left(\frac{p_o}{p_1}\right)^{\frac{1}{a}} - 1\right]$$

$$\text{(3)}\quad H'_b = -h\,s_o$$

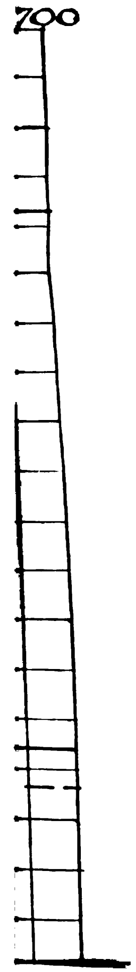
$$\text{(4)}\quad H'_{s_0} = -\ [h\,(a-1) - 1]\,p_o - p_1 - h\,b$$

$$= -\left\{[h\,(a-1)-1]\left(\frac{s_1}{s_o}\right)^a + 1\right\}p_1 - h\,b$$

where h must be taken per unit volume and must be measured in inch-lbs. or other mechanical units and *not* in "heat" (i.e. lb. Fahr.) units.

Now that portion of the total cost which arises from fuel and other working expenses in proportion to fuel may be taken, according to a straight-line law, as an initial constant plus a quantity of money proportionate to this heat flux. The factor of this last term, taken per lb. of working fluid used, is, in what follows, called η.

The parts of the cost due to bulk and weight of plant and to provision against high stress are mainly capital costs; but these are to be reduced to annual costs by converting them into charges for interest, depreciation, maintenance, and repairs, with what additions to working cost as may be considered due to mere size of plant and to keeping joints and working surfaces in good condition in spite of high working stress. They are then to be reduced from annual charges to charges per working stroke of the engine by dividing by


700

the number of working strokes made per year—that is, by dividing by a factor proportional to the rotary speed of the engine.

In such calculations, unless the conditions of working be abnormal, it is common to assume 270 working days of 10 hours each, or 2,700 hours, = 162,000 minutes, per year. 160,000 minutes per year would correspond to $266\frac{2}{3}$ days of 10 hours each. If n be the number of revolutions per minute and the engine be double acting, then the annual capital charges may be divided by 320,000 n to reduce them to per working stroke.

When they are so reduced to per working stroke, they must be compared with the final cylinder volume in order to make them useful for the purpose of calculation of economical design. It is not essential to make this comparison by dividing the one by the other, and in no case would the quotient be found constant for varying sizes. But in the absence of a diagram this plan of considering their ratio is that giving the simplest conception of the relation. Dividing the costs by the volume in cubic inches swept through by the piston per working stroke, we get them reduced to per cubic inch of cylinder volume filled by the working fluid at the end of its useful resilient expansion. In compound and triple expansion engines, the volume of the low pressure cylinder must be used for this reduction.

In Chapter III are given many diagrams and tables of the *prime* costs of various classes of steam, gas and oil engine and boilers, as well as of complete plants inclusive of buildings. These will serve as data for the above calculations when exact particulars for any case in hand cannot be obtained.

Along with these diagrams of prime cost are given, in the case of those referring to complete plants, other diagrams of annual costs, separate curves being given for fuel, wages and lubrication, etc., and capital charges. Other curves in the same diagrams give the total running costs reduced to "per brake horse-power-hour." As one horse-power-hour is $23\frac{3}{4}$ million inch-lbs. of work, the figures given by these last-named curves are of the same nature as would be obtained by dividing the total cost of one working stroke of an engine by the work measured from its indicator diagram. This latter in pence per inch-lb. equals the cost in pence per indicated horse-power-hour divided by 23·76 million.

For the purposes of the present chapter there is here added in Fig. 63 a diagram of four pairs of curves. Each pair refers to one class of plant, the data being the same as those of Figs. 25 to 32. The full line of each pair gives the ratio of total annual cost to the

capital outlay on engine and boiler (and condenser in case of condensing steam engines) alone. The dotted line of each pair gives the ratio of annual total capital charges to the capital outlay on engine and boiler alone. These curves will be found very useful as it is always easy to ascertain the prices of engines and boilers.

Not only the curves of total cost, but also those of the items making up this total, are seen from these diagrams to deviate considerably from straight lines so long as the cylinder size, or the horsepower, is small; but as the sizes become larger they all more or less rapidly tend to become straighter.

In respect to size the diagrams and formulas attached show clearly that the law—

$$(\text{Constant})_1 + (\text{Const.})_2 \times \text{Size} - \frac{(\text{Const.})_3}{(\text{Const.})_4 + \text{Size}}$$

follows actual variations of cost very closely. The omission of the third, or hyperbolic, term, leaves this a straight line. Such a straight line with properly adjusted constants applies fairly closely to very considerable ranges of the curves. Although the complete formula should be used in settling, for any purpose, absolute values throughout a complete range of manufacture; still when it is only desired to investigate *variations* throughout any moderate range the simpler straight-line law may be thoroughly useful.

In respect to the influence of working pressure on costs the evidence obtained direct from practice is very far from complete. The law—

$$(\text{Const.})_1 + (\text{Const.})_2 \times \text{Pressure}$$

is the most for which numerical data have so far been found. In this formula, however, it must be understood that the two constants do not remain the same for different *sizes* of the same style of plant.

When further information on this point is collected from the makers and users of engines, boilers, etc., there is little doubt that it will be found that the law suiting most cost variations, excluding those which are erratic and unreasonable, is of the form

$$C_1 + C_2 p + \frac{C_3}{C_4 - p}$$

where C_3 is a positive constant and where C_4 is a limiting high pressure near which the mechanical difficulties of high pressure become so excessively expensive that plant using such pressure is a commercial impossibility. For small pressures this law nearly coin-

cides with the straight-line $\left(C_1 + \frac{C_3}{C_4}\right) + C_2\,p$. The actual curve lies above this line, and its height above it is at first zero, but increases with the pressure, at first slowly, but more rapidly at high pressures. Here again, however, it must not be expected that the constants of this formula should be the same for small and large sizes.

Combining now the two laws of cost-variation with size and with pressure, the simplest law would be $(k_1 + k_2\,s_2 + k_3\,p_1)$, where s_2 is the final volume and p_1 the initial pressure. This, however, makes the variation with s_2 the same for all pressures, and the variation with p_1 the same for all sizes, which is markedly incorrect. This formula may be improved by the addition of a fourth term $k_4\,s_2\,p_1$; but so modified it also is found unsatisfactory.

The most complete and satisfactory formula is one with 9 constants, namely,

Equation XV.—$$K = k_1 + k_2\,s_2 + k_3\,p_1 - \frac{k_4 + k_5\,p_1}{k_6 + s_2} + \frac{k_7 + k_8\,s_2}{k_9 - p_1}$$

The 9 constants can be found by examination of 9 given costs at 3 different sizes, with 3 different pressures at each size. For any one pressure this formula reduces to one for various sizes of precisely the same shape as above seen to be correct; while for any one size it reduces to one for various pressures of the same shape as above said to be most probable.

It makes the two rates of cost-variation with size and with pressure

Equation XVI—$$\left\{\begin{aligned} K'_{s_2} &= k_2 + \frac{k_4 + k_5\,p_1}{(k_6 + s_2)^2} + \frac{k_8}{k_9 - p_1} \\ \text{and } K'_{p_1} &= k_3 + \frac{k_7 + k_8\,s_2}{(k_9 - p_1)^2} - \frac{k_5}{k_6 + s_2} \end{aligned}\right.$$

The second differential coefficient, showing the curvature of the surface diagram, is—

$$K''_{s_2 p_1} = K''_{p_1 s_2} = \frac{k_5}{(k_4 + s_2)^2} + \frac{k_8}{(k_9 - p_1)^2}$$

To find the constants from any particular set of given costs, 3 curves should be drawn for each of 3 differing pressures with s_2 as base. Then measure the 6 slopes at two different volumes, and take the 2 differences of slope at each of these two volumes, i.e.

4 differences of slope. The last equation for $K''_{s_2 p_1}$ gives the means of finding k_4, k_5, k_8 and k_9 from these 4 differences of slope. Next inserting these in the equation for K'_{s_2}, the difference between two slopes at two volumes well apart for the middle pressure will give k_6; and from this the slope at either volume will give k_2. Next insert the values thus found for k_2, k_4, k_5, k_6, k_8, k_9, in the equation for K; and measurement of K at any one volume and at the three pressures, will yield the values of the remaining 3 constants k_1, k_3 and k_7.

Unfortunately there are not yet to hand data from authentic sources sufficient to determine the curvature of the surface diagram in respect to pressure, and the terms involving this cannot at present be utilized. This involves the omission in Equation XV for K of the last term $\dfrac{k_7 + k_8 s_2}{k_9 - p_1}$, and in Equation XVI the omission in K'_{s_2} of the last term $\dfrac{k_8}{k_9 - p_1}$ and in K'_{p_1} of the middle term $\dfrac{k_7 + k_8 s_2}{(k_9 - p_1)^2}$.

These omissions reduce the number of constants required to 6, which can be found from *two* K, s_2 curves for *two* different pressures. For each size s_2 the diagram for variation of pressure is now reduced to a straight line at the slope $\left(k_3 - \dfrac{k_5}{k_6 + s_2}\right)$, a slope which increases with the volume s_2. This straight line starts from the height $\left(k_1 + k_2 s_2 - \dfrac{k_4}{k_6 + s_2}\right)$, a height which falls below the s_2 straight-line $(k_1 + k_2 s_2)$ after the same manner as the corresponding slope falls below the asymptotic slope k_3.

These omissions thus simplify the last two equations to

$$\text{Equation XVA.—}K = k_1 + k_2 s_2 + k_3 p_1 - \frac{k_4 + k_5 p_1}{k_6 + s_2}.$$

$$\text{Equation XVIA—}\begin{cases} K'_{s_2} = k_2 + \dfrac{k_4 + k_5 p_1}{(k_6 + s_2)^2} \\ K'_{p_1} = k_3 - \dfrac{k_5}{k_6 + s_2}. \end{cases}$$

No further simplification than this is desirable unless in XV the first three terms alone are used, when in XVI the two rates will reduce to k_2 and k_3.

The formation of these apparently clumsy cost-formulas, with the proper constants inserted, are useful for two purposes. The first, a sufficiently important one, is to enable manufacturers to "rationalize" their price-lists. Price-lists as published at present are often extremely irregular. Such irregularities arise in part from errors in cost account keeping and from erratic loading with establishment charges and profits; but, in part, also from strictly local difficulties of workmanship. For instance, in a particular workshop it may be comparatively cheap to bore cylinders up to a certain size, and above that size a sudden change in cost occurs. The first source of irregularity ought not to exist; the second cannot be avoided except in very large works. But manufacturers ought to recognize that their own special local difficulties ought not to influence their price lists. The price should depend on the cost of manufacture and on the state of the competitive market conjointly; and the latter influence ought, in reason, to smooth out all irregularities in the price-list due to special workshop facilities or absence of facilities; that is, should do so, of course, within the range of size for which the workshop manufactures and competes in the market.

The use of a formula such as here described removes all such erratic excrescences in the price-list, which must be obstructive to good business. Moreover the graphic use of the formula is quite easy, as will be seen below. If the constants be carefully got out, no more reliable or more rapid and easier method of estimating for new requirements can be conceived. It is absolutely safe, not only to yield the reasonable profit desired but also to secure a good standing in competition with other quotations—except as against such as by accident and error secure a losing or profitless contract.

The second use to which the formula is applicable is the calculation of most commercially economic combinations of the elements of design, which is the main purpose of the present treatise.

Once the nine constants of Equation XV, or the six of Equation XVA are known, the subjoined graphic construction in Fig. 64 makes the application to all combinations of size and pressure very easy and very rapid, and obviates the need of numerical calculation from a somewhat forbidding algebraic symbolism. The diagram having its base elements, dependent on the 9 or 6 constants, once set out on good evenly divided squared paper, of, say, 10 or 20 inches square, or 200 or 500 mm. square, remains permanently ready to be used for the calculation of every possible future need (within its range of size and pressure), each such calculation occupying only a couple of minutes or so.

The diagram may be applied either to engines, boilers, condensers, gas-producers, etc., etc., separately, or to complete plants of combinations of these.

In Fig. 64 the pressure scale runs to 200 lbs./in.^{2} and the volume scale to 100,000 cubic inches. The full and the dotted lines are parts of the permanent diagram. The four long dash (— — —) lines are the only construction lines needed to obtain the results for any desired particular pair of size and pressure values $p_1\,s_2$. Two of these lines are simply the lines of the squared-paper and do not need to be drawn. The other two need be "constructed," not by actual drawing, but simply by laying the edge of the set square in the correct direction through the "origin" O.

In Fig. 64 the four construction lines are shown for one pair of values $p_1\,s_2$, shown by the point marked P. Along the base is laid off negatively k_6; and downwards from its extremity is plotted $\frac{k_4}{k_5}$. These two plottings give the "pole" σ. Similarly $\frac{k_7}{k_8}$ is plotted negatively along the base, and k_9 upwards; these two giving the "pole" π. k_5 and k_8 are plotted from O, k_5 positively along the base and k_8 downwards; and from their extremities are drawn in full-line a vertical and a horizontal through the whole range of the diagram. 100,000 k_2 and 200 k_3 are plotted vertically and horizontally at $s_2 = 100{,}000$ and $p_1 = 200$; and the two points so plotted are joined by full lines with O. k_1 should be marked off to scale on any convenient part of the diagram. This completes the permanent part of the necessary construction; but, as in Fig. 64, it is useful to add an expansion-curve running from $s_2 = 5{,}000$ to $s_2 = 100{,}000$. The curve drawn in Fig. 64 is hyperbolic.

The horizontal construction line through P gives $k_3\,p_1$ read from the oblique full-line through O and 200 k_3, as marked. The vertical construction line through P gives similarly $k_2\,s_2$ on the full-line from O to 100,000 k_2. Through O is to be drawn a construction line parallel to σ P, and this cuts off $\frac{k_4 + k_5\,p_1}{k_6 + s_2}$ upon the vertical at $s_2 = k_5$. Through O another construction line is to be drawn parallel to π P, and this cuts off $\frac{k_7 + k_8\,s_2}{k_9 - p_1}$ on the horizontal at depth k_8.

The five parts of K are now obtained on the diagram and may be added (the fourth part is negative) either by help of the dividers

upon the diagram scale, or else by reading them off numerically and adding arithmetically. If in Equation XV the constants k_7, k_8 and k_9 for the last term be unobtainable, the pole π cannot be found on the diagram, and that part of the construction falls out. In constructing a permanent diagram for subsequent frequent use, however, it is very evident that it is worth spending some time and trouble to obtain these constants, because they add materially to the correctness and reliability of all results read from the figure.

In using this diagram careful attention must be paid to the scales. There is a vertical pressure scale, a horizontal volume scale, and a third scale of costs which is to be plotted and read both vertically and horizontally.

To the vertical pressure scale are to be plotted $\frac{k_4}{k_5}$ and k_9, these being actually pressures. To the horizontal volume scale are to be plotted k_6 and $\frac{k_7}{k_8}$, these being really volumes. 200 lbs./in.^{2} $\times$ k_3 is a cost, as also is 100,000 cub. ins. $\times$ k_2; and these two quantities must be plotted to the horizontal and vertical cost scales, which two scales ought, of course, to give the same number of £ per inch of scale length. k_1, being a cost, must be plotted to the same scale.

There remain k_5 and k_8, which are the only two plotted quantities whose scales are not obvious.

As regards the scale of k_5, the ratio of any cost to this constant is a ratio of a pressure to a volume. Therefore the ratio of unit cost to unity on the scale of k_5 is the ratio of unit pressure to unit volume. Therefore take the oblique line through O, which gives this ratio—that is passing through the point, say, 200 on the pressure scale at volume 200. At height 1 on the vertical cost scale, this oblique line gives the horizontal distance which must be taken as unit for the scale to which k_5 must be plotted. In Fig. 64 this oblique line would pass through the top edge of the diagram at a horizontal distance of $\frac{200}{100000} = \frac{1}{500}$th of the total horizontal length of the diagram. This line is too nearly vertical to permit of accurate reading of the k_5 unit from the paper; but as the height of 1 on the cost scale is some known easy fraction of the whole diagram height, it is easy to calculate it arithmetically. Otherwise, if the pressure scale be $1'' = m$ lbs./sq. in.; the volume scale be $1'' = n$ cub. ins.; and the cost scale $1'' = q$ £: then k_5 is to be plotted to scale $1'' = \frac{n\,q}{m}$.

Similarly the ratio of a cost to k_8 is the ratio of a volume to a pressure; and, therefore, k_8 is to be plotted to the scale $1'' = \frac{mq}{n}$. Under the diagram will be found exact instructions for easy graphic constructions giving these scales accurately, avoiding the difficulty of using very acute angles of intersection and using the main diagonal of the diagram instead.

Not much examination of this diagram Fig. 64 is needed to recognize how simply and readily it indicates to the eye how variation of either p_1 or s_2 changes the cost. Suppose P raised without horizontal shifting. A mere glance shows in what proportions this increases the two positive and one negative parts of K affected by p_1. Equally easy is it to note the changes in the parts affected by s_2 when P is moved either way horizontally.

The addition to the diagram of the 5 standard expansion curves enables one to calculate very quickly the changes of cost resulting from using higher or lower initial pressures, or from greater or smaller grades of expansion, along any one curve; that is, confining the pairs of selected p_1 and s_2 to one such curve. It must be understood, however, that the use of the diagram extends to any combination whatever of pressure and volume without any restricting conditions.

This graphic method is preferable to any numerical calculation, because it offers no temptation to neglect the due influence of any of the nine constants, it being just as easy to make use of all of them in the diagram as not. It should be understood that it is better to use even very rough, well rounded-off approximations than to neglect any of them. For the problem of maximum economy, k_3, k_4. and k_6 are those most important to get fairly accurate. The value of k_1 does not enter into the solution of this problem.

It should be understood that this formula and diagram may be applied equally well to prime costs or capital outlays, and to annual charges based on capital expenditure. Of course, the costs read from the diagram are different in the two cases, not only in numerical magnitude, but also in kind. In the one case they are sums of money; in the other they may be sums of money per year, or money per year per horse-power, or money per unit of work, or money per unit weight of working fluid used; and the unit of time involved may be a year or an hour, or a minute. The arithmetic magnitudes, and the kinds, of the constants k will differ correspondingly.

We have now, theoretically, laid down all the elements upon which must be based the solution of the problem of maximum com-

mercial economy; and in previous chapters we have given very numerous examples of the physical and commercial data entering into these theoretical expressions. We have thus arrived at the stage at which it is possible to state this problem in its full terms, and to attempt its general all-round solution. It cannot be expected that this general solution should be extremely simple, because commercial economy is influenced by many concurrent factors. It is essentially a complex thing, and no theory of it which was very simple and elementary could possibly be true.

Chapter IX

Partial Limit Values of Bulk, Initial Pressure, and Back Pressure for Maximum Economy

In Equations XI, XIII and XV of Chapter VIII we have the values of the indicator-diagram work done, there called W; of the heat-flux needed to obtain this work, which we have named H; and of the capital charges involved, these being indicated by K. In this and the following chapters it must be understood that we take W, H and K as corresponding quantities. W may be the work done per year, or per hour, or per minute, or per stroke of the engine, or per lb. of working fluid used, or per any other unit deemed convenient. But whatever unit W is referred to, H and K must be for the same quantity of work. Note that, in what follows therefore, K does not mean a total capital outlay. It means a capital or "establishment" charge based upon fixed capital plant. It is part of the "working capital" referred to in Chapter I, which is the total expenditure in a single "turn-over period." It is that part of the working capital arising from fixed outlay. The remainder of the working capital is the expense of fuel, lubrication, wages, etc.

It is desired to put the whole current costs into two classes alone. Therefore, salaries or portions of salaries, management or part of it, rent, rates, taxes, insurance, etc., etc.; all that arises directly or indirectly from the fixed capital outlay, all that is needed to maintain the plant in good condition, to secure its safety, and to keep it in activity *irrespective of the quantity of work actually done*, that is irrespective of whether the plant works at full, or half, or quarter power, or is overloaded, is to be placed in the category of capital charges and is to be reckoned in with K.

K is already money, and requires no affixed factor to reduce it to money.

The remainder of the expenses involved in producing power, including stokers', engine drivers' or other wages, part or possibly the whole of engineers' salaries, fuel, lubrication and other materials, are here taken as slumped together; and their sum is taken as directly proportional to H. H is a quantity of heat, and to make it comparable with money costs it must be multiplied by a factor whose nature is $\frac{\text{Money}}{\text{Heat.}}$ This factor will be named η. But this factor will be here taken of such magnitude as to include in η H all the above expenses thrown along with the fuel or heat. η H means the same thing as that named F in Chapter I, where F was intended to suggest the word "Fuel." Figs. 25 to 32, Chapter III, give for four distinct classes of plant examples of the ratio of this total to the cost of fuel, while the fuel-cost is almost strictly proportional to H. The proportion differs according to the style of boiler and furnace, or of gas producer, and according to whether "economizers" and feed water heaters and "superheaters" are used. It also differs according to the "load-factor" and the proportions taken from the above tables and diagrams are, it must be remembered, for 2,700 full working hours per year. But for each set of such working conditions the proportion of the whole to H is fairly constant.

As regards W, if we wish to find the "profit" arising from the power production, the money value of W must be taken and from it deducted the whole costs of production. In what follows ω will indicate the factor to be used to convert the work W into its money value ω W. The factor ω is of the same *kind* as η, because work and heat or any sort of energy have the same physical "dimensions." But, as explained in Chapter I, if we deal merely with the ratio of value of product to cost of production, then it is not necessary to evaluate W in money, but the ratio may be taken simply as so many ft.-lbs. or inch-lbs. of work per unit of time per £1 or per 1 penny cost of working capital.

Lord Kelvin demonstrated a method of economic design of the section of copper conductors for electric power transmission. A useful lesson may be learnt from a summary of his method and reasoning. Taking a foot, or yard, or mile length of cable, and calling the sectional area of the copper S, he assumes a uniform increase of capital price per square inch of copper section. Suitably reducing this to a capital charge per second, the capital expense per second per yard length may be written $K + k S$. The electric resistance is inversely as the copper section, and this resistance

involves loss of power—the c^2 R loss, as it is called, where c is the current passing and R is $\frac{\rho}{S}$, the "specific resistance" per yard length being ρ. This power costs money to generate, and its loss is part of the cost of delivering power at the end of this yard length. If ω be the cost of unit of power per second, this money loss or cost is $\omega \rho \frac{c^2}{S}$. These are the two elements of cost which vary according to the size of copper section used. The first of them increases and the second decreases with S. It is the sum of them $\left(K + k S + \omega \rho \frac{c^2}{S}\right)$ that is important. Lord Kelvin advises that this sum should be made a minimum. It is very large for very small section, and falls as more copper is put in the section up to a limit beyond which it increases. The rate at which it varies with the section is $\left(k - \omega \rho \frac{c^2}{S^2}\right)$, which is first negative, and then for larger S becomes positive, passing through zero value at $S = c \sqrt{\frac{\omega \rho}{k}}$, at which limit the minimum occurs. At this limit $\omega \rho \frac{c^2}{S} = k S$, so that the minimum total cost is $K + 2 k S = K + 2 \omega \rho \frac{c^2}{S}$. It is what may be called a mathematical accident that it makes the variable parts of the capital charge and of the power-loss charge equal: in this special case they are equal, but in the general application of the principle it is the equality of the plus and minus rates of change of the two parts that determines the limit, and this does not generally make the parts themselves equal.

This solution is perfectly correct under the conditions to which it is applied. But as it solves a quite restricted, and not the general, problem of industrial economy, it is necessary to note what the restrictions are. The solution is effected under the restriction that S alone varies: the current c, the power delivered, and everything except the size of the cable, is supposed to be unaffected by change of S. But in practical application one must make sure not only that it is possible to vary S without varying these other things, but also that the conditions which make this possible are actually adhered to. If we use C to represent the whole cost (capital charge plus working expenses), and ω W the gross income

from the production of the power W; then, according to Chapter I, the "Commercial Economy Coefficient" is $\frac{\omega W}{C T}$, in which T is the "time of turn-over." So far as the determination of maximum limits is concerned, it makes no difference whether we consider this fraction or what might be termed the "profit-making coefficient" $\frac{\omega W - C T}{C T}$, because this differs from the other only by the constant -1.

It is preferable, on the score of simplicity, to consider $\frac{\omega W}{C T}$, because from this we may omit the factor ω which is constant. The complete economy problem may therefore be stated as the effort to make $\frac{W}{C T}$ a maximum.

Now Lord Kelvin's electric-cable corresponds to those cases in which the economic investigation is confined to adjusting C alone to its best value without any accompanying change in W or T. Of course in these cases the minimum C gives the maximum $\frac{W}{C T}$. It is, no doubt, possible in many cases to vary C alone without change in W or T, and in such cases the application of the Kelvin principle is entirely legitimate. But it is doubtful whether true economy would lead main-line electric cables to be designed for a fixed power-delivery. They have to be designed to be suitable for an ultimate maximum demand; but it is certain that throughout the first and probably the major portion of their life they will never be used to supply this ultimate maximum demand.

How necessary it is to carefully consider whether the restrictions imposed by any mathematical solution of this sort correspond with actual conditions of work, is illustrated by examining in the case of electric power transmission whether the income-earning product (or W), which is the electric power delivered, is affected by change of cable section. If the power be generated at the central station at the voltage E, and if it be required to deliver it at the voltage e at a prescribed distance which makes the circuit resistance (exclusive of that between the e terminals) $\frac{L \rho}{S}$; then the current must be $\frac{(E - e) S}{L \rho}$ and the power delivered $W = \frac{e (E - e) S}{L \rho}$.

If E and e are fixed, and cannot be adjusted to compensate

for variation of S, then the assumption in the Kelvin economic solution that W is, or can be made to be, unaffected by change of S entirely falls to the ground, and the solution becomes incorrect.

Adhering to our main proposition that what is economically desirable is to make $\frac{W}{C\,T}$ a maximum, we have to examine how this is affected by any proposed change in the design. The general solution must proceed on the assumption that the change affects all three factors W, C T. If in the case of any particular proposed change, it is found that one or more of these remains unaffected, then the particular solution is derived from the general by putting in this latter the corresponding variations of zero value.

In our nomenclature W'_p means the rate at which W varies with p. Similarly W′ without suffix is to be used to indicate any rate of variation of W without specifying the particular change, it may be of p, or of s, or of b, etc., etc., which causes W to vary. W′ means a special rate of variation, but the particular cause of it is not filled in in the symbol—it is left blank to be filled in with the name of any such cause. Calling the economy coefficient $\rho \left(= \frac{W}{C\,T}\right)$ and applying similar meanings to ρ', C′, and T′; we find the complete general formula for the variation of ρ to be

$$\text{Equation XVII—}\begin{cases} \rho' = \dfrac{W}{C\,T}\left(\dfrac{W'}{W} - \dfrac{C'}{C} - \dfrac{T'}{T}\right) \\ \text{or } \dfrac{\rho'}{\rho} = \dfrac{W'}{W} - \dfrac{C'}{C} - \dfrac{T'}{T}, \end{cases}$$

and for variation from any one particular cause, the limit at which the maximum value of ρ is reached is to be found when $\rho' = o$;

$$\text{Equation XVIII—}\left\{\text{or } \frac{W'}{W} = \frac{C'}{C} + \frac{T'}{T} \text{ gives maximum } \rho,\right.$$

provided positive values of ρ' precede negative values of ρ'.

If the variation proposed has no effect upon the time of turnover T, then $T' = o$, and these equations become—

$$\text{Equation XVIIA—}\left\{\frac{\rho'}{\rho} = \frac{W'}{W} - \frac{C'}{C},\right.$$

and

$$\text{Equation XVIIIA—}\left\{\frac{W}{W'} = \frac{C}{C'} \text{ at maximum } \rho.\right.$$

In the last expression the inverted ratios are preferred for the sake of greater ease in the graphic solution given below.

In Chapter I it was shown that C T equals the "working capital" when W equals the product per unit of time. It is, as there said, often simpler to consider ρ in terms of working capital rather than in separate terms of cost and time of turn-over. If the working capital C T be called, C_w, then —

$$\rho = \frac{W}{C_w}$$

$$\text{Equation XVII}_B\text{—}\begin{cases} \text{and } \dfrac{\rho'}{\rho} = \dfrac{W'}{W} - \dfrac{C'_w}{C_w} \\ \text{and } \dfrac{W}{W'} = \dfrac{C_w}{C'_w} \text{ at maximum } \rho. \end{cases}$$

(Equation XVIII B is the second line.)

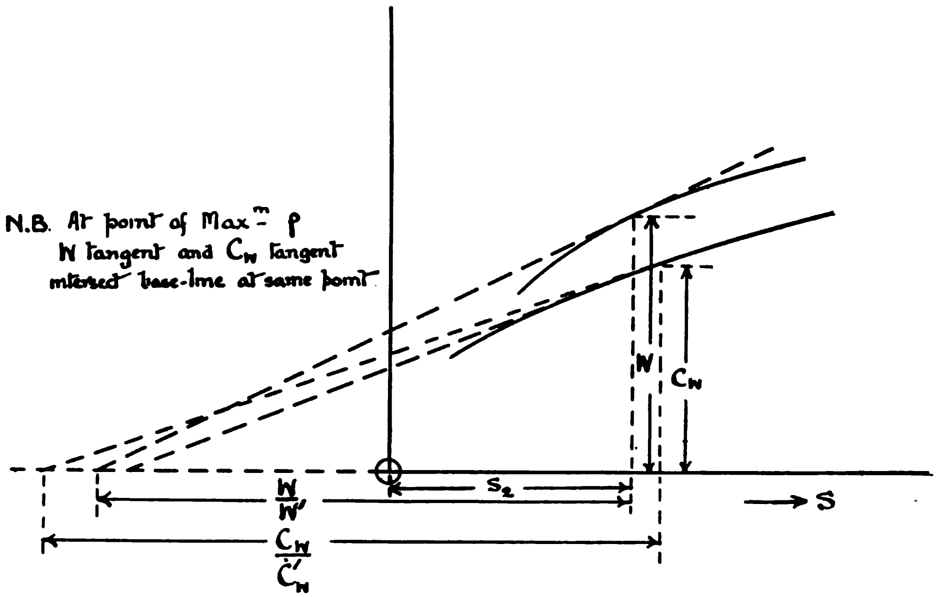


FIG. 65.—Graphic Construction for Solution of Equation for Maximum Economy Coefficient with Output and Cost alone variable.

In those special cases where no change in W is involved or permitted, which correspond to the Kelvin rule, maximum economy is reached when—

$$\text{Equation XVIII}_C\text{—}\begin{cases} \dfrac{C'}{C} = -\dfrac{T'}{T} \\ \text{or } C'_w = 0. \end{cases}$$

In Fig. 65 above is shown a graphic construction enabling these

equations for maximum economy coefficient to be solved easily and quickly. The diagram illustrates the method for investigation of the effect of varying s_2, and for determination of the s_2 which gives maximum ρ.

On a horizontal base line on which various s_2's are measured, the corresponding values of W and C_w are plotted vertically and two curves drawn through the points so as to show the continuous

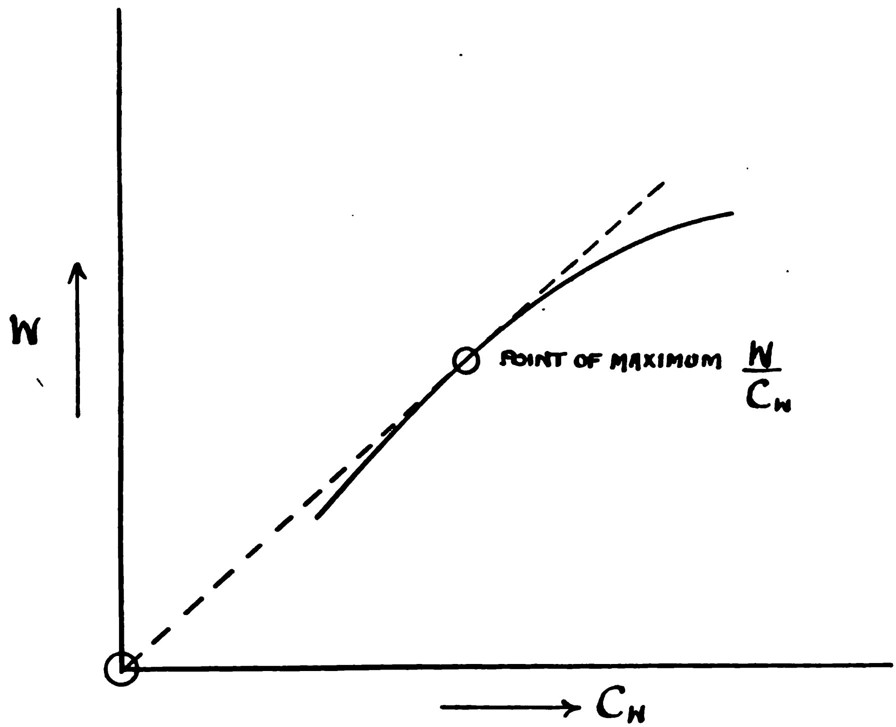


Fig. 66.—Alternative Construction for Solution of Equation for Maximum Commercial Economy Coefficient with Output and Cost alone variable.

variations of W and of C_w. If a tangent be drawn to the W curve at any point, and produced to cut the base line, then what geometricians call the "sub-tangent" measures $\frac{W}{W'}$ as marked on the diagram. Similarly $\frac{C_w}{C'_w}$ is found measured on the base by drawing a tangent to the C_w curve. At the position giving maximum ρ

these two sub-tangents must be equal;—of course, for tangents drawn from points vertically one over the other in the two curves—and therefore they must cut the base in the same point. In the diagram this is illustrated by drawing two tangents to the C_w curve from two points a little before and after the point from which the W tangent is drawn, which two C_w tangents cut the base on opposite sides of the intersection of the W tangent. This is the best test of having found the correct position at which the two tangent-base intersections coincide. After the curves are drawn a few trial pairs of tangents lead quickly to the discovery of the correct position.

In carrying out the construction it should be remembered, what has been previously pointed out, that since $\rho' = o$ at this correct position, and equals nearly o for some distance on either side of it, it is of no practical importance whatever to find the correct position with minute accuracy. A small error in either direction makes no difference in the useful result.

In the case in which one wishes to work with the total cost C and the time of turn-over T, and in which $T' = 0$, precisely the same construction with a C curve substituted for the C_w curve serves to solve this problem of maximum commercial economy coefficient. Fig. 66 gives another diagram for the same purpose, which in some respects is still simpler. Here after calculating, say, 5 corresponding values of W and C_w for 5 different values of s_2, no other influential factor except s_2 being varied, these values of C_w are plotted as horizontal ordinates while the values of W are plotted as the corresponding vertical co-ordinates, and a curve drawn through the points so obtained. Next draw from the origin a tangent to this curve. The touching point of this tangent clearly corresponds to a maximum or a minimum value of $\frac{W}{C_w}$, according as the curve curves downwards or upwards.

If, however, T′ be not zero, then either a (C T) curve may be calculated and drawn and treated exactly as the C_w curve is in Fig. 65, or else three curves of W, C and T may be used in the manner shown in the next diagram, Fig. 67. At any horizontal position run a vertical line through the three curves and up to some convenient height such as 100 or 1,000 on the vertical scale. The solution is not affected by the numerical simplicity of the value of this height, so that any height on the diagram will serve. It ought to be as high as is convenient, and the top edge of the diagram is as good as any other height. From the top of this line draw three

lines (drawn in Fig. 67 with long dashes — — —) perpendicular to the curve tangents at the intersections of the vertical line with the

TEST OF MAXIMUM COMMERCIAL ECONOMY COEFFICIENT.

$\frac{W}{CT}$ maximum when

$$\frac{W'}{W} = \frac{C'}{C} + \frac{T'}{T}$$

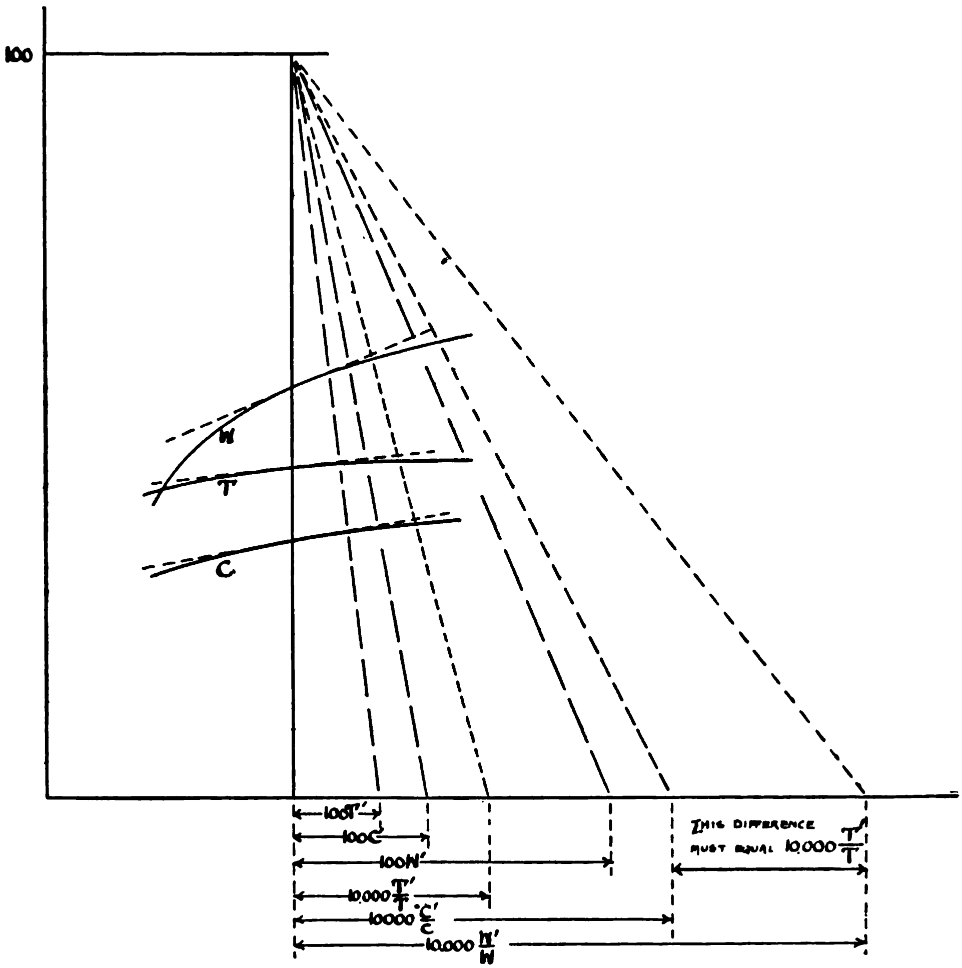


FIG. 67.—Test of Maximum Commercial Economy Coefficient with Output, Cost and Time of Turnover variable.

curves, and extend these to cut the base-line. Parallel to the lines joining these intersections on the base-line with the curve inter-

sections of the vertical line, draw three new lines from the same pole, namely the 100 height on the vertical line; and extend these three new lines down to cut the base-line. Their intercepts on the base-line measured from the foot of the vertical are $10{,}000\,\frac{W'}{W}$, $10{,}000\,\frac{C'}{C}$ and $10{,}000\,\frac{T'}{T}$, the factor 10,000 being the square of the height 100 of the pole chosen upon the vertical. The value of this factor is of no consequence because it is common to all three, and the test of the maximum is that the first should equal the sum of the two others. Taking $10{,}000\,\frac{T'}{T}$ in the dividers, or reading it on the scaled squared paper, it is to be noted whether it equals, is smaller than, or is greater than, the distance between the feet of the $10{,}000\,\frac{W'}{W}$ and the $10{,}000\,\frac{C'}{C}$ lines. By repeating this simple and rapid construction at three or four horizontal positions one quickly discovers the approximate position of maximum ρ. In such graphic constructions as these, although in the descriptive diagram the construction-lines are necessarily drawn in through their full length, in practical work they should be drawn through only that part needed to give the required intersection.

Since, however, this construction involves the plotting of the three curves, and this plotting involves knowledge of some of the values of W, C and T, those who are not *au fait* in graphic construction may prefer to solve the problem by arithmetically calculating a number of values of $\rho = \frac{W}{K\,T}$ and continuing such trials until they find that they have passed the maximum. The great advantage of the graphic method is that from three or four calculations and plottings of W, C and T the curves can be very well drawn, and these at once give the *continuous* variations of all the quantities. The graphic method not only gives a more rapid and more accurate solution, but it also puts the whole range of the influential facts very clearly before the eye and the mind.

Another graphic method which may be followed is to calculate four or five values of $\rho = \frac{W}{C\,T}$ arithmetically, to plot these as ordinates to a ρ-curve, to draw in this curve and select its highest point on the diagram. To do this fairly it is essential that one at least of the calculated values of ρ be on the opposite side of the maximum

to the other calculated values. It is desirable that one calculated value should be near the maximum and that two others should lie on either side of it.

These methods apply to the profitable limits of variation resulting from change in any one influential factor. Examples of such factors are final bulk s_2, initial pressure p_1, and back pressure b. Whatever the factor be, the variations of W, C and T with respect to this factor alone are to be taken in applying the above methods. The rates of variation W'_{s_2}, W'_{p_1}, etc., etc., have been already given in Equations XII, XIV and XVI of Chapter VIII. But the methods just given apply equally well to finding the extent to which any possible influence should be allowed to act in order to reach maximum economy. The influences of high speed, of superheating steam, of steam-jacketing, of better quality of coal fuel, of greater skill in stoking, of oil enrichment or other improvement in quality of gas fuel, of any really operative change so long as that change can be made continuous and gradual in its increase or decrease, can be investigated in the same way. In some cases the investigation will not yield any limits of maximum economy, namely, in those in which the change is throughout an improvement or else throughout bad in its effect on economy. Generally, however, there are at least extreme limits beyond which any improvement ceases and is altered to loss of economy.

In our problem of power-generation by expenditure of heat, the whole cost C is conveniently considered in two parts, or—

$$C = K + \eta H,$$

and therefore—

$$C' = K' + \eta H',$$

η being taken as constant.

The various values of H′ for different kinds of important change are given in Equation XIV. In Equation XVI are given K′ for the bulk, or s_2, variation, and for the pressure, or p_1, variation. The capital charges in condensing steam engines do vary with back pressure b, the kinds of condenser giving better vacuum being more expensive than those giving small vacuum. We have not, however, included in our formula for K any term showing the condensation capital costs ; and, therefore, in XVI, K'_b does not appear. It should not be supposed, however, because an algebraic formula cannot be constructed to show such an influence as that of back pressure without giving the formula so complex a shape as to make it useless, that, therefore, the present scientific method of investi-

gating commercial economy cannot be used. The great advantage possessed by the graphic methods here recommended over all other methods is that they are completely independent of formulas; and that the most complex variation, which it would be hopeless to try to reduce to algebraic treatment, is as easily represented by a plotted curve as is the simplest variation. So soon as the capital costs are known of three, or four, or better five, comparable condensing plants giving a fair range of different vacuums, then the curve of capital costs can be plotted on squared paper co-ordinated with b, and from this K'_b can be obtained at all points of this range.

It is safe to put $K'_{s_0} = o$ in all cases. This means that for given final bulk s_2, it does not matter, so far as capital costs are concerned, whether the resilience be generated at small or large volume. This comparison must, of course, be restricted within the range of one general style of plant: it does not hold in comparing two entirely different styles of plant, as, for example, steam engines and gas engines. Moreover it does not mean that large generating bulk is not of great deleterious importance in respect of capital outlay, because, as previously remarked, large s_0 necessitates large s_2 per horse-power. It only means that when s_2 is fixed no decrease of capital outlay results from diminishing s_0.

We have, then, already investigated all the elements in the maximum-commercial-economy equation—

$$\text{Equation XIX—}\left\{\frac{W'}{W} = \frac{C'}{C} = \frac{K' + \eta H'}{K + \eta H},\right.$$

to complete which the term $\frac{T'}{T}$ should be added on the right if T' be not zero in the variation investigated. There is no advantage gained by re-writing this in the complex terms of the previous equations giving these elements, because no further simplification can be effected algebraically. Equation XIX expressed fully in terms of the elements is too complicated for algebraic solution; but the graphic solutions already given are eminently simple and easy, and moreover are not very laborious after the suitable values of the coefficients have been ascertained and inserted.

In Chapter VII there were investigated limits of final bulk, equivalent to limits of expansion, giving maximum to what was there called "bulk-economy." These limits are not only technically interesting, but also commercially useful; but they were not, of course, given as full solutions of the commercial-economy problem. They were based upon the restricted view (1) of economy

in relation to capital outlay alone, irrespective of fuel and other working costs, and (2) of capital outlay supposed to vary with final cylinder bulk according to the straight-line law $(S + s_2)$. It is only now that the stage has been reached enabling a complete all-embracing law of economy to be studied.

The maximum limits investigated in this chapter are, in mathematical phraseology, called "partial" limits. Each is found by use of a "partial" rate of variation or differential coefficient. This is a rate of variation resulting from one kind of change only, such as from change of p_1 alone, or of s_2 alone.

The "partial" values of $C' = K' + \eta H'$ of Equation XIX for the separate variations of (1) final bulk s_2, (2) initial pressure p_1, (3) back pressure b, and (4) original volume s_o are, by combination of Equations XIV and XVI,

Equation XX—

$$(1)\ C'_{s_2} = k_2 + \frac{k_4 + k_5 p_1}{(k_6 + s_2)^2} + \frac{k_8}{k_9 - p}$$

$$(2)\ C'_{p_1} = k_3 + \frac{k_7 + k_8 s_2}{(k_9 - p_1)^2} - \frac{k_5}{k_6 + s_2} + (s_1 - s_o)$$

$$(3)\ C'_b = K'_b - h\, s_o$$

$$(4)\ C'_{s_o} = -\left\{[h(a-1) - 1]\left(\frac{s_1}{s_o}\right)^a + 1\right\}p_1 - h\, b.$$

In (4) the part $K'_{so} = o$ always. In (3) $K'_b = o$ for non-condensing steam-engines always; and for condensing engines, if the condensing plant be considered separately from the boiler and engines, $K'_b = o$ for boiler and engine, but is not zero for the condensing plant.

In the next chapter will be investigated methods of combining these partial solutions, so as to obtain the perfectly general solution. A definite numerical solution of any one partial limit-equation, for example that giving the best limit of initial pressure p_1, can only be obtained, of course, for given numerical values of s_2, b, etc., etc. But in the general combination of the partial equations, there is to be supposed inserted in each the best limit-values of the variables of the other equations, which best values are only obtainable by solution of these other equations. The whole set of equations then becomes mutually interdependent: they must be solved as a group of "simultaneous" equations.

Before proceeding to this general solution, we will, for the sake

of further illustration of the subject of this chapter, revert to the electric cable problem and reconsider its partial solution for variation of copper section S. The power delivered from the far end of the line L may be measured as $W = e\,c$, e being the delivery voltage. Thus in terms of e, c and S, and eliminating the time-factor T, the commercial-economy coefficient would be $\dfrac{e\,c}{L\left\{K + k\,S + \omega\,\rho\,\dfrac{c^2}{S}\right\}}$;

and if e and c are specified and unalterable, then for variation of S the Kelvin criterion of maximum economy is the correct solution.

But if the generating voltage E and e be specified and unalterable, then $c = \dfrac{E - e}{L\,\rho}\,S$; and in this case—

$$\frac{W}{C} = \frac{E - e}{L\,\rho} \cdot \frac{e\,S}{K + \left\{k + \dfrac{\omega}{\rho}\left(\dfrac{E - e}{L}\right)^2\right\} S},$$

which always increases with S, approaching asymptotically the limit $\dfrac{E - e}{L\,\rho} \cdot \dfrac{e}{k + \dfrac{\omega}{\rho}\left(\dfrac{E - e}{L}\right)^2}$ as S becomes very large, but giving no limit of maximum economy. This therefore gives no rule for economic proportionment of section to current. The practical meaning of the result is that, so far as quantity of copper in the mains is concerned, large installations for transmission of large powers have a better commercial economy than small ones; the rate of improvement, however, dwindling off towards zero as very large sizes are approached.

Again if E, the generating voltage, and the current c, be specified and fixed, the work delivered is $W = c\left(E - \dfrac{L\,\rho\,c}{S}\right)$, and the value of S which makes $\dfrac{W}{C}$ a maximum is given by one of the two roots of the quadratic equation—

$$S^2 - 2\frac{L\,\rho\,c}{E}\,S = \frac{\rho\,c}{k}\left(\omega\,c + \frac{K\,L}{E}\right),$$

the other root giving a minimum.

Chapter X

Combinations of Best Values for Maximum Commercial Economy.

Comparing Equation XX of last chapter, which gives the "partial" values of C' with Equations XI for W and XV and XIII which give $K + \eta H = C$, it will be recognized that it would be hopeless to attempt to solve by algebraic processes a combination of simultaneous equations of the form $\frac{W'}{W} = \frac{C'}{C}$; and it would become still more impracticable if the term $\frac{T'}{T}$ were added to the right-hand side.

Nevertheless, there is at least one general relation among these simultaneous equations the consideration of which serves a useful and practical purpose. Suppose that s and p are two influential quantities which can be varied independently, and that there is sought their best combination for economy. Then, omitting the term $\frac{T'}{T}$, the two simultaneous equations giving this best combination, are—

$$\begin{cases} \dfrac{W'_s}{W} = \dfrac{C'_s}{C} \\ \dfrac{W'_p}{W} = \dfrac{C'_p}{C} \end{cases}$$

from which by dividing one by the other is obtained—

$$W'_s \, C'_p = W'_p \, C'_s.$$

The special values of s and p which form the combination yielding best economy must, when inserted in this last equation, make this

equation a true one. This, therefore, furnishes a very useful preliminary test as to whether there exists any such maximum as is sought for; useful because the rates W'_s, etc., etc., are always simpler than the quantities W, etc. etc., themselves. If by formation and inspection of this equation, it is found that no values of s and p can satisfy it, then this shows at once that the complex maximum does not exist, and that it will be waste of labour to try to find it.

To illustrate the meaning of this result, suppose that W and C were truly represented by the straight-line laws $W = w_1 + w_2 s + w_3 p$, and $C = c_1 + c_2 s + c_3 p$; then $W'_s = w_2$, $W'_p = w_3$, $C'_s = c_2$ and $C'_p = c_3$. This would necessitate for a complex maximum $w_2 c_3 = w_3 c_2$. In special cases it might happen that these constants had values making this equation true, but the equality would not depend on any particular values of s and p: it would be true for all combinations of any values of s and p. Thus a combination of two straight-line laws for W and C gives no maximum to the ratio $\frac{W}{C}$. This is easy to recognize if only one variable be involved: it is here shown to be true for a combination of two variables.

In the same manner it is easily proved to be true of the combination of any number of variables, so long as the first power only of each variable enters into the expressions for W and C.

This negative result is a very important one. It should be thoroughly realized that, however convenient straight-line approximations to the various laws governing engineering practice may be for many extremely useful purposes, still they are of no use at all as aids in the discovery of those combinations which yield maximum commercial economy. It is the curvature of the diagrams representing graphically these laws that determine these best combinations. If there be no curvature there exists no maximum to discover. If there be hardly any curvature throughout certain ranges, then it is not likely that the maximum will be discovered within these ranges: it is more likely to be found beyond them, perhaps immediately beyond them where the curvature begins to be marked.

Moreover, since, as has been already pointed out, the variation of the economy is nearly nothing for some little distance on either side of the maximum position, therefore, if the curves involved be very flat there is the less practical need of determining this maximum position with great accuracy—a considerable deviation from it

in either direction will make no difference of consequence in the useful result. But an endeavour should always be made to ensure that such deviation should be in the direction dictated by minor economic considerations that have been neglected in the mathematical solution for the sake of simplicity.

Several graphic methods of solution are available. Naturally they are simpler to understand, and easier to use when only two variables are considered. They will, therefore, in the first place be described as for the two variations of bulk s_2 and initial pressure p_1.

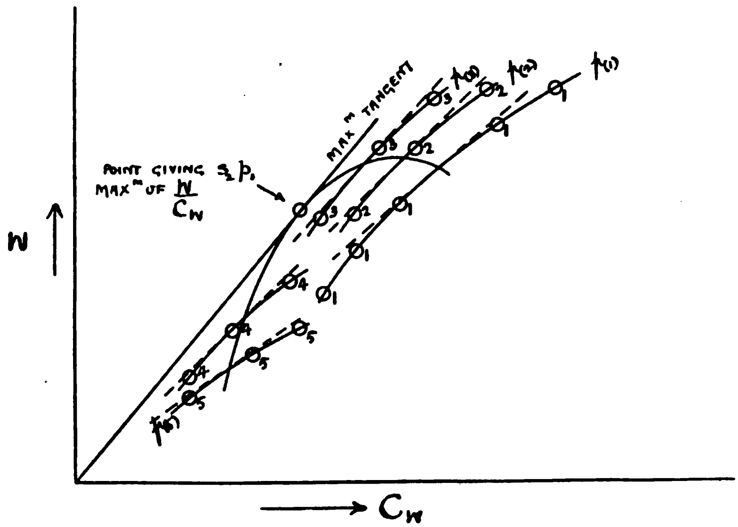


FIG. 68.—Graphic determination of Maximum Commercial Economy Coefficient with variation of Output and of Cost caused by two independent variables, e.g. size and pressure.

NOTE.—Each curve co-ordinates W and C_W for varying values of s_2 and for one particular p_1. The 5 curves are for 5 different values of p_1. The tangent points give best values of $\frac{W}{C_W}$ and s_2 for each particular p_1. The tangent point of the curve passing through the 5 tangent points gives the best values of $\frac{W}{C_W}$ and s_2 and p_1, for combined variation of s_2 and p_1.

The most readily intelligible of these methods is shown in the diagram, Fig. 66, of Chapter IX, where the co-ordinates are W and C_W. On this method is based the construction now shown in Fig. 68. Five curves are drawn each for one particular value of p_1, and for varying value of s_2. The height of the first curve 1 is calculated for 5 values of s_2, giving 5 pairs of values of W and C_W, the plotting of

which gives the 5 points marked by small circles. To this curve is drawn a tangent from the origin O, and the touching point of this tangent gives the best values of s_2, W and C_w for this particular p_1. For the next curve 2, it is sufficient to calculate 3 points, because curve 1 has already not only given the general family shape of the curves but also indicated the approximate position in which the maximum point of curve 2 will lie and therefore the approximate values of s_2 which will give three points near the touching point of the tangent to it drawn from O. Similarly three points are sufficient to calculate for each of the remaining three curves 3, 4 and 5. Five tangents from O being drawn to these five curves, and their touching

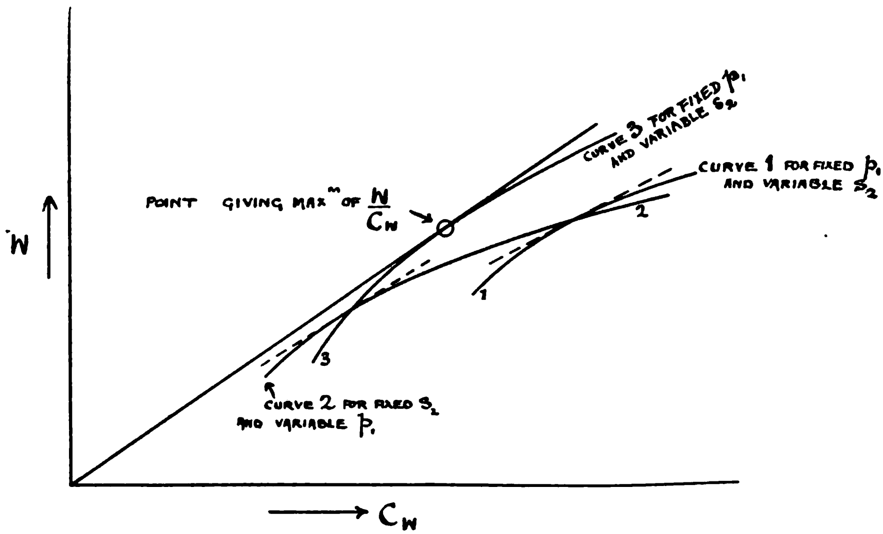


FIG. 69.—Alternative Construction to Figure 68.

points marked, a traversing curve is to be drawn through these five touching points. Finally through O draw a tangent to this traversing curve or "locus of touching points." The touching point of the tangent to this locus gives the solution of the problem; that is, it gives the maximum value of $\frac{W}{C_w}$ for all combinations of s_2 and p_1, and at the same time the values of s_2 and p_1 yielding this maximum.

If the five primitive curves and their tangent points do not give a sufficient length of the traversing curve to enable a tangent from O to be drawn to it, then it must be extended by drawing one or more additional primitive curves.

The next diagram, Fig. 69, is also founded on the method of Fig. 68, but instead of the successive curves being drawn each for a particular p_1 they are drawn alternately for a particular p_1 and a particular s_2. Firstly, curve 1 is drawn for one value of p_1, and various values of s_2, and the tangent from O is drawn to it. This gives the best value of s_2 with this special p_1. Secondly, curve 2 is drawn for this just found value of s_2 and various values of p_1. This curve 2 therefore runs through the tangent-touching-point of curve 1, and in general will cut through curve 1 at this point. A tangent from O is now to be drawn to curve 2, which tangent will be at a slightly steeper inclination than the tangent to curve 1; that is, will give a slightly greater ratio of W to C_w. This touching point to curve 2 gives the best p_1 to combine with the s_2 of this curve. Thirdly, there is drawn curve 3 for this last found best value of p_1, and various values of s_2, this curve 3 cutting through curve 2 at this last found touching point. A tangent through O is drawn to curve 3, and gives a greater value of W to C_w than either of the previous two tangents, it being somewhat steeper. This alternate process may be repeated as long as any measurably important increase of steepness in the tangent drawn through O results from the repetition. But it will be generally found that a pair of s_2 curves with one p_1 curve between them are enough to give a close approximation to the desired result. If these three curves are not sufficient it is better to draw five curves. The first curve and the last curve drawn should be for variable s_2 and special fixed p_1, because the influence of s_2 upon the economy is more marked than that of p_1. Thus in this kind of graphic solution an odd number of curves should be drawn.

Both of these constructions give the maximum value of $\frac{W}{C_w}$, the commercial economy coefficient, very accurately; but they do not indicate with the same precision the values of s_2 and p_1 which yield this maximum. This want of precision is evidenced by the long distances between the positions of the three tangent points compared with the very small differences between the inclinations of the three tangents. But on this very account the constructions are theoretically and practically valuable in demonstrating what has been more than once previously urged, namely that the condition of attaining a close approximation to maximum economy leaves freedom for a very considerable range of choice of the variables. A deviation from the mathematically exact proper values of these variables produces little change in the economy coefficient.

A construction giving s_2 and p_1 more definitely is shown in the next diagram, Fig. 70. In this, values of $\rho = \frac{W}{C_w}$ are calculated for five curves, each curve for one particular p_1 and for variable s_2. In each curve the height is made equal to ρ, and the horizontal ordinate s_2. The highest point of each curve is to be determined by drawing a horizontal tangent to it. For curve 1, five points should be calculated as shown by the five small circles. But this first curve being thus carefully drawn and showing the general family shape of the whole set, and moreover showing whereabouts the maximum height of the next one, curve 2, will lie, it is sufficient

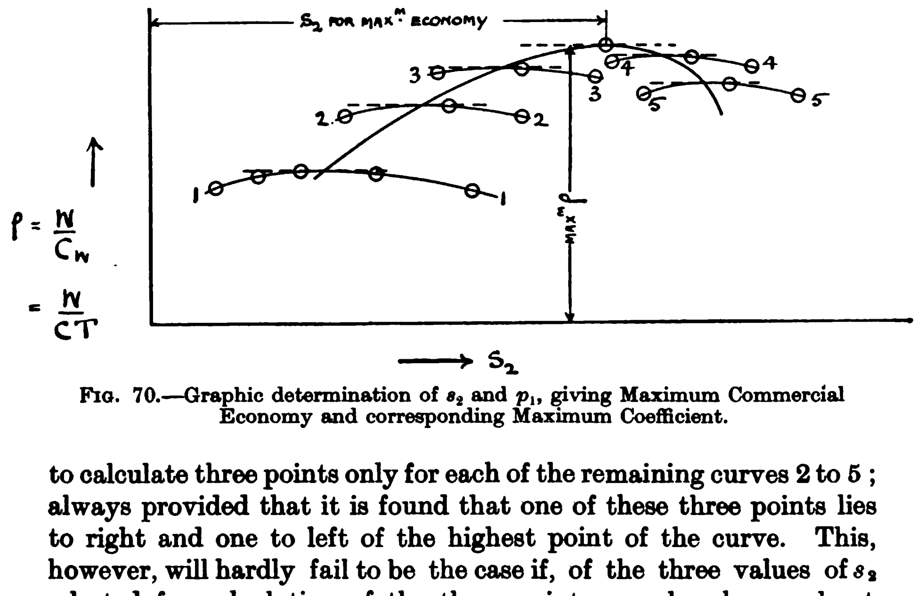


FIG. 70.—Graphic determination of s_2 and p_1, giving Maximum Commercial Economy and corresponding Maximum Coefficient.

to calculate three points only for each of the remaining curves 2 to 5; always provided that it is found that one of these three points lies to right and one to left of the highest point of the curve. This, however, will hardly fail to be the case if, of the three values of s_2 selected for calculation of the three points, one be chosen about equal to that giving the top of curve 1, and the other two be chosen, one less, and one considerably more than this. The five highest points being carefully marked a traversing curve is to be drawn through them. This traversing curve will rise to a highest point, and then fall, thus showing a maximum height. To ensure this care must be taken to let it run through the five highest points of the primitive curves *in the order of magnitude of the values of* p_1, for those curves. If the five points do not carry the traversing curve through its maximum, then a sixth and possibly a seventh primitive curve must be drawn for other values of p_1 beyond the first range of five in the direction giving higher ρ. It may be worth noting

that after the first two primitive curves have been drawn, the p_1 for the third curve must be chosen beyond the range of the first two p_1's in that direction giving higher ρ; and the progression of p_1 for fourth and fifth curves must be continued in the direction so determined by the first two. If the fifth curve is found to fall lower than the fourth, then a sixth does not need to be drawn. A line drawn through the highest points of the first two curves also gives useful guidance in the choice of the three values of s_2 for the plotting of the third curve; and a curve sketched through the maxima of these three curves gives similar guidance in the construction of the fourth.

A horizontal tangent to the traversing curve gives the highest value of ρ yielded by combined variation of s_2 and p_1, and at the same time the values of s_2 and p_1 yielding this maximum. The value of s_2 is read from the horizontal scale of the diagram, and that of p_1 by interpolation between the values for the two nearest primitive curves.

Yet another graphic method, in which the vertical ordinate is $\rho = \frac{W}{CT}$, is that already very fully described in Fig. 5 of Chapter I, and which, therefore, needs only short mention here. In Fig. 5 the two variables were called s and r. For the present application they are s_2 and p_1, the method of procedure remaining precisely the same. The two diagrams used alternately have s_2 for the horizontal ordinate of one of them, and p_1 for that of the other, while ρ is the vertical ordinate in both. First, a curve for a particular p_1 is drawn for variable s_2 and the highest points of it determined by drawing a horizontal tangent. It may be mentioned in passing that all these graphic calculations should be carried out on squared paper, and if so no actual drawing of either horizontal or vertical lines is required. Next with the s_2 of the highest point of curve 1 as a fixed constant, curve 2 is drawn in the other diagram for variable p_1; and its highest point found, which gives a "best" p_1. This p_1 is used as constant in curve 3 drawn for variable s_2 in the first diagram, and the highest point of curve 3 determined. At each alternate move from one diagram to the other, there will be an increase of maximum height in the new curve drawn; but this increase will rapidly become immeasurably small. In general, if the process be skilfully carried out, only three curves need be drawn to obtain a sufficiently close approximation to the maximum desired.

This construction is probably in nearly all circumstances the best described here. It gives not only the maximum value of ρ, but also

those of s_2 and p_1, yielding this maximum, with precision and good definition in the drawing.

Still another pair of graphic modes of solution may be derived from the construction shown in Fig. 65, in Chapter IX. Here there is a horizontal scale for s_2, and a vertical cost scale for the two curves W and C_W, each pair of such curves being for one particular value of p_1. Fig. 71 gives the graphic solution based on this method. Draw on the diagram five such pairs of W and C_W curves, each pair for a different fixed p_1. In Fig. 71 the C_W curves are drawn in full

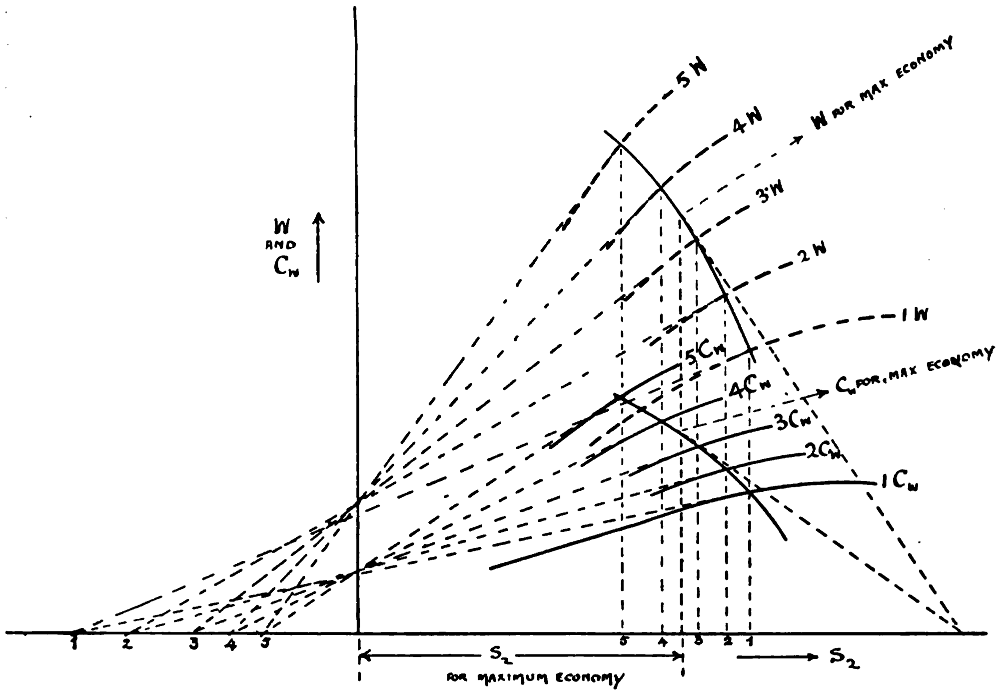


FIG. 71.—Alternative Method to Figure 70.

line, while, for the sake of distinction, the W curves are dotted. By the construction shown, namely by finding where the two tangents meet on the base line, a value of s_2 giving maximum $\rho = \dfrac{W}{C_W}$ is found for each pair of curves, that is, five values of s_2 corresponding to five special values of p_1, and at the same time giving five pairs of heights for the corresponding W and C_W. Join the five points giving these values of W by a fair curve, and similarly run another fair curve through the points giving the corresponding C_W.

This derived pair of curves give all the "best" corresponding values of W, C_w and s_2 for variable p_1. Treat this derived pair of curves in the same way as the primitive curves were treated, that is, find at which horizontal position the two tangents to the two curves intersect the base line in the same point. This horizontal position is the solution of the economy problem, giving the best W, C_w, and s_2 directly as read off the diagram scales, and the best p_1 by interpolation between the p_1 of the two nearest primitive curves. In Fig. 71 the solution falls between the 3rd and 4th primitive curves, and the two derived curves slope downwards to the right. It must not be supposed that they necessarily or always have this right-hand downward slope.

The other graphic solution using the same base construction follows the plan of alternating between s_2 and p_1 curves. It is desirable to carry it out in two separate diagrams. In one of these the horizontal ordinate is s_2; in the other it is p_1. In both the vertical scale is for W and C_w. Begin by plotting in the first diagram two curves of W and C_w for one special fixed p_1. Find where the tangents to these two curves meet in the same point in the base line, and measure the corresponding s_2. Next in the second diagram and using this last found s_2 as a fixed constant, plot two W and C_w curves for variable p_1. Find the position where their two tangents have coincident intersections on the base line of this diagram. This position gives the "best" p_1 for this special value of s_2. Now, returning to the first diagram, use this last found p_1 as a fixed constant for the construction of a second pair of W and C_w curves on the base of variable s_2; and find the "best" position on this new pair by the same tangent method. If the values found from it differ much from those found on the second diagram, it will be necessary to continue the process through the construction of two more pairs of the curves in the two diagrams; but the probability is that this will not be required to obtain a sufficiently close approximation to the desired result. In any case more than five pairs of curves should not be needed.

In this last method both s_2 and p_1 at their best values may be read directly off their scales in their respective diagrams. It, therefore, gives them with considerable definiteness and precision.

In several of these graphic solutions p_1 is to be read by interpolation between two known values. A simple and useful instrument for making such interpolation is made by ruling eleven parallel lines at uniform small distances apart (i.e. enclosing ten distance divisions) on a small strip of tracing paper or tracing cloth. It is

best to rule the 1st, 6th, and 11th line in red, and the others in fine sharp black lines. By laying this obliquely across any line of given length, the given length may be made to occupy exactly the whole space between the two outside red lines, and the intermediate black lines, then divide the given length into ten equal parts. The condition of this being possible is that the given length to be decimally divided is not less than the perpendicular distance between the outside red lines on the tracing cloth. It is, therefore, desirable, in making such an instrument, to rule on the cloth three sets of parallel lines, with three different outside widths; the narrow set to be used for the fine decimal subdivision of very small lengths, and the wider sets being more convenient for subdividing larger spaces. This simple little instrument completely obviates all difficulty in interpolating readings of results lying between curves in such diagrams

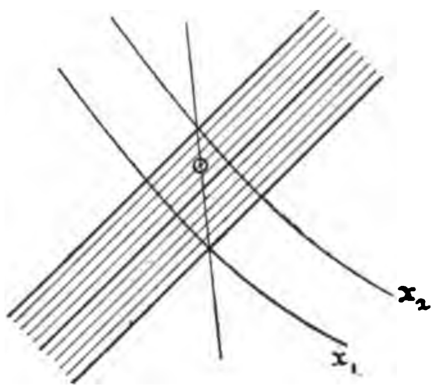


FIG. 72.—Interpolation Device.

as have just been described. The above Fig. 72 shows how it is applied to the interpolation for the point marked by a small circle lying between two curves giving known values. The point lies $\frac{7}{10}$ths of the distance from lower to higher curve. If the lower curve be for any value x_1 and the higher curve for x_2; then the point corresponds to the value $x_1 + {\cdot}7\,(x_2 - x_1)$.

These graphic solutions certainly require some little time and patience to be expended in order to obtain them; but, except for this labour, they offer no difficulty of any kind, provided that for each case of practical application the requisite "constants" have been ascertained. In practical work the diagrams are easily kept clear to the eye by using different-coloured inks for the sets of curves of different kinds. When a draughtsman has become

In terms of the speed the limit is reached when

Equation XXII— $$\mathrm{N}\,n - 1 = \frac{2\,(\tau_c - \tau_s)}{\sqrt{\tau_c\,(\tau_o + \tau_c)}}$$

which corresponds to the excess of furnace temperature over chimney temperature—

$$\tau_f - \tau_c = \sqrt{\tau_c\,(\tau_o + \tau_c)}$$

as previously found.

At this limit the minimum cost per unit of heat utilized by transmission to water and steam is found to be proportional to

$$\tau_o + 2\,\tau_c + 2\,\sqrt{\tau_c\,(\tau_o + \tau_c)}$$
$$= \tau_c\left(\frac{\tau_o}{\tau_c} + 2 + 2\,\sqrt{\frac{\tau_o}{\tau_c} + 1}\right).$$

This, by comparison with the last expression, is seen to be identical with $(\tau_o + 2\,\tau_f)$, the limit of low cost which was previously found by differentiating with respect to τ_f instead of, as here, with respect to n. This limit of low cost is here expressed in terms of chimney temperature, and the expression shows very plainly in what manner it is lowered by low chimney temperature, that is, by more complete utilization of the heat. For any one ratio of the constant τ_o to τ_c, it varies in simple proportion to the chimney temperature.

It would be more difficult to express by way of formula this limit of low cost in terms of speed, because, as seen above, the corresponding speed limit depends upon the steam temperature as well as upon the chimney temperature. The clearest way to make the relation plain is to tabulate $\frac{\mathrm{N}\,n - 1}{\tau_c - \tau_s} = \frac{2}{\tau_f - \tau_c}$ for different chimney temperatures, and different values of the constant τ_o. This is done in the following Table LXXIV, and graphically in Fig. 74.

The corresponding furnace temperatures are the same as those already given in the last table and diagram for the same chimney temperatures. The limit of low cost itself may be thought of most simply as proportional to $(\tau_o + 2\,\tau_f)$, but $\left(\tau_o + 2\,\tau_c + 2\sqrt{\tau_c\,(\tau_o + \tau_c)}\right)$ is really more easily calculated. This quantity is inserted in the second line of each section of the table; but in the third line of each section it is reduced to the base 1 as a standard of comparison at 300° F. chimney temperature above atmosphere. It must be noted

that it is this comparison given on the third line that is alone of important meaning; because, in the first place, the figures on the second line are not costs, but only quantities in proportion to which the cost per unit of heat varies, and, in the second place, the proportion alters entirely with the constant τ_o, that is, it is different in the different sections of the table. These sections must be considered separately so far as the lines of Cost Ratio are concerned, both in this and the previous Table LXXIII; and it is for this reason that the ratio is started from 1 as a standard of reference at $300° = \tau_c$ in each section.

In Fig. 74, which gives this table of results in diagram form, the horizontal ordinate is $\frac{10^5}{\tau_c - \tau_s}$ (N n – 1), and there are two sets of curves whose heights read τ_c, the chimney temperature, and the comparative cost. This latter, it should be remembered, is the cost factor η in the total cost (K + η H), which is the divisor in the economy-coefficient ρ; or, more exactly speaking, η is proportional to it. The object of the diagram is to show how this is affected by speed of working. The horizontal ordinate of the diagram is not this speed, but its variation is the same as that of the speed—exactly so, if steam temperature be raised with chimney temperature so as to keep $(\tau_c - \tau_s)$ constant.

It will be seen that the τ_c curves, and the η curves in this diagram are quite similar in general shape. The variations of these two quantities with speed follow almost the same law. Each decreases rapidly as the speed increases, the rate of decrease, however, diminishing, so that the curve has a hyperbolic character. In fact, if one shifts the vertical axis of the diagram to 35 upon the horizontal scale of Fig. 74 the η curves become nearly true curves of reciprocals, the 35 constant being the same for all the curves.

But it must not be forgotten that the speed here referred to is an adjusted speed, adjusted along with the furnace temperature to suit the chimney temperature and so as to secure minimum η. It must not be imagined that mere variation of speed in itself results in this lower cost of heat. This being an important and not, at first sight, easily grasped point, it may be put in another way. Equation XXI shows that the cost of heat goes up continuously with rise of chimney temperature; but is also affected by speed, at first decreasing, and then again rising, as the speed is raised. Selecting the speed that gives the lowest cost, we find—see Equation XXII and the diagram—that this best speed decreases rapidly as the chimney temperature is raised. Thus, as the lower costs of heat correspond with the lower chimney temperatures, they also

TABLE LXXIV

Economic Adjustment of Speed to Excess of Chimney over Steam Temperature **and corresponding Cost per Unit of Heat Utilized.**

Cost Initial Constant τ_o		τ_c = Chimney Temperature above Atmosphere.					
		300	400	500	600	700	800
		320	267	231	204	183	167
1000		**2850**	**3296**	**3732**	**4160**	**4582**	**5000**
		1	**1·16**	**1·31**	**1·46**	**1·61**	**1·76**
	100,000 Times Ratio of	241	204	179	160	145	134
2000	$\{N \times \text{Speed} - 1\}$ to	**4262**	**4760**	**5236**	**5698**	**6050**	**6593**
		1	**1·12**	**1·23**	**1·34**	**1·45**	**1·55**
	Excess Chimney over Steam Temperature	201	171	152	136	124	115
3000	$=\left(\frac{Nn-1}{\tau_c-\tau_s}\right)10^5$	**5590**	**6132**	**6646**	**7140**	**7618**	**8087**
		1	**1·10**	**1·19**	**1·28**	**1·36**	**1·45**
	and Comparative Cost per Unit	176	151	133	120	110	102
4000	**of Heat Utilized**	**6872**	**7454**	**8000**	**8522**	**9028**	**9520**
		1	**1·08**	**1·16**	**1·24**	**1·31**	**1·38**
		159	136	120	109	100	93
5000		**8122**	**8740**	**9316**	**9866**	**10395**	**10908**
		1	**1·08**	**1·15**	**1·22**	**1·28**	**1·34**

correspond with higher best speeds. Equation XX, or the simple formula $(\tau_o + 2\,\tau_f)$, show that the lower costs of heat also correspond with lower furnace temperatures when these are selected so as to give lowest cost with prescribed chimney temperature.

In the diagram, the dotted curves traversing the η curves are each for one chimney temperature. The lowest is a straight line for $\tau_c = 300$ at level 1, which was taken as the standard of comparison for the costs at other temperatures. The other dotted curves are for the successive chimney temperatures 400, 500, 600, 700 and 800. These curves rise with greater speed.

In further elucidation of these interesting results, it is finally needful to point out that n, the speed of working, is not simply the

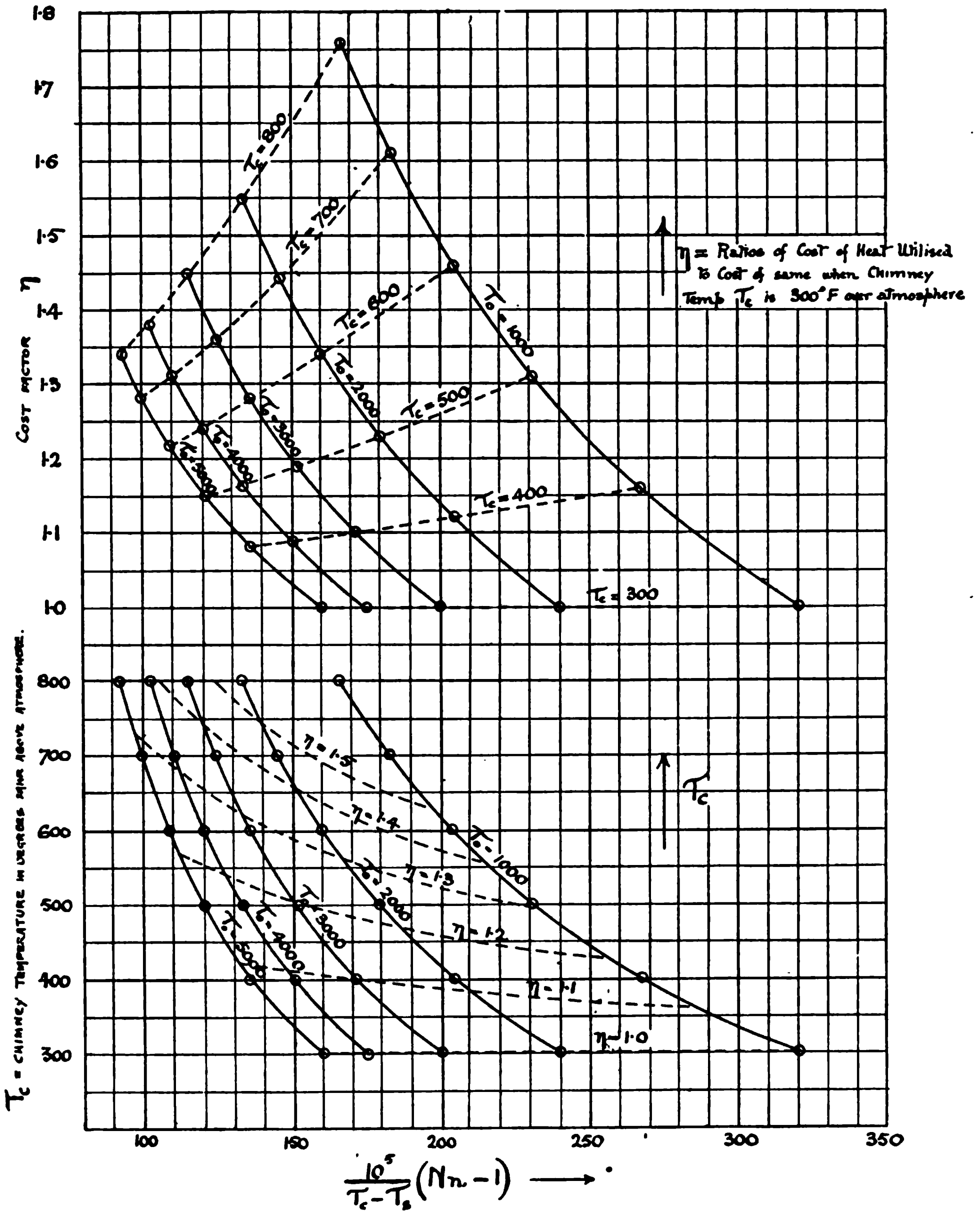


Fig. 74.—Working Speed and Minimum Heat Cost for various Chimney and Steam Temperatures.

engine speed, either in revolutions per minute or in linear piston velocity. It is the working speed of the entire plant: the speed at which heat is generated in the furnace by burning coal or other fuel; the speed at which heat is conducted or otherwise transmitted into the working fluid, whether that be water and steam, or the air-mixed products of gas combustion or of oil-combustion; the speed at which this heat flux generates resilience; and finally the speed at which this resilience is spent in doing mechanical work in the engine. In steady working, if these various speeds be reduced to one common measure of energy per unit of time, they differ only because of the necessary or wasteful losses at each stage of the whole process of power-production in useful form; and, these losses being at steady rates, all these speeds bear constant mutual proportions. Our rate n may be supposed to apply to any of them, or all of them: its application to one or other only alters its scale, so to speak; and the laws determining the best economic ratios between the elements of the design do not in any way depend upon scales of measurement. As applied to such different measures of working speed, the factor N naturally takes different values, and even changes in *kind*, i.e. in physical dimensions. Its value as to magnitude and kind is most readily recognized from the terms of Equation XX, from which it is seen that $\mathrm{N}\,n$ exceeds by unity twice the ratio of excess of chimney over steam temperature to excess of furnace over chimney temperature. This ratio is a small one, ranging from $\frac{1}{10}$ to $\frac{5}{10}$ at the worst. $\mathrm{N}\,n$ is a pure number.

In the commercial economy coefficient—

$$\rho=\frac{\mathrm{W}}{\mathrm{C}}=\frac{\mathrm{W}\,n}{\mathrm{K}+n\,\eta\,\mathrm{H}},$$

where W and H are measured per working stroke, or per lb. of working fluid, and K is measured per minute, or per year, while n is the number of working strokes, or number of pounds of working fluid, per minute or per year; it seems at first sight an easy task to adjust the speed so as to make ρ a maximum. If W, H and η remained constant while n were varied, this might be done without any difficulty. But we have just seen that η at any rate varies largely with n.

Equation XXI, page 285, shows in what manner it varies, first decreasing with n, and then again rising as n is further raised beyond the limit investigated above. The factor of H is $n\,\eta$, and, although in Equation XXI the divisor and the multiplier involving n are $(\mathrm{N}\,n-1)$, and not simply n, one may conclude

that $n\ \eta$ varies nearly as $\eta_0 n^i$ where η_0 and i are positive constants.

If, then, ρ is found to have the form—

$$\rho = \frac{W n}{K + \eta_0 n^i H},$$

then, W, K, η_0 and i being kept constant, the rate of change of ρ with n is—

$$\rho'_n \ (K + \eta_0 n^i H)^2 = W \left\{K - \eta_0 (i - 1) n^i H\right\},$$

which, if $i > 1$, gives a maximum ρ when $\eta_0 (i-1) n^i H = K$ or $n = \sqrt[i]{\frac{K}{\eta_0 (i-1) H}}$.

This maximum ρ equals

Equation XXIII— ρ max. $= \frac{i-1}{i} \frac{W}{K} \sqrt[i]{\frac{K}{\eta_0 (i-1) H}}$.

The result given on page 279 agrees with this, for $i = 2$.

This maximum, however, does not exist unless $i > 1$; and it cannot be obtained concurrently with the minimum heat cost found previously. There is no general relation between the two speeds giving minimum η and maximum ρ; at any rate the general relation is too complex to make it useful to write it out here. It is, however, of practical importance to observe that the speed of working giving greatest commercial economy is always considerably greater than that giving minimum heat cost. The proportion in which it is greater depends upon the ratio of the capital charges to the working expenditure and becomes higher as this ratio is greater.

The η curves in Fig. 74 are nearly rectangular hyperbolas. Along any one of these curves the product $n\ \eta$ varies comparatively little. Each curve gives the relation between η and n when n is adjusted to give minimum η. If then it were determined always to work at that speed giving least heat cost, then the divisor of ρ would change hardly at all by concurrent change of η and n. The numerator remains proportional to the n so adjusted; and, therefore, the commercial economy ρ is increased continuously by all changes which diminish η and increase n concurrently so as to maintain the above adjustment. Under these conditions there is no limit of concurrent heat-cost and speed which would yield a maximum commercial economy.

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